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- 1 EITHER: State or imply non-modular inequality $(2(x-3))^2 > (3x+1)^2$, or corresponding quadratic equation, or pair of linear equations $2(x-3) = \pm(3x+1)$ B1
 Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations M1
 Obtain critical values $x = -7$ and $x = 1$ A1
 State answer $-7 < x < 1$ A1
- OR: Obtain critical value $x = -7$ or $x = 1$ from a graphical method, or by inspection, or by solving a linear equation or inequality B1
 Obtain critical values $x = -7$ and $x = 1$ B2
 State answer $-7 < x < 1$ B1 [4]
 [Do not condone: $<$ for $<.$]
- 2 Use law for the logarithm of a power, a quotient, or a product correctly at least once M1
 Use $\ln e = 1$ or $e = \exp(1)$ M1
 Obtain a correct equation free of logarithms, e.g. $1 + x^2 = ex^2$ A1
 Solve and obtain answer $x = 0.763$ only A1 [4]
 [For the solution $x = 0.763$ with no relevant working give B1, and a further B1 if 0.763 is shown to be the only root.]
 [Treat the use of logarithms to base 10 with answer 0.333 only, as a misread.]
 [SR: Allow iteration, giving B1 for an appropriate formula, e.g. $x_{n+1} = \exp((\ln(1 + x_n^2) - 1)/2)$, M1 for using it correctly once, A1 for 0.763, and A1 for showing the equation has no other root but 0.763.]
- 3 Attempt use of $\cos(A+B)$ formula to obtain an equation in $\cos \theta$ and $\sin \theta$ M1
 Use trig formula to obtain an equation in $\tan \theta$ (or $\cos \theta$, $\sin \theta$ or $\cot \theta$) M1
 Obtain $\tan \theta = 1/(4 + \sqrt{3})$ or equivalent (or find $\cos \theta$, $\sin \theta$ or $\cot \theta$) A1
 Obtain answer $\theta = 9.9^\circ$ A1
 Obtain $\theta = 189.9^\circ$, and no others in the given interval A1 [5]
 [Ignore answers outside the given interval. Treat answers in radians as a misread (0.173, 3.31).]
 [The other solution methods are *via* $\cos \theta = \pm(4 + \sqrt{3}) / \sqrt{1 + (4 + \sqrt{3})^2}$ or $\sin \theta = \pm 1 / \sqrt{1 + (4 + \sqrt{3})^2}$.]
- 4 (i) Make recognisable sketch of a relevant graph over the given range B1
 Sketch the other relevant graph on the same diagram and justify the given statement B1 [2]
- (ii) Consider sign of $4x^2 - 1 - \cot x$ at $x = 0.6$ and $x = 1$, or equivalent M1
 Complete the argument correctly with correct calculated values A1 [2]
- (iii) Use the iterative formula correctly at least once M1
 Obtain final answer 0.73 A1
 Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (0.725, 0.735) A1 [3]

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- 5 (i) State or imply $dx = 2 \cos \theta d\theta$, or $\frac{dx}{d\theta} = 2 \cos \theta$, or equivalent B1
 Substitute for x and dx throughout the integral M1
 Obtain the given answer correctly, having changed limits and shown sufficient working A1 [3]
- (ii) Replace integrand by $2 - 2 \cos 2\theta$, or equivalent B1
 Obtain integral $2\theta - \sin 2\theta$, or equivalent B1√
 Substitute limits correctly in an integral of the form $a\theta \pm b \sin 2\theta$, where $ab \neq 0$ M1
 Obtain answer $\frac{1}{3}\pi - \frac{\sqrt{3}}{2}$ or exact equivalent A1 [4]
 [The f.t. is on integrands of the form $a + c \cos 2\theta$, where $ac \neq 0$.]
- 6 (i) State modulus is 2 B1
 State argument is $\frac{1}{6}\pi$, or 30° , or 0.524 radians B1 [2]
- (ii) (a) State answer $3\sqrt{3} + i$ B1
- (b) EITHER: Multiply numerator and denominator by $\sqrt{3} - i$, or equivalent M1
 Simplify denominator to 4 or numerator to $2\sqrt{3} + 2i$ A1
 Obtain final answer $\frac{1}{2}\sqrt{3} + \frac{1}{2}i$, or equivalent A1
 OR 1: Obtain two equations in x and y and solve for x or for y M1
 Obtain $x = \frac{1}{2}\sqrt{3}$ or $y = \frac{1}{2}$ A1
 Obtain final answer $\frac{1}{2}\sqrt{3} + \frac{1}{2}i$, or equivalent A1
 OR 2: Using the correct processes express iz^*/z in polar form M1
 Obtain $x = \frac{1}{2}\sqrt{3}$ or $y = \frac{1}{2}$ A1
 Obtain final answer $\frac{1}{2}\sqrt{3} + \frac{1}{2}i$, or equivalent A1 [4]
- (iii) Plot A and B in relatively correct positions B1
 EITHER: Use fact that angle $AOB = \arg(iz^*) - \arg z$ M1
 Obtain the given answer A1
 OR 1: Obtain $\tan \hat{AOB}$ from gradients of OA and OB and the correct $\tan(A - B)$ formula M1
 Obtain the given answer A1
 OR 2: Obtain $\cos \hat{AOB}$ by using correct cosine formula or scalar product M1
 Obtain the given answer A1 [3]

