

Page 4	Mark Scheme: Teachers version	Syllabus	Paper
	GCE AS/A LEVEL – October/November 2011	9709	33
1	<p>Either</p> <p>Obtain correct unsimplified version of x or x^2 term in expansion of $(2+x)^{-2}$ or $(1 + \frac{1}{2}x)^{-2}$</p> <p>Correct first term 4 from correct work</p> <p>Obtain $-4x$</p> <p>Obtain $+3x^2$</p> <p>Or</p> <p>Differentiate and evaluate $f(0)$ and $f'(0)$ where $f'(x) = k(2+x)^{-3}$</p> <p>State correct first term 4</p> <p>Obtain $-4x$</p> <p>Obtain $+3x^2$</p>	M1 B1 A1 A1 M1 B1 A1 A1	[4]
2	<p>Use correct quotient or product rule or equivalent</p> <p>Obtain $\frac{(1+e^{2x}) \cdot 2e^{2x} - e^{2x} \cdot 2e^{2x}}{(1+e^{2x})^2}$ or equivalent</p> <p>Substitute $x = \ln 3$ into attempt at first derivative and show use of relevant logarithm property at least once in a correct context</p> <p>Confirm given answer $\frac{9}{50}$ legitimately</p>	M1 A1 M1 A1	[4]
3	<p>(i) State or imply $R = 17$</p> <p>Use correct trigonometric formula to find α</p> <p>Obtain 61.93° with no errors seen</p> <p>(ii) Evaluate $\cos^{-1} \frac{12}{R}$ ($= 45.099$)</p> <p>Obtain answer 107.0°</p> <p>Carry out correct method for second answer</p> <p>Obtain answer 16.8° and no others between 0° and 360°</p>	B1 M1 A1 M1 A1 M1 A1	[3] [4]

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4	<p>(i) Separate variables and attempt integration on both sides</p> <p>Obtain $2N^{0.5}$ on left-hand side or equivalent</p> <p>Obtain $-60e^{-0.02t}$ on right-hand side or equivalent</p> <p>Use 0 and 100 to evaluate a constant or as limits in a solution containing terms $aN^{0.5}$ and $be^{-0.02t}$</p> <p>Obtain $2N^{0.5} = -60e^{-0.02t} + 80$ or equivalent</p> <p>Conclude with $N = (40 - 30e^{-0.02t})^2$ or equivalent</p> <p>(ii) State number approaches 1600 or equivalent, following expression of form $(c + de^{-0.02t})^n$</p>	<p>M1*</p> <p>A1</p> <p>A1</p> <p>DM1*</p> <p>A1</p> <p>A1</p> <p>B1√</p>	<p>[6]</p> <p>[1]</p>
5	<p>(i) Either</p> <p>Use integration by parts and reach an expression $kx^2 \ln x \pm n \int x^2 \cdot \frac{1}{x} dx$</p> <p>Obtain $\frac{1}{2}x^2 \ln x - \int \frac{1}{2}x dx$ or equivalent</p> <p>Obtain $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$</p> <p>Or</p> <p>Use Integration by parts and reach an expression $kx(x \ln x - x) \pm m \int x \ln x - x dx$</p> <p>Obtain $I = (x^2 \ln x - x^2) - I + \int x dx$</p> <p>Obtain $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$</p> <p>Substitute limits correctly and equate to 22, having integrated twice</p> <p>Rearrange and confirm given equation $a = \sqrt{\frac{87}{2 \ln a - 1}}$</p> <p>(ii) Use iterative process correctly at least once</p> <p>Obtain final answer 5.86</p> <p>Show sufficient iterations to 4 d.p. to justify 5.86 or show a sign change in the interval (5.855, 5.865)</p> <p>(6 → 5.8030 → 5.8795 → 5.8491 → 5.8611 → 5.8564)</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>DM1*</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>[5]</p> <p>[3]</p>

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6	<p>(i) Use correct method for finding modulus of their w^2 or w^3 or both</p> <p>Obtain $w^2 = 2$ and $w^3 = 2\sqrt{2}$ or equivalent</p> <p>Use correct method for finding argument of their w^2 or w^3 or both</p> <p>Obtain $\arg(w^2) = -\frac{1}{2}\pi$ or $\frac{3}{2}\pi$ and $\arg(w^3) = \frac{1}{4}\pi$</p> <p>(ii) Obtain centre $-\frac{1}{2} - \frac{1}{2}i$ (their w^2)</p> <p>Calculate the diameter or radius using $w-w^2$ w21 or right-angled triangle or cosine rule or equivalent</p> <p>Obtain radius $\frac{1}{2}\sqrt{10}$ or equivalent</p> <p>Obtain $z + \frac{1}{2} + \frac{1}{2}i = \frac{1}{2}\sqrt{10}$ or equivalent</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1ft</p> <p>B1ft</p> <p>M1</p> <p>A1</p> <p>A1ft</p>	<p>[4]</p> <p>[4]</p>
7	<p>(i) Substitute $x = \frac{1}{2}$ and equate to zero</p> <p>or divide by $(2x-1)$, reach $\frac{a}{2}x^2 + kx + \dots$ and equate remainder to zero</p> <p>or by inspection reach $\frac{a}{2}x^2 + bx + c$ and an equation in b/c</p> <p>or by inspection reach $Ax^2 + Bx + a$ and an equation in A/B</p> <p>Obtain $a = 2$</p> <p>Attempt to find quadratic factor by division or inspection or equivalent</p> <p>Obtain $(2x-1)(x^2+2)$</p> <p>(ii) State or imply form $\frac{A}{2x-1} + \frac{Bx+C}{x^2+2}$, following factors from part (i)</p> <p>Use relevant method to find a constant</p> <p>Obtain $A = -4$, following factors from part (i)</p> <p>Obtain $B = 2$</p> <p>Obtain $C = 5$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1cwo</p> <p>B1√</p> <p>M1</p> <p>A1√</p> <p>A1</p> <p>A1</p>	<p>[4]</p>

