1	Use correct quotient or product rule	M1	
	Obtain correct derivative in any form	A1	
	Justify the given statement	A1	**[3]**
2	*EITHER:* State or imply non-modular equation $2^2\left(3^x-1\right)^2=\left(3^x\right)^2$, or pair of equations $2\left(3^x-1\right)=\pm 3^x$	M1	
	Obtain $3^x = 2$ and $3^x=\frac{2}{3}$ (or $3^{x+1} = 2$)	A1	
	OR: Obtain $3^x = 2$ by solving an equation or by inspection	B1	
	Obtain $3^x=\frac{2}{3}$ (or $3^{x+1} = 2$) by solving an equation or by inspection	B1	
	Use correct method for solving an equation of the form $3^x = a$ (or $3^{x+1} = a$), where $a > 0$	M1	
	Obtain final answers 0.631 and –0.369	A1	**[4]**
3	*EITHER:* Integrate by parts and reach $kx^{\frac{1}{2}}\ln x - m\int x^{\frac{1}{2}}.\frac{1}{x}\,dx$	M1*	
	Obtain $2x^{\frac{1}{2}}\ln x - 2\int \frac{1}{x^{\frac{1}{2}}}\,dx$, or equivalent	A1	
	Integrate again and obtain $2x^{\frac{1}{2}}\ln x - 4x^{\frac{1}{2}}$, or equivalent	A1	
	Substitute limits $x = 1$ and $x = 4$, having integrated twice	M1(dep*)	
	Obtain answer $4(\ln 4 - 1)$, or exact equivalent	A1	
	OR1: Using $u = \ln x$, or equivalent, integrate by parts and reach $kue^{\frac{1}{2}u} - m\int e^{\frac{1}{2}u}\,du$	M1*	
	Obtain $2ue^{\frac{1}{2}u} - 2\int e^{\frac{1}{2}u}\,du$, or equivalent	A1	
	Integrate again and obtain $2ue^{\frac{1}{2}u} - 4e^{\frac{1}{2}u}$, or equivalent	A1	
	Substitute limits $u = 0$ and $u = \ln 4$, having integrated twice	M1(dep*)	
	Obtain answer $4\ln 4 - 4$, or exact equivalent	A1	
	OR2: Using $u=\sqrt{x}$, or equivalent, integrate and obtain $ku\ln u - m\int u.\frac{1}{u}du$	M1*	
	Obtain $4u\ln u - 4\int 1du$, or equivalent	A1	
	Integrate again and obtain $4u\ln u - 4u$, or equivalent	A1	
	Substitute limits $u = 1$ and $u = 2$, having integrated twice or quoted $\int \ln u\,du$ as $u\ln u \pm u$	M1(dep*)	
	Obtain answer $8\ln 2 - 4$, or exact equivalent	A1	
	OR3: Integrate by parts and reach $I=\frac{x\ln x \pm x}{\sqrt{x}} + k\int \frac{x\ln x \pm x}{x\sqrt{x}}\,dx$	M1*	
	Obtain $I=\frac{x\ln x - x}{\sqrt{x}} + \frac{1}{2}I - \frac{1}{2}\int \frac{1}{\sqrt{x}}\,dx$	A1	
	Integrate and obtain $I = 2\sqrt{x}\ln x - 4\sqrt{x}$, or equivalent	A1	
	Substitute limits $x = 1$ and $x = 4$, having integrated twice	M1(dep*)	
	Obtain answer $4\ln 4 - 4$, or exact equivalent	A1	**[5]**

4		Use correct product or quotient rule at least once	M1*	
		Obtain $\frac{dx}{dt} = e^{-t}\sin t - e^{-t}\cos t$ or $\frac{dy}{dt} = e^{-t}\cos t - e^{-t}\sin t$, or equivalent	A1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
		Obtain $\frac{dy}{dx} = \frac{\sin t - \cos t}{\sin t + \cos t}$, or equivalent	A1	
		EITHER: Express $\frac{dy}{dx}$ in terms of tan t only	M1(dep*)	
		Show expression is identical to $\tan\left(t - \frac{1}{4}\pi\right)$	A1	
		OR: Express $\tan\left(t - \frac{1}{4}\pi\right)$ in terms of tan t	M1	
		Show expression is identical to $\frac{dy}{dx}$	A1	**[6]**
5	**(i)**	Use Pythagoras	M1	
		Use the sin2A formula	M1	
		Obtain the given result	A1	**[3]**
	(ii)	Integrate and obtain a $k\ln\sin\theta$ or $m\ln\cos\theta$ term, or obtain integral of the form $p\ln\tan\theta$	M1*	
		Obtain indefinite integral $\frac{1}{2}\ln\sin\theta - \frac{1}{2}\ln\cos\theta$, or equivalent, or $\frac{1}{2}\ln\tan\theta$	A1	
		Substitute limits correctly	M1(dep)*	
		Obtain the given answer correctly having shown appropriate working	A1	**[4]**
6	**(i)**	State or imply $AB = 2r\cos\theta$ or $AB^2 = 2r^2 - 2r^2\cos(\pi - 2\theta)$	B1	
		Use correct formula to express the area of sector ABC in terms of r and θ	M1	
		Use correct area formulae to express the area of a segment in terms of r and θ	M1	
		State a correct equation in r and θ in any form	A1	
		Obtain the given answer	A1	**[5]**
		[SR: If the complete equation is approached by adding two sectors to the shaded area above BO and OC give the first M1 as on the scheme, and the second M1 for using correct area formulae for a triangle AOB or AOC, and a sector AOB or AOC.]		
	(ii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.95	A1	
		Show sufficient iterations to 4 d.p. to justify 0.95 to 2 d.p., or show there is a sign change in the interval (0.945, 0.955)	A1	**[3]**

