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	GCE A LEVEL – October/November 2012	9709	33

1 State or imply $\ln e = 1$ B1

Apply at least one logarithm law for product or quotient correctly (or exponential equivalent) M1

Obtain $x+5=ex$ or equivalent and hence $\dfrac{5}{e-1}$ A1 [3]

2 **(i)** State or imply $R = 25$ B1

Use correct trigonometric formula to find α M1

Obtain $16.26°$ **with no errors seen** A1 [3]

(ii) Evaluate of $\sin^{-1}\dfrac{17}{R}$ $(= 42.84\ldots°)$ M1

Obtain answer $59.1°$ A1 [2]

3 **(i)** Either Use correct quotient rule or equivalent to obtain

$\dfrac{dx}{dt} = \dfrac{4(2t+3)-8t}{(2t+3)^2}$ or equivalent B1

Obtain $\dfrac{dy}{dt} = \dfrac{4}{2t+3}$ or equivalent B1

Use $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$ or equivalent M1

Obtain $\dfrac{1}{3}(2t+3)$ or similarly simplified equivalent A1

Or Express t in terms of x or y e.g. $t = \dfrac{3x}{4-2x}$ B1

Obtain Cartesian equation e.g. $y = 2\ln\left(\dfrac{6}{2-x}\right)$ B1

Differentiate and obtain $\dfrac{dy}{dx} = \dfrac{2}{2-x}$ M1

Obtain $\dfrac{1}{3}(2t+3)$ or similarly simplified equivalent A1 [4]

(ii) Obtain $2t = 3$ or $t = \dfrac{3}{2}$ B1

Substitute in expression for $\dfrac{dy}{dx}$ and obtain 2 B1 [2]

4 Separate variables correctly and integrate one side — M1
Obtain $\ln y = ...$ or equivalent — A1
Obtain $= 3\ln(x^2+4)$ or equivalent — A1
Evaluate a constant or use $x = 0, y = 32$ as limits in a solution containing terms $a \ln y$ and $b \ln\left(x^2+4\right)$ — M1
Obtain $\ln y = 3\ln(x^2+4) + \ln 32 - 3\ln 4$ or equivalent — A1
Obtain $y = \frac{1}{2}\left(x^2+4\right)$ or equivalent — A1 [6]

5 (i) Either: Use correct product rule — M1
Obtain $3e^{-2x} - 6xe^{-2x}$ or equivalent — A1
Substitute $-\frac{1}{2}$ and obtain 6e — A1
Or: Take ln of both sides and use implicit differentiation correctly — M1
Obtain $\frac{dy}{dx} = y\left(\frac{1}{x} - 2\right)$ or equivalent — A1
Substitute $-\frac{1}{2}$ and obtain 6e — A1 [3]

(ii) Use integration by parts to reach $kxe^{-2x} \pm \int ke^{-2x}dx$ — M1
Obtain $-\frac{3}{2}xe^{-2x} + \int\frac{3}{2}e^{-2x}\,dx$ or equivalent — A1
Obtain $-\frac{3}{2}xe^{-2x} - \frac{3}{4}e^{-2x}$ or equivalent — A1
Substitute correct limits correctly — DM1
Obtain $-\frac{3}{4}$ with no errors or inexact work seen — A1 [5]

6 (i) Find y for $x = -2$ — M1
Obtain 0 and conclude that $\alpha = -2$ — A1 [2]

(ii) Either: Find cubic factor by division or inspection or equivalent — M1
Obtain $x^3 + 2x - 8$ — A1
Rearrange to confirm given equation $x = \sqrt[3]{8-2x}$ — A1
Or: Derive cubic factor from given equation and form product with $(x - \alpha)$ — M1
$(x+2)\left(x^3+2x-8\right)$ — A1
Obtain quartic $x^4 + 2x^3 + 2x^2 - 4x - 16\ (= 0)$ — A1
Or: Derive cubic factor from given equation and divide the quartic by the cubic — M1
$\left(x^4 + 2x^3 + 2x^2 - 4x - 16\right) \div \left(x^3 + 2x - 8\right)$ — A1
Obtain correct quotient and zero remainder — A1 [3]

(iii) Use the given iterative formula correctly at least once — M1
Obtain final answer 1.67 — A1
Show sufficient iterations to at least 4 d.p. to justify answer 1.67 to 2 d.p. or show there is a change of sign in interval (1.665, 1.675) — A1 [3]

7 (i) State or imply $du = 2\cos 2x\, dx$ or equivalent — B1

Express integrand in terms of u and du — M1

Obtain $\int \frac{1}{2}u^3\left(1-u^2\right) du$ or equivalent — A1

Integration to obtain an integral of the form $k_1u^4 + k_2u^6, k_1, k_2 \neq 0$ — M1

