

Page 4	Mark Scheme	Syllabus	Paper
	GCE AS/A LEVEL – October/November 2012	9709	31

- 1 *EITHER* State or imply non-modular inequality $(3(x-1))^2 < (2x+1)^2$
or corresponding quadratic equation, or pair of linear equations $3(x-1) = \pm(2x+1)$ B1
Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations M1
Obtain critical values $x = \frac{2}{5}$ and $x = 4$ A1
State answer $\frac{2}{5} < x < 4$ A1
- OR* Obtain critical value $x = \frac{2}{5}$ or $x = 4$ from a graphical method, or by inspection, or by solving a linear equation or inequality B1
Obtain critical values $x = \frac{2}{5}$ and $x = 4$ B2
State answer $\frac{2}{5} < x < 4$ B1 [4]
[Do not condone \leq for $<$.]
- 2 *EITHER* Use laws of indices correctly and solve for 5^x or for 5^{-x} or for 5^{x-1} M1
Obtain 5^x or for 5^{-x} or for 5^{x-1} in any correct form, e.g. $5^x = \frac{5}{1-1/5}$ A1
Use correct method for solving $5^x = a$, or $5^{-x} = a$, or $5^{x-1} = a$, where $a > 0$ M1
Obtain answer $x = 1.14$ A1
- OR* Use an appropriate iterative formula, e.g. $x_{n+1} = \frac{\ln(5^{x-1}+5)}{\ln 5}$, correctly, at least once M1
Obtain answer 1.14 A1
Show sufficient iterations to at least 3 d.p. to justify 1.14 to 2 d.p., or show there is a sign change in the interval (1.135, 1.145) A1
Show there is no other root A1 [4]
[For the solution $x = 1.14$ with no relevant working give B1, and a further B1 if 1.14 is shown to be the only solution.]
- 3 Attempt use of $\sin(A+B)$ and $\cos(A-B)$ formulate to obtain an equation in $\cos \theta$ and $\sin \theta$ M1
Obtain a correct equation in any form A1
Use trig. formula to obtain an equation in $\tan \theta$ (or $\cos \theta$, $\sin \theta$ or $\cot \theta$) M1
Obtain $\tan \theta = \frac{\sqrt{6}-1}{1-\sqrt{2}}$, or equivalent (or find $\cos \theta$, $\sin \theta$ or $\cot \theta$) A1
Obtain answer $\theta = 105.9^\circ$, and no others in the given interval A1 [5]
[Ignore answers outside the given material]
- 4 (i) Obtain correct unsimplified terms in x and x^3 B1 + B1
Equate coefficients and solve for a M1
Obtain final answer $a = \frac{1}{\sqrt{2}}$, or exact equivalent A1 [4]
- (ii) Use correct method and value of a to find the first two terms of the expansion $(1+ax)^{-2}$ M1
Obtain $1 - \sqrt{2}x$, or equivalent A1 \checkmark
Obtain term $\frac{3}{2}x^2$ A1 \checkmark [3]
[Symbolic coefficients, e.g. $\binom{-2}{1}a$, are not sufficient for the first B marks]
[The f.t. is solely on the value of a .]

Page 5	Mark Scheme	Syllabus	Paper
	GCE AS/A LEVEL – October/November 2012	9709	31

