1	*EITHER*	State or imply non-modular inequality $(3(x-1))^2 < (2x+1)^2$ or corresponding quadratic equation, or pair of linear equations $3(x-1) = \pm(2x+1)$	B1	
		Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations	M1	
		Obtain critical values $x = \frac{2}{5}$ and $x = 4$	A1	
		State answer $\frac{2}{5} < x < 4$	A1	
	OR	Obtain critical value $x = \frac{2}{5}$ or $x = 4$ from a graphical method, or by inspection, or by solving a linear equation or inequality	B1	
		Obtain critical values $x = \frac{2}{5}$ and $x = 4$	B2	
		State answer $\frac{2}{5} < x < 4$	B1	[4]
		[Do not condone $\le$ for $<$.]		
2	*EITHER*	Use laws of indices correctly and solve for 5^x or for 5^{-x} or for 5^{x-1}	M1	
		Obtain 5^x or for 5^{-x} or for 5^{x-1} in any correct form, e.g. $5^x = \frac{5}{1-{}^1/_5}$	A1	
		Use correct method for solving $5^x = a$, or $5^{-x} = a$, or $5^{x-1} = a$, where $a > 0$	M1	
		Obtain answer $x = 1.14$	A1	
	OR	Use an appropriate iterative formula, e.g. $x_{n+1} = \frac{\ln(5^{x-1}+5)}{\ln 5}$, correctly, at least once	M1	
		Obtain answer 1.14	A1	
		Show sufficient iterations to at least 3 d.p. to justify 1.14 to 2 d.p., or show there is a sign change in the interval (1.135, 1.145)	A1	
		Show there is no other root	A1	[4]
		[For the solution $x = 1.14$ with no relevant working give B1, and a further B1 if 1.14 is shown to be the only solution.]		
3		Attempt use of sin $(A + B)$ and cos $(A - B)$ formulate to obtain an equation in cos θ and sin θ	M1	
		Obtain a correct equation in any form	A1	
		Use trig. formula to obtain an equation in tan θ (or cos θ, sin θ or cot θ)	M1	
		Obtain $\tan\theta = \frac{\sqrt{6}-1}{1-\sqrt{2}}$, or equivalent (or find cost θ, sin θ or cot θ)	A1	
		Obtain answer θ = 105.9°, and no others in the given interval	A1	[5]
		[Ignore answers outside the given material]		
4	**(i)**	Obtain correct unsimplified terms in x and x^3	B1 + B1	
		Equate coefficients and solve for a	M1	
		Obtain final answer $a = \frac{1}{\sqrt{2}}$, or exact equivalent	A1	[4]
	(ii)	Use correct method and value of a to find the first two terms of the expansion $(1 + ax)^{-2}$	M1	
		Obtain $1 - \sqrt{2}x$, or equivalent	A1 √	
		Obtain term $\frac{3}{2}x^2$	A1 √	[3]
		[Symbolic coefficients, e.g. $\binom{-2}{1}a$, are not sufficient for the first B marks]		
		[The f.t. is solely on the value of a.]		

5 **(i)** Use correct quotient or chain rule — M1
Obtain the given answer correctly having shown sufficient working — A1 [2]

(ii) Use a valid method, e.g. multiply numerator and denominator by sec x + tan x, and a version of Pythagoras to justify the given identity — B1 [1]

(iii) Substitute, expand $(\sec x + \tan x)^2$ and use Pythagoras once — M1
Obtain given identity — A1 [2]

(iv) Obtain integral $2\tan x - x + 2\sec x$ — B1
Use correct limits correctly in an expression of the form $a\tan x + bx + c\sec x$, or equivalent, where $abc \neq 0$ — M1
Obtain the given answer correctly — A1 [3]

6 Separate variables correctly and attempt integration of one side — B1
Obtain term $\ln x$ — B1
State or imply $\frac{1}{1-y^2} \equiv \frac{A}{1-y} + \frac{B}{1+y}$ and use a relevant method to find A or B — M1
Obtain $A = \frac{1}{2}$, $B = \frac{1}{2}$
Integrate and obtain $-\frac{1}{2}\ln(1-y) + \frac{1}{2}\ln(1+y)$, or equivalent — A1 √
[If the integral is directly stated as $k_1 \ln\left(\frac{1+y}{1-y}\right)$ or $k_2 \ln\left(\frac{1-y}{1+y}\right)$ give M1, and then A2 for $k_1 = \frac{1}{2}$ or $k_2 = -\frac{1}{2}$]
Evaluate a constant, or use limits $x = 2$, $y = 0$ in a solution containing terms $a\ln x$, $b\ln(1-y)$ and $c\ln(1+y)$, where $abc \neq 0$ — M1
[This M mark is not available if the integral of $1/(1-y^2)$ is initially taken to be of the form $k\ln(1-y^2)$]
Obtain solution in any correct form, e.g. $\frac{1}{2}\ln\left(\frac{1+y}{1-y}\right) = \ln x - \ln 2$ — A1
Rearrange and obtain $y = \frac{x^2-4}{x^2+4}$, or equivalent, free of logarithms — A1 [8]