Use limits 0 and 1 or (if reverting to x) 0 and $\frac{1}{4}\pi$ correctly — DM1

Obtain $\frac{1}{24}$, or equivalent — A1 [6]

(ii) Use 40 and upper limit from part **(i)** in appropriate calculation — M1

Obtain $k = 10$ with no errors seen — A1 [2]

8 (i) State or imply general point of either line has coordinates $(5 + s, 1 - s, -4 + 3s)$ or $(p + 2t, 4 + 5t, -2 - 4t)$ — B1

Solve simultaneous equations and find s and t — M1

Obtain $s = 2$ and $t = -1$ or equivalent in terms of p — A1

Substitute in third equation to find $p = 9$ — A1

State point of intersection is $(7, -1, 2)$ — A1 [5]

(ii) Either Use scalar product to obtain a relevant equation in a, b, c

e.g. $a - b + 3c = 0$ or $2a + 5b - 4c = 0$ — M1

State two correct equations in a, b, c — A1

Solve simultaneous equations to obtain at least one ratio — DM1

Obtain $a : b : c = -11 : 10 : 7$ or equivalent — A1

Obtain equation $-11x + 10y + 7z = -73$ or equivalent with integer coefficients — A1

Or 1 Calculate vector product of $\begin{pmatrix}1\\-1\\3\end{pmatrix}$ and $\begin{pmatrix}2\\5\\-4\end{pmatrix}$ — M1

Obtain two correct components of the product — A1

Obtain correct $\begin{pmatrix}-11\\10\\7\end{pmatrix}$ or equivalent — A1

Substitute coordinates of a relevant point in $\mathbf{r}.\mathbf{n} = d$ to find d — DM1

Obtain equation $-11x + 10y + 7z = -73$ or equivalent with integer coefficients — A1

Or 2 Using relevant vectors, form correctly a two-parameter equation for the plane — M1

Obtain $\mathbf{r} = \begin{pmatrix}5\\1\\-4\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\3\end{pmatrix} + \mu\begin{pmatrix}2\\5\\-4\end{pmatrix}$ or equivalent — A1

State three equations in x, y, z, λ, μ — A1

Eliminate λ and μ — DM1

Obtain $11x - 10y - 7z = 73$ or equivalent with integer coefficients — A1 [5]

9 **(i)** State or imply form $\frac{A}{3-x}+\frac{Bx+C}{1+x^2}$ B1

Use relevant method to determine a constant M1

Obtain $A = 6$ A1

Obtain $B = -2$ A1

Obtain $C = 1$ A1 [5]

(ii) Either Use correct method to obtain first two terms of expansion

of $(3-x)^{-1}$ or $\left(1-\frac{1}{3}x\right)^{-1}$ or $\left(1+x^2\right)^{-1}$ M1

Obtain $\frac{A}{3}\left(1+\frac{1}{3}x+\frac{1}{9}x^2+\frac{1}{27}x^3\right)$ A1

Obtain $(Bx + C)(1 - x^2)$ A1

Obtain sufficient terms of the product $(Bx + C)(1 - x^2)$, $B, C \neq 0$ and add the two expansions M1

Obtain final answer $3-\frac{4}{3}x-\frac{7}{9}x^2+\frac{56}{27}x^3$ A1

Or Use correct method to obtain first two terms of expansion

of $(3-x)^{-1}$ or $\left(1-\frac{1}{3}x\right)^{-1}$ or $\left(1+x^2\right)^{-1}$ M1

Obtain $\frac{1}{3}\left(1+\frac{1}{3}x+\frac{1}{9}x^2+\frac{1}{27}x^3\right)$ A1

Obtain $(1 - x^2)$ A1

Obtain sufficient terms of the product of the three factors M1

Obtain final answer $3-\frac{4}{3}x-\frac{7}{9}x^2+\frac{56}{27}x^3$ A1 [5]

10 **(a)** Expand and simplify as far as $iw^2 = -8i$ or equivalent B1

Obtain first answer $i\sqrt{8}$, or equivalent B1

Obtain second answer $-i\sqrt{8}$, or equivalent and no others B1 [3]

(b) **(i)** Draw circle with centre in first quadrant M1

Draw correct circle with interior shaded or indicated A1 [2]

(ii) Identify ends of diameter corresponding to line through origin and centre M1

Obtain $p = 3.66$ and $q = 7.66$ A1

Show tangents from origin to circle M1

Evaluate $\sin^{-1}\left(\frac{1}{4}\sqrt{2}\right)$ M1

Obtain $\alpha=\frac{1}{4}\pi-\sin^{-1}\left(\frac{1}{4}\sqrt{2}\right)$ or equivalent and hence 0.424 A1

Obtain $\beta=\frac{1}{4}\pi+\sin^{-1}\left(\frac{1}{4}\sqrt{2}\right)$ or equivalent and hence 1.15 A1 [6]