- 5 (i) Use correct quotient or chain rule M1
Obtain the given answer correctly having shown sufficient working A1 [2]
- (ii) Use a valid method, e.g. multiply numerator and denominator by $\sec x + \tan x$, and a version of Pythagoras to justify the given identity B1 [1]
- (iii) Substitute, expand $(\sec x + \tan x)^2$ and use Pythagoras once M1
Obtain given identity A1 [2]
- (iv) Obtain integral $2 \tan x - x + 2 \sec x$ B1
Use correct limits correctly in an expression of the form $a \tan x + bx + c \sec x$, or equivalent, where $abc \neq 0$ M1
Obtain the given answer correctly A1 [3]
- 6 Separate variables correctly and attempt integration of one side B1
Obtain term $\ln x$ B1
State or imply $\frac{1}{1-y^2} \equiv \frac{A}{1-y} + \frac{B}{1+y}$ and use a relevant method to find A or B M1
Obtain $A = \frac{1}{2}$, $B = \frac{1}{2}$
Integrate and obtain $-\frac{1}{2} \ln(1-y) + \frac{1}{2} \ln(1+y)$, or equivalent A1 \checkmark
[If the integral is directly stated as $k_1 \ln\left(\frac{1+y}{1-y}\right)$ or $k_2 \ln\left(\frac{1-y}{1+y}\right)$ give M1, and then A2 for $k_1 = \frac{1}{2}$ or $k_2 = -\frac{1}{2}$]
Evaluate a constant, or use limits $x = 2, y = 0$ in a solution containing terms $a \ln x, b \ln(1-y)$ and $c \ln(1+y)$, where $abc \neq 0$ M1
[This M mark is not available if the integral of $1/(1-y^2)$ is initially taken to be of the form $k \ln(1-y^2)$]
Obtain solution in any correct form, e.g. $\frac{1}{2} \ln\left(\frac{1+y}{1-y}\right) = \ln x - \ln 2$ A1
Rearrange and obtain $y = \frac{x^2 - 4}{x^2 + 4}$, or equivalent, free of logarithms A1 [8]
- 7 (i) EITHER: State or imply $\frac{1}{x} + \frac{1}{y} \frac{dy}{dx}$ as derivative of $\ln xy$, or equivalent B1
State or imply $3y^2 \frac{dy}{dx}$ as derivative of y^3 , or equivalent B1
Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$ M1
Obtain the given answer A1
OR Obtain $xy = \exp(1+y^3)$ and state or imply $y + x \frac{dy}{dx}$ as derivative of xy B1
State or imply $3y^2 \frac{dy}{dx} \exp(1+y^3)$ as derivative of $(1+y^3)$ B1
Equate derivatives and solve for $\frac{dy}{dx}$ M1
Obtain the given answer A1 [4]
[The M1 is dependent on at least one of the B marks being earned]
- (ii) Equate denominator to zero and solve for y M1*
Obtain $y = 0.693$ only A1
Substitute found value in the equation and solve for x M1(dep*)
Obtain $x = 5.47$ only A1 [4]

Page 6	Mark Scheme	Syllabus	Paper
	GCE AS/A LEVEL – October/November 2012	9709	31

- 8 (i) Use correct product or quotient rule and use chain rule at least once M1
 Obtain derivative in any correct form A1
 Equate derivative to zero and solve an equation with at least two non-zero terms for real x M1
 Obtain answer $x = \frac{1}{\sqrt{2}}$, or exact equivalent A1 [4]
- (ii) State a suitable equation, e.g. $\alpha = \sqrt{(\ln(4 + 8\alpha^2))}$ B1
 Rearrange to reach $e^{\alpha^2} = 4 + 8\alpha^2$ B1
 Obtain $\frac{1}{2} = e^{-\frac{1}{2}\alpha^2} \sqrt{(1 + 2\alpha^2)}$, or work *vice versa* B1 [3]
- (iii) Use the iterative formula correctly at least once M1
 Obtain final answer 1.86 A1
 Show sufficient iterations to 4 d.p. to justify 1.86 to 2 d.p., or show there is a sign change in the interval (1.855, 1.865) A1 [3]
- 9 (i) EITHER Substitute $x = 1 + \sqrt{2}i$ and attempt the expansions of the x^2 and x^4 terms M1
 Use $i^2 = -1$ correctly at least once B1
 Complete the verification A1
 State second root $1 - \sqrt{2}i$ B1
 OR 1 State second root $1 - \sqrt{2}i$ B1
 Carry out a complete method for finding a quadratic factor with zeros $1 \pm \sqrt{2}i$ M1
 Obtain $x^2 - 2x + 3$, or equivalent A1
 Show that the division of $p(x)$ by $x^2 - 2x + 3$ gives zero remainder and complete the verification A1
 OR 2 Substitute $x = 1 + \sqrt{2}i$ and use correct method to express x^2 and x^4 in polar form M1
 Obtain x^2 and x^4 in any correct polar form (allow decimals here) B1
 Complete an exact verification A1
 State second root $1 - \sqrt{2}i$, or its polar equivalent (allow decimals here) B1 [4]
- (ii) Carry out a complete method for finding a quadratic factor with zeros $1 \pm \sqrt{2}i$ M1*
 Obtain $x^2 - 2x + 3$, or equivalent A1
 Attempt division of $p(x)$ by $x^2 - 2x + 3$ reaching a partial quotient $x^2 + kx$, or equivalent M1 (dep*)
 Obtain quadratic factor $x^2 - 2x + 2$ A1
 Find the zeros of the second quadratic factor, using $i^2 = -1$ M1 (dep*)
 Obtain roots $-1 + i$ and $-1 - i$ A1 [6]
 [The second M1 is earned if inspection reaches an unknown factor $x^2 + Bx + C$ and an equation in B and/or C , or an unknown factor $Ax^2 + Bx + (6/3)$ and an equation in A and/or B]
 [If part (i) is attempted by the OR 1 method, then an attempt at part (ii) which uses or quotes relevant working or results obtained in part (i) should be marked using the scheme for part (ii)]