7 **(i)** *EITHER*: State or imply $\frac{1}{x} + \frac{1}{y}\frac{dy}{dx}$ as derivative of $\ln xy$, or equivalent — B1
State or imply $3y^2\frac{dy}{dx}$ as derivative of y^3, or equivalent — B1
Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$ — M1
Obtain the given answer — A1

OR: Obtain $xy = \exp(1 + y^3)$ and state or imply $y + x\frac{dy}{dx}$ as derivative of xy — B1
State or imply $3y^2\frac{dy}{dx}\exp(1 + y^3)$ as derivative of $(1 + y^3)$ — B1
Equate derivatives and solve for $\frac{dy}{dx}$ — M1
Obtain the given answer — A1 [4]
[The M1 is dependent on at least one of the B marks being earned]

(ii) Equate denominator to zero and solve for y — M1*
Obtain $y = 0.693$ only — A1
Substitute found value in the equation and solve for x — M1(dep*)
Obtain $x = 5.47$ only — A1 [4]

8	**(i)**	Use correct product or quotient rule and use chain rule at least once	M1	
		Obtain derivative in any correct form	A1	
		Equate derivative to zero and solve an equation with at least two non-zero terms for real x	M1	
		Obtain answer $x = \frac{1}{\sqrt{2}}$, or exact equivalent	A1	[4]
	(ii)	State a suitable equation, e.g. $\alpha = \sqrt{(\ln(4 + 8\alpha^2))}$	B1	
		Rearrange to reach $e^{\alpha^2} = 4 + 8\alpha^2$	B1	
		Obtain $\frac{1}{2} = e^{-\frac{1}{2}\alpha^2}\sqrt{(1 + 2\alpha^2)}$, or work *vice versa*	B1	[3]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.86	A1	
		Show sufficient iterations to 4 d.p. to justify 1.86 to 2 d.p., or show there is a sign change in the interval (1.855, 1.865)	A1	[3]

9	**(i)**	*EITHER*	Substitute $x = 1 + \sqrt{2}\,i$ and attempt the expansions of the x^2 and x^4 terms	M1	
			Use $i^2 = -1$ correctly at least once	B1	
			Complete the verification	A1	
			State second root $1 - \sqrt{2}\,i$	B1	
		OR 1	State second root $1 - \sqrt{2}\,i$	B1	
			Carry out a complete method for finding a quadratic factor with zeros $1 \pm \sqrt{2}\,i$	M1	
			Obtain $x^2 - 2x + 3$, or equivalent	A1	
			Show that the division of p(x) by $x^2 - 2x + 3$ gives zero remainder and complete the verification	A1	
		OR 2	Substitute $x = 1 + \sqrt{2}\,i$ and use correct method to express x^2 and x^4 in polar form	M1	
			Obtain x^2 and x^4 in any correct polar form (allow decimals here)	B1	
			Complete an exact verification	A1	
			State second root $1 - \sqrt{2}\,i$, or its polar equivalent (allow decimals here)	B1	[4]

(ii)	Carry out a complete method for finding a quadratic factor with zeros $1 \pm \sqrt{2}\,i$	M1*	
	Obtain $x^2 - 2x + 3$, or equivalent	A1	
	Attempt division of p(x) by $x^2 - 2x + 3$ reaching a partial quotient $x^2 + kx$, or equivalent	M1 (dep*)	
	Obtain quadratic factor $x^2 - 2x + 2$	A1	
	Find the zeros of the second quadratic factor, using $i^2 = -1$	M1 (dep*)	
	Obtain roots $-1 + i$ and $-1 - i$	A1	[6]

[The second M1 is earned if inspection reaches an unknown factor $x^2 + Bx + C$ and an equation in B and/or C, or an unknown factor $Ax^2 + Bx + (6/3)$ and an equation in A and/or B]

[If part **(i)** is attempted by the *OR 1* method, then an attempt at part **(ii)** which uses or quotes relevant working or results obtained in part **(i)** should be marked using the scheme for part **(ii)**]

