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<p><b>1</b> <math>k^3 \times \left(\frac{1}{3}x\right)^2 \times 10</math> (or correct factorials)</p> $10 \times k^3 \times \frac{1}{9} = 30 \Rightarrow k = 3$	<p>B2</p> <p>B1</p> <p>[3]</p>	<p>B1 for 2/3 terms correct</p> <p>cao</p>
<p><b>2</b> (i) <math>5[8 + 9 \times 4]</math> 220</p> <p>(ii) <math>\frac{4(2^{10} - 1)}{2 - 1}</math> 4092</p>	<p>M1</p> <p>A1</p> <p>[2]</p> <p>M1</p> <p>A1</p> <p>[2]</p>	<p>Use correct formula with <math>a=4, d=4</math></p> <p>Use correct formula with <math>a=4, r=2</math> or <math>\frac{1}{2}</math></p> <p>4090 without 4092 A0</p>
<p><b>3</b> (i) <math>2x^5 + 3x^2 = 2x \Rightarrow 2x^5 + 3x^2 - 2x = 0</math>  <math>[x(2x^4 + 3x^2 - 2) = 0]</math>  <math>2x^4 + 3x^2 - 2 = 0</math></p> <p>(ii) <math>(x^2 + 2)(2x^2 - 1) = 0</math></p> <p><math>x = \pm \frac{1}{\sqrt{2}}</math> only</p> $\left(\frac{1}{\sqrt{2}}, \frac{2}{\sqrt{2}}\right), \left(\frac{-1}{\sqrt{2}}, \frac{-2}{\sqrt{2}}\right)$	<p>M1</p> <p>A1</p> <p>[2]</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[3]</p>	<p>First line essential</p> <p><b>AG</b> Factorising needed for A1</p> <p>Reasonable attempt at solving a quadratic in <math>x^2</math></p> <p>For a correct pair of solutions, either 2 <math>x^2</math>'s or 1 <math>x</math> and 1 <math>y</math></p> <p>SC (<math>\pm 0.707, \pm 1.41</math>) AWRT B1</p>
<p><b>4</b> (i) <math>10^2 \sin 0.8 = 71.7</math></p> <p>(ii) sector(s) = <math>(2) \times \frac{1}{2} \times 10^2 \times 0.8 = (2) \times 40</math> Total area = 80</p> <p>(iii) arc(s) = <math>(2) \times 10 \times 0.8</math> <math>16 + 20 = 36</math></p>	<p>M1A1</p> <p>[2]</p> <p>M1</p> <p>A1</p> <p>[2]</p> <p>M1</p> <p>A1</p> <p>[2]</p>	<p>Completely correct method for a triangle</p> <p>Correct formula used for a sector</p> <p>Correct formula used for an arc</p>
<p><b>5</b> (i) <math>3\cos^2 x + 8\cos x + 4 = 0</math>  <math>(3\cos x + 2)(\cos x + 2) = 0</math></p> $\cos x = -\frac{2}{3}$ <p>(ii) <math>\cos(\theta + 70) = -\frac{2}{3}, \quad \theta = 61.8</math>  <math>\theta + 70 = 131.8</math> (or 228.2)  <math>\theta = 158.2</math></p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>Use of <math>c^2 + s^2 = 1</math></p> <p>Factorising, formula or completing the square needed</p> <p><b>AG</b> Ignore <math>\cos x = -2</math> also offered</p> <p>SC B1 if <math>-2/3</math> and <math>-2</math> seen</p>