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8	(i) Differentiate y to obtain $3\sin^2 t \cos t - 3\cos^2 t \sin t$ o.e.	B1	[3]
	Use $\frac{dy}{dx} = \frac{dy}{dt} / \frac{dt}{dx}$	M1	
	Obtain given result $-3\sin t \cos t$	A1cwo	
	(ii) Identify parameter at origin as $t = \frac{3}{4}\pi$	B1	[2]
	Use $t = \frac{3}{4}\pi$ to obtain $\frac{3}{2}$	B1	
	(iii) Rewrite equation as equation in one trig variable e.g. $\sin 2t = -\frac{2}{3}$, $9 \sin^4 x - 9 \sin^2 x + 1 = 0$, $\tan^2 x + 3 \tan x + 1 = 0$	B1	[4]
	Find at least one value of t from equation of form $\sin 2t = k$ o.e.	M1	
	Obtain 1.9	A1	
	Obtain 2.8 and no others	A1	

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9	(i) Calculate scalar product of direction of l and normal to p	M1	
	Obtain $4 \times 2 + 3 \times (-2) + (-2) \times 1 = 0$ and conclude accordingly	A1	[2]
	(ii) Substitute $(a, 1, 4)$ in equation of p and solve for a	M1	
	Obtain $a = 4$	A1	[2]
	(iii) Either		
	Attempt use of formula for perpendicular distance using $(a, 1, 4)$	M1	
	Obtain at least $\frac{2a-2+4-10}{\sqrt{4+4+1}} = 6$	A1	
	Obtain $a = 13$	A1	
	Attempt solution of $\frac{2a-8}{3} = -6$	M1	
	Obtain $a = -5$	A1	
	Or		
	Form equation of parallel plane and substitute $(a, 1, 4)$	M1	
	Obtain $\frac{2a+2}{3} - \frac{10}{3} = 6$	A1	
	Obtain $a = 13$	A1	
	Solve $\frac{2a+2}{3} - \frac{10}{3} = -6$	M1	
	Obtain $a = -5$	A1	
	Or		
	State a vector from a pt on the plane to $(a, 1, 4)$ e.g.	B1	
$\begin{pmatrix} a-5 \\ 1 \\ 4 \end{pmatrix}$ or $\begin{pmatrix} a \\ 1 \\ -6 \end{pmatrix}$			
Calculate the component of this vector in the direction of the unit normal and equate to 6 : $\frac{1}{3} \begin{pmatrix} a-5 \\ 1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = 6$	M1		
Obtain $a = 13$	A1		
Solve $\frac{1}{3} \begin{pmatrix} a-5 \\ 1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = -6$	M1		
Obtain $a = -5$	A1		

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	<p>Or</p> <p>State or imply perpendicular line $\mathbf{r} = \begin{pmatrix} a \\ 1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$</p> <p>Substitute components for p and solve for μ</p> <p>Obtain $\mu = \frac{8-2a}{9}$</p> <p>Equate distance between $(a, 1, 4)$ and foot of perpendicular to ± 6</p> <p>Obtain $\frac{3(8-2a)}{9} = \pm 6$ or equivalent and hence -5 and 13</p>	B1	
		M1	
		A1	
		M1	
		A1	[5]
10	<p>(i) State or imply $\frac{du}{dx} = \sec^2 x$</p> <p>Express integrand in terms of u and du</p> <p>Integrate to obtain $\frac{u^{n+1}}{n+1}$ or equivalent</p> <p>Substitute correct limits correctly to confirm given result $\frac{1}{n+1}$</p> <p>(ii) (a) Use $\sec^2 x = 1 + \tan^2 x$ twice</p> <p>Obtain integrand $\tan^4 x + \tan^2 x$</p> <p>Apply result from part (i) to obtain $\frac{1}{3}$</p> <p>Or</p> <p>Use $\sec^2 x = 1 + \tan^2 x$ and the substitution from (i)</p> <p>Obtain $\int u^2 du$</p> <p>Apply limits correctly and obtain $\frac{1}{3}$</p> <p>(b) Arrange, perhaps implied, integrand to $t^9 + t^7 + 4(t^7 + t^5) + t^5 + t^3$</p> <p>Attempt application of result from part (i) at least twice</p> <p>Obtain $\frac{1}{8} + \frac{4}{6} + \frac{1}{4}$ and hence $\frac{25}{24}$ or exact equivalent</p>	B1	
		M1	
		A1	
		A1	[4]
		M1	
		A1	
		A1	[3]
		M1	
		A1	
		B1	
		M1	
		A1	[3]