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- 7 (i) State correct equation in any form, e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k} + \lambda(2\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$ B1 [1]
- (ii) *EITHER*: Equate a relevant scalar product to zero and form an equation in λ M1
OR 1: Equate derivative of OP^2 (or OP) to zero and form an equation in λ M1
OR 2: Use Pythagoras in OAP or OBP and form an equation in λ M1
 State a correct equation in any form A1
 Solve and obtain $\lambda = -\frac{1}{6}$ or equivalent A1
 Obtain final answer $\overline{OP} = \frac{2}{3}\mathbf{i} + \frac{5}{3}\mathbf{j} + \frac{7}{3}\mathbf{k}$, or equivalent A1 [4]
- (iii) *EITHER*: State or imply \overline{OP} is a normal to the required plane M1
 State normal vector $2\mathbf{i} + 5\mathbf{j} + 7\mathbf{k}$, or equivalent A1√
 Substitute coordinates of a relevant point in $2x + 5y + 7z = d$ and evaluate d M1
 Obtain answer $2x + 5y + 7z = 26$, or equivalent A1
OR 1: Find a vector normal to plane AOB and calculate its vector product with a direction vector for the line AB M1*
 Obtain answer $2\mathbf{i} + 5\mathbf{j} + 7\mathbf{k}$, or equivalent A1
 Substitute coordinates of a relevant point in $2x + 5y + 7z = d$ and evaluate d M1(dep*)
 Obtain answer $2x + 5y + 7z = 26$, or equivalent A1
OR 2: Set up and solve simultaneous equations in a, b, c derived from zero scalar products of $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ with (i) a direction vector for line AB , (ii) a normal to plane OAB M1*
 Obtain $a : b : c = 2 : 5 : 7$, or equivalent A1
 Substitute coordinates of a relevant point in $2x + 5y + 7z = d$ and evaluate d M1(dep*)
 Obtain answer $2x + 5y + 7z = 26$, or equivalent A1
OR 3: With $Q(x, y, z)$ on plane, use Pythagoras in OPQ to form an equation in x, y and z M1*
 Form a correct equation A1√
 Reduce to linear form M1(dep*)
 Obtain answer $2x + 5y + 7z = 26$, or equivalent A1
OR 4: Find a vector normal to plane AOB and form a 2-parameter equation with relevant vectors, e.g., $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k} + \lambda(2\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}) + \mu(8\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})$ M1*
 State three correct equations in x, y, z, λ and μ A1
 Eliminate λ and μ M1(dep*)
 Obtain answer $2x + 5y + 7z = 26$, or equivalent A1 [4]

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- 8 (i) State or imply the form $\frac{A}{1+x} + \frac{Bx+C}{1+2x^2}$ B1
 Use any relevant method to evaluate a constant M1
 Obtain one of $A = -1, B = 2, C = 1$ A1
 Obtain a second value A1
 Obtain the third value A1 [5]
- (ii) Use correct method to obtain the first two terms of the expansion of $(1+x)^{-1}$ or $(1+2x^2)^{-1}$ M1
 Obtain correct expansion of each partial fraction as far as necessary A1√ + A1√
 Multiply out fully by $Bx + C$, where $BC \neq 0$ M1
 Obtain answer $3x - 3x^2 - 3x^3$ A1 [5]
- [Symbolic binomial coefficients, e.g., $\binom{-1}{1}$ are not sufficient for the first M1. The f.t. is on A, B, C .]
 [If B or C omitted from the form of fractions, give B0M1A0A0A0 in (i); M1A1√A1√ in (ii), max 4/10.]
 [If a constant D is added to the correct form, give M1A1A1A1 and B1 if and only if $D = 0$ is stated.]
 [If an extra term $D/(1+2x^2)$ is added, give B1M1A1A1, and A1 if $C + D = 1$ is resolved to $1/(1+2x^2)$.]
 [In the case of an attempt to expand $3x(1+x)^{-1}(1+2x^2)^{-1}$, give M1A1A1 for the expansions up to the term in x^2 , M1 for multiplying out fully, and A1 for the final answer.]
 [For the identity $3x \equiv (1+x+2x^2+2x^3)(a+bx+cx^2+dx^3)$ give M1A1; then M1A1 for using a relevant method to find two of $a = 0, b = 3, c = -3$ and $d = -3$; and then A1 for the final answer in series form.]
- 9 (i) Use correct product rule M1
 Obtain correct derivative in any form A1
 Equate derivative to zero and find non-zero x M1
 Obtain $x = \exp(-\frac{1}{3})$, or equivalent A1
 Obtain $y = -1/(3e)$, or any ln-free equivalent A1 [5]
- (ii) Integrate and reach $kx^4 \ln x + l \int x^4 \cdot \frac{1}{x} dx$ M1
 Obtain $\frac{1}{4}x^4 \ln x - \frac{1}{4} \int x^3 dx$ A1
 Obtain integral $\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4$, or equivalent A1
 Use limits $x = 1$ and $x = 2$ correctly, having integrated twice M1
 Obtain answer $4 \ln 2 - \frac{15}{16}$, or exact equivalent A1 [5]

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- 10 (i)** State or imply $\frac{dx}{dt} = k(20 - x)$ B1
 Show that $k = 0.05$ B1 [2]
- (ii)** Separate variables correctly and integrate both sides B1
 Obtain term $-\ln(20 - x)$, or equivalent B1
 Obtain term $\frac{1}{20}t$, or equivalent B1
 Evaluate a constant or use limits $t = 0, x = 0$ in a solution containing terms $a \ln(20 - x)$ and bt M1*
 Obtain correct answer in any form, e.g. $\ln 20 - \ln(20 - x) = \frac{1}{20}t$ A1 [5]
- (iii)** Substitute $t = 10$ and calculate x M1(dep*)
 Obtain answer $x = 7.9$ A1 [2]
- (iv)** State that x approaches 20 B1 [1]