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<p>6 (i) Scalar product = <math>15-8+3</math>  <math>10 =  \mathbf{OA}   \mathbf{OB}  \cos \theta</math>  <math> \mathbf{OA}  = \sqrt{26}</math>, <math> \mathbf{OB}  = \sqrt{38}</math>  Angle <math>BOA = 71.4</math> or <math>71.5</math>  or <math>1.25</math> radians</p> <p>(ii) <math>\mathbf{a} + \frac{1}{2}(\mathbf{b}-\mathbf{a})</math> or <math>\mathbf{b} + \frac{1}{2}(\mathbf{a}-\mathbf{b})</math> or <math>\frac{1}{2}(\mathbf{a}+\mathbf{b})</math>  <math>-2\mathbf{b} + \text{their } \mathbf{c}</math> oe  <math>-6\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}</math></p>	M1 M1 M1 A1 [4]  M1 M1 A2,1,0 [4]	Use of $x_1x_2 + y_1y_2 + z_1z_2$ Correct magnitude for either Linking everything correctly cao    
<p>7 (i) <math>y = m(x - 2)</math> oe</p> <p>(ii) <math>x^2 - 4x + 5 = mx - 2m \Rightarrow x^2 - x(4 + m) + 5 + 2m = 0</math>  <math>(4 + m)^2 - 4(5 + 2m) = 0 \Rightarrow m^2 - 4 = 0</math>  <math>m = \pm 2</math>  <math>m = 2 \Rightarrow x^2 - 6x + 9 = 0 \Rightarrow x = 3</math>    <math>m = -2 \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow x = 1</math>  <math>(3, 2), (1, 2)</math></p> <p><b>OR</b> <math>m = 2x - 4</math>    <math>y = mx - 2m, y = x^2 - 4x + 5</math></p> <p>(iii) <math>(x - 2)^2 + 1, (2, 1)</math></p>	B1 [1]  M1 DM1 A1 DM1 A1 A1 [6]  M1 M1  M1 A1 A1 A1  B1,B1 [2]	Accept $y = mx + c$ , $c = -2m$  Apply $b^2 - 4ac$  Substitute their $m$ and attempt to solve for $x$ Allow for a pair of $x$ values or 1 $x$ and 1 $y$ .  <b>Eliminating 2 variables from 3 equations.</b> <b>Obtaining a quadratic in <math>x</math> or <math>y</math>.</b>  <b>Solving their quadratic correctly.</b> A pair of $x$ values or 1 $x$ and 1 $y$ ..  $m=2, -2$ also needed for final mark.  

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<p><b>8 (i)</b> <math>f'(3) = 0 \Rightarrow 18 + 3k - 12 = 0</math>  <math>k = -2</math>  <math>(x - 3)(x + 2) = 0</math>  <math>x = -2</math>, (Allow also = 3)</p> <p><b>(ii)</b> <math>f'(x) = 4x - 2</math>  <math>f'(3) &gt; 0</math> hence min at <math>P</math>  <math>f'(-2) &lt; 0</math> hence max at <math>Q</math></p> <p><b>(iii)</b> <math>f(x) = \frac{2}{3}x^3 - x^2 - 12x (+ c)</math>  Sub <math>(3, -10) \rightarrow -10 = 18 - 9 - 36 + c</math>  <math>c = 17</math></p>	M1 A1 M1 A1  [4]  B1  B1  [2]  B2,1,0 M1 A1  [4]	AG   3 min, -2 max independent of $f'(x)$  Accept anywhere in question Dependent on $c$ present Condone $y =$ , or equation =
<p><b>9 (i)</b> <math>f^{-1}(x) = \frac{1}{2}x - \frac{3}{2}</math>    <math>2x + 3 = \frac{1}{2}x - \frac{3}{2} \Rightarrow x = -3</math></p> <p><b>(ii)</b> 2 lines approximately correct,  reflected in <math>y=x</math> &amp; meeting at <math>(-3, -3)</math></p> <p><b>(iii)</b> <math>gf(x) = (2x + 3)^2 - 6(2x + 3)</math>  <math>4x^2 - 9</math>  <math>4x^2 - 9 \leq 16 \Rightarrow x^2 \leq \frac{25}{4}</math>    <math>-\frac{5}{2} \leq x \leq 0</math></p>	B1  M1A1  [3]   B3,2,1,0  [3]  M1 A1  M1  A1A1  [5]	   Can be implied by graph or in writing. Ignore lines extended   Solving any quadratic to do with $f$ and $g \leq 16$ , to $x =$  Condone $<$ and $>$

