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- 1 Rearrange as $e^{2x} - e^x - 6 = 0$, or $u^2 - u - 6 = 0$, or equivalent B1
 Solve a 3-term quadratic for e^x or for u M1
 Obtain simplified solution $e^x = 3$ or $u = 3$ A1
 Obtain final answer $x = 1.10$ and no other A1 [4]
- 2 *EITHER*: Use chain rule M1
 obtain $\frac{dx}{dt} = 6 \sin t \cos t$, or equivalent A1
 obtain $\frac{dy}{dt} = -6 \cos^2 t \sin t$, or equivalent A1
 Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ M1
 Obtain final answer $\frac{dy}{dx} = -\cos t$ A1
OR: Express y in terms of x and use chain rule M1
 Obtain $\frac{dy}{dx} = k(2 - \frac{x}{3})^{\frac{1}{2}}$, or equivalent A1
 Obtain $\frac{dy}{dx} = -(2 - \frac{x}{3})^{\frac{1}{2}}$, or equivalent A1
 Express derivative in terms of t M1
 Obtain final answer $\frac{dy}{dx} = -\cos t$ A1 [5]
- 3 (i) *EITHER*: Attempt division by $x^2 - x + 1$ reaching a partial quotient of $x^2 + kx$ M1
 Obtain quotient $x^2 + 4x + 3$ A1
 Equate remainder of form lx to zero and solve for a , or equivalent M1
 Obtain answer $a = 1$ A1
OR: Substitute a complex zero of $x^2 - x + 1$ in $p(x)$ and equate to zero M1
 Obtain a correct equation in a in any unsimplified form A1
 Expand terms, use $i^2 = -1$ and solve for a M1
 Obtain answer $a = 1$ A1 [4]
 [SR: The first M1 is earned if inspection reaches an unknown factor $x^2 + Bx + C$ and an equation in B and/or C , or an unknown factor $Ax^2 + Bx + 3$ and an equation in A and/or B . The second M1 is only earned if use of the equation $a = B - C$ is seen or implied.]
- (ii) State answer, e.g. $x = -3$ B1
 State answer, e.g. $x = -1$ and no others B1 [2]
- 4 Separate variables and attempt integration of at least one side M1
 Obtain term $\ln(x + 1)$ A1
 Obtain term $k \ln \sin 2\theta$, where $k = \pm 1, \pm 2$, or $\pm \frac{1}{2}$ M1
 Obtain correct term $\frac{1}{2} \ln \sin 2\theta$ A1
 Evaluate a constant, or use limits $\theta = \frac{1}{12} \pi$, $x = 0$ in a solution containing terms $a \ln(x + 1)$ and $b \ln \sin 2\theta$ M1
 Obtain solution in any form, e.g. $\ln(x + 1) = \frac{1}{2} \ln \sin 2\theta - \frac{1}{2} \ln \frac{1}{2}$ (f.t. on $k = \pm 1, \pm 2$, or $\pm \frac{1}{2}$) A1√
 Rearrange and obtain $x = \sqrt{(2 \sin 2\theta) - 1}$, or simple equivalent A1 [7]

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- 5 (i) Make recognisable sketch of a relevant graph over the given interval
Sketch the other relevant graph and justify the given statement B1 [2]
B1
- (ii) Consider the sign of $\sec x - (3 - \frac{1}{2}x^2)$ at $x = 1$ and $x = 1.4$, or equivalent M1
Complete the argument with correct calculated values A1 [2]
- (iii) Convert the given equation to $\sec x = 3 - \frac{1}{2}x^2$ or work *vice versa* B1 [1]
- (iv) Use a correct iterative formula correctly at least once M1
Obtain final answer 1.13 A1
Show sufficient iterations to 4 d.p. to justify 1.13 to 2 d.p., or show there is a sign change in the interval (1.125, 1.135) A1 [3]
[SR: Successive evaluation of the iterative function with $x = 1, 2, \dots$ scores M0.]
- 6 (i) State or imply $R = \sqrt{10}$ B1
Use trig formulae to find α M1
Obtain $\alpha = 71.57^\circ$ with no errors seen A1 [3]
[Do not allow radians in this part. If the only trig error is a sign error in $\cos(x - \alpha)$ give M1A0]
- (ii) Evaluate $\cos^{-1}(2/\sqrt{10})$ correctly to at least 1 d.p. ($50.7684\dots^\circ$) (Allow 50.7° here) B1✓
Carry out an appropriate method to find a value of 2θ in $0^\circ < 2\theta < 180^\circ$ M1
Obtain an answer for θ in the given range, e.g. $\theta = 61.2^\circ$ A1
Use an appropriate method to find another value of 2θ in the above range M1
Obtain second angle, e.g. $\theta = 10.4^\circ$, and no others in the given range A1 [5]
[Ignore answers outside the given range.]
[Treat answers in radians as a misread and deduct A1 from the answers for the angles.]
[SR: The use of correct trig formulae to obtain a 3-term quadratic in $\tan \theta$, $\sin 2\theta$, $\cos 2\theta$, or $\tan 2\theta$ earns M1; then A1 for a correct quadratic, M1 for obtaining a value of θ in the given range, and A1 + A1 for the two correct answers (candidates who square must reject the spurious roots to get the final A1).]