7 **(i)** State or imply partial fractions are of the form $\frac{A}{x-2}+\frac{Bx+C}{x^2+3}$ B1

Use a relevant method to determine a constant M1

Obtain one of the values $A=-1$, $B=3$, $C=-1$ A1

Obtain a second value A1

Obtain the third value A1 **[5]**

(ii) Use correct method to obtain the first two terms of the expansions of $(x-2)^{-1}$, $\left(1-\frac{1}{2}x\right)^{-1}$, $\left(x^2+3\right)^{-1}$ or $\left(1+\frac{1}{3}x^2\right)^{-1}$ M1

Substitute correct unsimplified expansions up to the term in x^2 into each partial fraction A1√+A1√

Multiply out fully by $Bx+C$, where $BC \neq 0$ M1

Obtain final answer $\frac{1}{6}+\frac{5}{4}x+\frac{17}{72}x^2$, or equivalent A1 **[5]**

[Symbolic binomial coefficients, e.g. $\binom{-1}{1}$ are not sufficient for the M1. The f.t. is on *A, B, C.*]

[In the case of an attempt to expand $(2x^2-7x-1)(x-2)^{-1}(x^2+3)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]

[If *B* or *C* omitted from the form of partial fractions, give B0M1A0A0A0 in **(i)**; M1A1√A1√ in **(ii)**]

8 **(a)** *EITHER:* Solve for u or for v M1

Obtain $u=\frac{2\text{i}-6}{1-2\text{i}}$ or $v=\frac{5}{1-2\text{i}}$, or equivalent A1

Either: Multiply a numerator and denominator by conjugate of denominator, or equivalent

Or: Set u or v equal to $x+\text{i}y$, obtain two equations by equating real and imaginary parts and solve for x or for y M1

OR: Using $a+\text{i}b$ and $c+\text{i}d$ for u and v, equate real and imaginary parts and obtain four equations in a, b, c and d M1

Obtain $b+2d=2$, $a+2c=0$, $a+d=0$ and $-b+c=3$, or equivalent A1

Solve for one unknown M1

Obtain final answer $u=-2-2\text{i}$, or equivalent A1

Obtain final answer $v=1+2\text{i}$, or equivalent A1 **[5]**

(b) Show a circle with centre $-\text{i}$ B1

Show a circle with radius 1 B1

Show correct half line from 2 at an angle of $\frac{3}{4}\pi$ to the real axis B1

Use a correct method for finding the least value of the modulus M1

Obtain final answer $\frac{3}{\sqrt{2}}-1$, or equivalent, e.g. 1.12 (allow 1.1) A1 **[5]**

9	(i)	*EITHER:*	Obtain a vector parallel to the plane, e.g. $\overrightarrow{AB} = -2\mathbf{i} + 4\mathbf{j} - \mathbf{k}$	B1	
			Use scalar product to obtain an equation in *a, b, c,* e.g. $-2a + 4b - c = 0$, $3a - 3b + 3c = 0$, or $a + b + 2c = 0$	M1	
			Obtain two correct equations in *a, b, c*	A1	
			Solve to obtain ratio $a : b : c$	M1	
			Obtain $a : b : c = 3 : 1 : -2$, or equivalent	A1	
			Obtain equation $3x + y - 2z = 1$, or equivalent	A1	
		OR1:	Substitute for two points, e.g. A and B, and obtain $2a - b + 2c = d$ and $3b + c = d$	B1	
			Substitute for another point, e.g. C, to obtain a third equation and eliminate one unknown entirely from the three equations	M1	
			Obtain two correct equations in three unknowns, e.g. in *a, b, c*	A1	
			Solve to obtain their ratio, e.g. $a : b : c$	M1	
			Obtain $a : b : c = 3 : 1 : -2$, $a : c : d = 3 : -2 : 1$, $a : b : d = 3 : 1 : 1$ or $b : c : d = -1 : -2 : 1$	A1	
			Obtain equation $3x + y - 2z = 1$, or equivalent	A1	
		OR2:	Obtain a vector parallel to the plane, e.g. $\overrightarrow{BC} = 3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$	B1	
			Obtain a second such vector and calculate their vector product e.g. $(-2\mathbf{i} + 4\mathbf{j} - \mathbf{k}) \times (3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k})$	M1	
			Obtain two correct components of the product	A1	
			Obtain correct answer, e.g. $9\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$	A1	
			Substitute in $9x + 3y - 6z = d$ to find d	M1	
			Obtain equation $9x + 3y - 6z = 3$, or equivalent	A1	
		OR3:	Obtain a vector parallel to the plane, e.g. $\overrightarrow{AC} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$	B1	
			Obtain a second such vector and form correctly a 2-parameter equation for the plane	M1	
			Obtain a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} + 4\mathbf{k} + \lambda(-2\mathbf{i} + 4\mathbf{j} - \mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$	A1	
			State three correct equations in x, y, z, λ, μ	A1	
			Eliminate λ and μ	M1	
			Obtain equation $3x + y - 2z = 1$, or equivalent	A1	**[6]**
	(ii)		Obtain answer $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, or equivalent	B1	**[1]**