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<p>10 (i) <math>\int (x+1)^{\frac{1}{2}} - (x+1)</math> or <math>\int (y^2 - 1) - (y - 1)</math></p> <p><math>\frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{1}{2}x^2 - x</math> or <math>\frac{1}{3}y^2 - \frac{1}{2}y^2</math></p> <p><math>\frac{2}{3} - \left(0 - \frac{1}{2} + 1\right)</math> or <math>\frac{1}{3} - \frac{1}{2}</math></p> <p><math>\frac{1}{6}</math></p>	<p>M1</p> <p>M1A1</p> <p>DM1</p> <p>A1</p> <p>[5]</p>	<p>Dealing with line as a triangle or integral with correct limits.</p> <p>Attempt at integral of curve.</p> <p>Applying limits <math>-1 \rightarrow 0</math> or <math>0 \rightarrow 1</math> to curve</p> <p><math>\pi</math> included loses last mark.</p>
<p>(ii) <math>V_1 = (\pi) \int (y^2 - 1)^2 = (\pi) \int y^4 - 2y^2 + 1</math></p> <p><math>(\pi) \left[ \frac{y^5}{5} - \frac{2y^2}{3} + y \right]</math></p> <p><math>(\pi) \left[ \frac{1}{5} - \frac{2}{3} + 1 \right]</math></p> <p><math>V_1 = \frac{8}{15(\pi)}</math> or 0.533(<math>\pi</math>) (AWRT)</p>	<p>M1</p> <p>A1</p> <p>DM1</p> <p>A1</p>	<p>Attempt at <math>\int x^2 dy</math> for curve</p> <p>Apply limits <math>0 \rightarrow 1</math></p>
<p>or <math>(\pi) \left[ \frac{y^3}{3} - y^2 + y \right]</math></p> <p><math>V_2 = \frac{1}{3}\pi</math></p> <p>Volume = <math>\frac{8}{15}\pi - \frac{1}{3}\pi = \frac{1}{5}\pi</math> (or 0.628)</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>[7]</p>	<p>Or <math>\frac{1}{3} \times \pi (\times 1^2 \times 1)</math></p> <p>Vol of cone or attempt to <math>\int x^2 dy</math> for line</p>
<p>OR <math>(y^4 - 2y^2 + 1) - (y^2 - 2y + 1)</math></p> <p><math>(\pi) \int y^4 - 3y^2 + 2y</math></p> <p><math>(\pi) [y^{\uparrow} 5 / 5 - y^{\uparrow} 3 + y^{\uparrow} 2]</math></p> <p><math>(\pi) \left[ \frac{1}{5} - 1 + 1 \right]</math></p> <p><math>\frac{1}{5}\pi</math></p>	<p>M1</p> <p>M1</p> <p>A1,A1,A1</p> <p>DM1</p> <p>A1</p>	<p>Attempt to <math>\int x^2 dy</math></p> <p>Attempt to <math>\int (x_1^2 - x_2^2)</math></p> <p>Apply limits <math>0 \rightarrow 1</math> dependent on first M1</p>

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$\int_{-1}^0 x+1 - \int_{-1}^0 (x+1)^2$ $\left[ \frac{x^2}{2} + x \right] - \left[ \frac{x+1^3}{3} \right]$ $\text{SC} = \left[ (0) - \left( \frac{1}{2} - 1 \right) \right] - \left[ \frac{1}{3} - 0 \right]$ $\frac{1}{2} - \frac{1}{3} = \frac{1}{6} \pi \quad (0.524)$	M1  M1  A1	SC MR integrating about x axis  Use of -1,0 as limits
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