Page 7	Mark Scheme	Syllabus	Paper
	GCE AS/A LEVEL – October/November 2012	9709	31

- 10 (i) *EITHER* Use scalar product of relevant vectors, or subtract point equations to form two equations in a, b, c , e.g. $a - 5b - 3c = 0$ and $a - b - 3c = 0$ M1*
- State two correct equations in a, b, c A1
- Solve simultaneous equations and find one ratio, e.g. $a : c$, or $b = 0$ M1 (dep*)
- Obtain $a : b : c = 3 : 0 : 1$, or equivalent A1
- Substitute a relevant point in $3x + z = d$ and evaluate d M1 (dep*)
- Obtain equation $3x + z = 13$, or equivalent A1
- OR 1* Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i} - 5\mathbf{j} - 3\mathbf{k}) \times (\mathbf{i} - \mathbf{j} - 3\mathbf{k})$ M2*
- Obtain 2 correct components of the product A1
- Obtain correct product, e.g. $12\mathbf{i} + 4\mathbf{k}$ A1
- Substitute a relevant point in $12x + 4z = d$ and evaluate d M1 (dep*)
- Obtain $3x + z = 13$, or equivalent A1
- OR 2* Attempt to form 2-parameter equation for the plane with relevant vectors M2*
- State a correct equation e.g. $\mathbf{r} = 3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k} + \lambda(\mathbf{i} - 5\mathbf{j} - 3\mathbf{k}) + \mu(\mathbf{i} - \mathbf{j} - 3\mathbf{k})$ A1
- State 3 equations in x, y, z, λ and μ A1
- Eliminate λ and μ M1 (dep*)
- Obtain equation $3x + z = 13$, or equivalent A1 [6]
- (ii) *EITHER* Find \overrightarrow{CP} for a point P on AB with a parameter t , e.g. $2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k} + t(-\mathbf{i} + \mathbf{j} + 3\mathbf{k})$ B1 ✓
- Either:* Equate scalar product $\overrightarrow{CP}, \overrightarrow{AB}$ to zero and form an equation in t
- Or 1:* Equate derivative for CP^2 (or CP) to zero and form an equation in t
- Or 2:* Use Pythagoras in triangle CPA (or CPB) and form an equation in t M1
- Solve and obtain correct value of t , e.g. $t = -2$ A1
- Carry out a complete method for finding the length of CP M1
- Obtain answer $3\sqrt{2}$ (4.24), or equivalent A1
- OR 1* State \overrightarrow{AC} (or \overrightarrow{BC}) and \overrightarrow{AB} in component form B1 ✓
- Using a relevant scalar product find the cosine of CAB (or CBA) M1
- Obtain $\cos CAB = -\frac{22}{\sqrt{11}\sqrt{62}}$, or $\cos CBA = \frac{33}{\sqrt{11}\sqrt{117}}$, or equivalent A1
- Use trig to find the length of the perpendicular M1
- Obtain answer $3\sqrt{2}$ (4.24), or equivalent A1
- OR 2* State \overrightarrow{AC} (or \overrightarrow{BC}) and \overrightarrow{AB} in component form B1 ✓
- Using a relevant scalar product find the length of the projection AC (or BC) on AB M1
- Obtain answer $2\sqrt{11}$ (or), $3\sqrt{11}$ or equivalent A1
- Use Pythagoras to find the length of the perpendicular M1
- Obtain answer $3\sqrt{2}$ (4.24), or equivalent A1
- OR 3* State \overrightarrow{AC} (or \overrightarrow{BC}) and \overrightarrow{AB} in component form B1 ✓
- Calculate their vector product, e.g. $(-2\mathbf{i} - 3\mathbf{j} - 7\mathbf{k}) \times (-\mathbf{i} + \mathbf{j} + 3\mathbf{k})$ M1
- Obtain correct product, e.g. $-2\mathbf{i} + 13\mathbf{j} - 5\mathbf{k}$ A1
- Divide modulus of the product by the modulus of \overrightarrow{AB} M1
- Obtain answer $3\sqrt{2}$ (4.24), or equivalent A1
- OR 4* State two of $\overrightarrow{AB}, \overrightarrow{BC}$ and \overrightarrow{AC} in component form B1 ✓
- Use cosine formula in triangle ABC to find $\cos A$ or $\cos B$ M1
- Obtain $\cos A = -\frac{44}{2\sqrt{11}\sqrt{62}}$, or $\cos B = \frac{66}{2\sqrt{11}\sqrt{117}}$ A1
- Use trig to find the length of the perpendicular M1
- Obtain answer $3\sqrt{2}$ (4.24), or equivalent A1 [5]
- [The f.t is on \overrightarrow{AB}]