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- 7 (i) Use a correct method to express \overrightarrow{OP} in terms of λ M1
Obtain the given answer A1 [2]
- (ii) *EITHER:* Use correct method to express scalar product of \overrightarrow{OA} and \overrightarrow{OP} , or \overrightarrow{OB} and \overrightarrow{OP} in terms of λ M1
Using the correct method for the moduli, divide scalar products by products of moduli and express $\cos AOP = \cos BOP$ in terms of λ , or in terms of λ and OP M1*
- OR:* Use correct method to express $OA^2 + OP^2 - AP^2$, or $OB^2 + OP^2 - BP^2$ in terms of λ M1
Using the correct method for the moduli, divide each expression by twice the product of the relevant moduli and express $\cos AOP = \cos BOP$ in terms of λ , or λ and OP M1*
- Obtain a correct equation in any form, e.g. $\frac{9 + 2\lambda}{3\sqrt{(9 + 4\lambda + 12\lambda^2)}} = \frac{11 + 14\lambda}{5\sqrt{(9 + 4\lambda + 12\lambda^2)}}$ A1
- Solve for λ M1(dep*)
Obtain $\lambda = \frac{3}{8}$ A1 [5]
- [SR: The M1* can also be earned by equating $\cos AOP$ or $\cos BOP$ to a sound attempt at $\cos \frac{1}{2} AOB$ and obtaining an equation in λ . The exact value of the cosine is $\sqrt{(13/15)}$, but accept non-exact working giving a value of λ which rounds to 0.375, provided the spurious negative root of the quadratic in λ is rejected.]
[SR: Allow a solution reaching $\lambda = \frac{3}{8}$ after cancelling identical incorrect expressions for OP to score 4/5. The marking will run M1M1A0M1A1, or M1M1A1M1A0 in such cases.]
- (iii) Verify the given statement correctly B1 [1]
- 8 (i) Use any relevant method to determine a constant M1
Obtain one of the values $A = 3, B = 4, C = 0$ A1
Obtain a second value A1
Obtain the third value A1 [4]
- (ii) Integrate and obtain term $-3 \ln(2 - x)$ B1√
Integrate and obtain term $k \ln(4 + x^2)$ M1
Obtain term $2 \ln(4 + x^2)$ A1√
Substitute correct limits correctly in a complete integral of the form $a \ln(2 - x) + b \ln(4 + x^2), ab \neq 0$ M1
Obtain given answer following full and correct working A1 [5]

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- 9 (i) Use product rule M1
 Obtain correct derivative in any form A1
 Equate derivative to zero and solve for x M1
 Obtain answer $x = e^{-\frac{1}{2}}$, or equivalent A1
 Obtain answer $y = -\frac{1}{2}e^{-1}$, or equivalent A1 [5]
- (ii) Attempt integration by parts reaching $kx^3 \ln x \pm k \int x^3 \cdot \frac{1}{x} dx$ M1*
 Obtain $\frac{1}{3}x^3 \ln x - \frac{1}{3} \int x^2 dx$, or equivalent A1
 Integrate again and obtain $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3$, or equivalent A1
 Use limits $x = 1$ and $x = e$, having integrated twice M1(dep*)
 Obtain answer $\frac{1}{9}(2e^3 + 1)$, or exact equivalent A1 [5]
- [SR: An attempt reaching $ax^2(x \ln x - x) + b \int 2x(x \ln x - x) dx$ scores M1. Then give the first A1 for $I = x^2(x \ln x - x) - 2I + \int 2x^2 dx$, or equivalent.]
- 10 (a) EITHER: Square $x + iy$ and equate real and imaginary parts to 1 and $-2\sqrt{6}$ respectively M1*
 Obtain $x^2 - y^2 = 1$ and $2xy = -2\sqrt{6}$ A1
 Eliminate one variable and find an equation in the other M1(dep*)
 Obtain $x^4 - x^2 - 6 = 0$ or $y^4 + y^2 - 6 = 0$, or 3-term equivalent A1
 Obtain answers $\pm(\sqrt{3} - i\sqrt{2})$ A1 [5]
- OR: Denoting $1 - 2\sqrt{6}i$ by $R\text{cis}\theta$, state, or imply, square roots are $\pm\sqrt{R}\text{cis}(\frac{1}{2}\theta)$
 and find values of R and either $\cos \theta$ or $\sin \theta$ or $\tan \theta$ M1*
 Obtain $\pm\sqrt{5}(\cos \frac{1}{2}\theta + i \sin \frac{1}{2}\theta)$, and $\cos \theta = \frac{1}{5}$ or $\sin \theta = -\frac{2\sqrt{6}}{5}$ or
 $\tan \theta = -2\sqrt{6}$ A1
 Use correct method to find an exact value of $\cos \frac{1}{2}\theta$ or $\sin \frac{1}{2}\theta$ M1(dep*)
 Obtain $\cos \frac{1}{2}\theta = \pm\sqrt{\frac{3}{5}}$ and $\sin \frac{1}{2}\theta = \pm\sqrt{\frac{2}{5}}$, or equivalent A1
 Obtain answers $\pm(\sqrt{3} - i\sqrt{2})$, or equivalent A1
 [Condone omission of \pm except in the final answers.]
- (b) Show point representing $3i$ on a sketch of an Argand diagram B1
 Show a circle with centre at the point representing $3i$ and radius 2 B1√
 Shade the interior of the circle B1√
 Carry out a complete method for finding the greatest value of $\arg z$ M1
 Obtain answer 131.8° or 2.30 (or 2.3) radians A1 [5]
 [The f.t. is on solutions where the centre is at the point representing $-3i$.]