(iii) *EITHER:* Use $\dfrac{\overrightarrow{OA}.\overrightarrow{OD}}{\left|\overrightarrow{OD}\right|}$ to find projection *ON* of *OA* onto *OD* — M1

Obtain $ON = \dfrac{4}{3}$ — A1

Use Pythagoras in triangle *OAN* to find *AN* — M1

Obtain the given answer — A1

OR1: Calculate the vector product of $\overrightarrow{OA}$ and $\overrightarrow{OD}$ — M1

Obtain answer 6**i** + 2**j** – 5**k** — A1

Divide the modulus of the vector product by the modulus of $\overrightarrow{OD}$ — M1

Obtain the given answer — A1

OR2: Taking general point *P* of *OD* to have position vector $\lambda(\mathbf{i}+2\mathbf{j}+2\mathbf{k})$, form an equation in λ by either equating the scalar product of $\overrightarrow{AP}$ and $\overrightarrow{OP}$ to zero, or using Pythagoras in triangle *OPA*, or setting the derivative of $\left|\overrightarrow{AP}\right|$ to zero — M1

Solve and obtain $\lambda = \dfrac{4}{9}$ — A1

Carry out method to calculate *AP* when $\lambda = \dfrac{4}{9}$ — M1

Obtain the given answer — A1

OR3: Use a relevant scalar product to find the cosine of *AOD* or *ADO* — M1

Obtain $\cos AOD = \dfrac{4}{9}$ or $\cos ADO = \dfrac{5}{3\sqrt{10}}$, or equivalent — A1

Use trig to find the length of the perpendicular — M1

Obtain the given answer — A1

OR4: Use cosine formula in triangle *AOD* to find cos *AOD* or cos *ADO* — M1

Obtain $\cos AOD = \dfrac{8}{18}$ or $\cos ADO = \dfrac{10}{6\sqrt{10}}$, or equivalent — A1

Use trig to find the length of the perpendicular — M1

Obtain the given answer — A1 **[4]**

10 **(i)** State or imply $V = \pi h^3$ — B1

State or imply $\dfrac{\mathrm{d}V}{\mathrm{d}t} = -k\sqrt{h}$ — B1

Use $\dfrac{\mathrm{d}V}{\mathrm{d}t} = \dfrac{\mathrm{d}V}{\mathrm{d}h}.\dfrac{\mathrm{d}h}{\mathrm{d}t}$, or equivalent — M1

Obtain the given equation — A1 **[4]**

[The M1 is only available if $\dfrac{\mathrm{d}V}{\mathrm{d}h}$ is in terms of *h* and has been obtained by a correct method.]

[Allow B1 for $\dfrac{\mathrm{d}V}{\mathrm{d}t} = k\sqrt{h}$ but withhold the final A1 until the polarity of the constant $\dfrac{k}{3\pi}$ has been justified.]

(ii)	Separate variables and integrate at least one side	M1	
	Obtain terms $\frac{2}{5}h^{\frac{5}{2}}$ and $-At$, or equivalent	A1	
	Use $t = 0, h = H$ in a solution containing terms of the form $ah^{\frac{5}{2}}$ and $bt + c$	M1	
	Use $t = 60$, $h = 0$ in a solution containing terms of the form $ah^{\frac{5}{2}}$ and $bt + c$	M1	
	Obtain a correct solution in any form, e.g. $\frac{2}{5}h^{\frac{5}{2}} = \frac{1}{150}H^{\frac{5}{2}}t + \frac{2}{5}H^{\frac{5}{2}}$	A1	
(ii)	Obtain final answer $t = 60\left(1 - \left(\frac{h}{H}\right)^{\frac{5}{2}}\right)$, or equivalent	A1	**[6]**
(iii)	Substitute $h = \frac{1}{2}H$ and obtain answer $t = 49.4$	B1	**[1]**