10	**(i)**	*EITHER*	Use scalar product of relevant vectors, or subtract point equations to form two equations in a,b,c, e.g. $a - 5b - 3c = 0$ and $a - b - 3c = 0$	M1*
			State two correct equations in a,b,c	A1
			Solve simultaneous equations and find one ratio, e.g. $a : c$, or $b = 0$	M1 (dep*)
			Obtain $a : b : c = 3 : 0 : 1$, or equivalent	A1
			Substitute a relevant point in $3x + z = d$ and evaluate d	M1 (dep*)
			Obtain equation $3x + z = 13$, or equivalent	A1
		OR 1	Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i} - 5\mathbf{j} - 3\mathbf{k}) \times (\mathbf{i} - \mathbf{j} - 3\mathbf{k})$	M2*
			Obtain 2 correct components of the product	A1
			Obtain correct product, e.g. $12\mathbf{i} + 4\mathbf{k}$	A1
			Substitute a relevant point in $12x + 4z = d$ and evaluate d	M1 (dep*)
			Obtain $3x + z =13$, or equivalent	A1
		OR 2	Attempt to form 2–parameter equation for the plane with relevant vectors	M2*
			State a correct equation e.g. $\mathbf{r} = 3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k} + \lambda(\mathbf{i} - 5\mathbf{j} - 3\mathbf{k}) + \mu(\mathbf{i} - \mathbf{j} - 3\mathbf{k})$	A1
			State 3 equations in x, y, z, λ and μ	A1
			Eliminate λ and μ	M1 (dep*)
			Obtain equation $3x + z =13$, or equivalent	A1 [6]
	(ii)	*EITHER*	Find $\overrightarrow{CP}$ for a point P on AB with a parameter t, e.g. $2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k} + t(-\mathbf{i} + \mathbf{j} + 3\mathbf{k})$	B1 √
			Either: Equate scalar product $\overrightarrow{CP}, \overrightarrow{AB}$ to zero and form an equation in t	
			Or 1: Equate derivative for CP^2 (or CP) to zero and form an equation in t	
			Or 2: Use Pythagoras in triangle CPA (or CPB) and form an equation in t	M1
			Solve and obtain correct value of t, e.g. $t = -2$	A1
			Carry out a complete method for finding the length of CP	M1
			Obtain answer $3\sqrt{2}$ (4.24), or equivalent	A1
		OR 1	State $\overrightarrow{AC}$ (or $\overrightarrow{BC}$) and $\overrightarrow{AB}$ in component form	B1 √
			Using a relevant scalar product find the cosine of CAB (or CBA)	M1
			Obtain cost $CAB = -\frac{22}{\sqrt{11}.\sqrt{62}}$, or cos $CBA = \frac{33}{\sqrt{11}.\sqrt{117}}$, or equivalent	A1
			Use trig to find the length of the perpendicular	M1
			Obtain answer $3\sqrt{2}$ (4.24), or equivalent	A1
		OR 2	State $\overrightarrow{AC}$ (or $\overrightarrow{BC}$) and $\overrightarrow{AB}$ in component form	B1 √
			Using a relevant scalar product find the length of the projection AC (or BC) on AB	M1
			Obtain answer $2\sqrt{11}$ (or), $3\sqrt{11}$ or equivalent	A1
			Use Pythagoras to find the length of the perpendicular	M1
			Obtain answer $3\sqrt{2}$ (4.24), or equivalent	A1
		OR 3	State $\overrightarrow{AC}$ (or $\overrightarrow{BC}$) and $\overrightarrow{AB}$ in component form	B1 √
			Calculate their vector product, e.g. $(-2\mathbf{i} - 3\mathbf{j} - 7\mathbf{k}) \times (-\mathbf{i} + \mathbf{j} + 3\mathbf{k})$	M1
			Obtain correct product, e.g. $-2\mathbf{i} + 13\mathbf{j} - 5\mathbf{k}$	A1
			Divide modulus of the product by the modulus of $\overrightarrow{AB}$	M1
			Obtain answer $3\sqrt{2}$ (4.24), or equivalent	A1
		OR 4	State two of $\overrightarrow{AB}$, $\overrightarrow{BC}$) and $\overrightarrow{AC}$ in component form	B1 √
			Use cosine formula in triangle ABC to find cos A or cos B	M1
			Obtain cos $A = -\frac{44}{2\sqrt{11}.\sqrt{62}}$, or cos $B = \frac{66}{2\sqrt{11}.\sqrt{117}}$	A1
			Use trig to find the length of the perpendicular	M1
			Obtain answer $3\sqrt{2}$ (4.24), or equivalent	A1 [5]
			[The f.t is on $\overrightarrow{AB}$]	