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1	$\int \left(x + \frac{1}{x}\right)^2 dx$ $= \frac{x^3}{3} - \frac{1}{x} + 2x + (c)$	B1 × 3	[3]	co. Omission of middle term of expansion can still get 2/3.
2	$(1 + ax)^6$ Term in $x = 6ax$ Equate with $-30 \rightarrow a = -5$ Term in $x^3 = \frac{6.5.4}{3!} a^3$ \rightarrow coefficient of -2500	B1 B1√ B1 B1√	[4]	co √ from his answer for $6ax$ co For $20 \times a^3$
3	$f: x \mapsto 2x + 3,$ $g: x \mapsto x^2 - 2x,$ $gf(x) = (2x + 3)^2 - 2(2x + 3)$ $= 4x^2 + 8x + 3$ $= 4(x + 1)^2 - 1$	M1 A1 $3 \times B1\sqrt$	[5]	Must be f into g, not g into f. co Allow all these as √ for either fg or gf.
4	(i) $\frac{\sin x \tan x}{1 - \cos x} = \frac{\sin^2 x}{\cos x(1 - \cos x)}$ $= \frac{1 - \cos^2 x}{\cos x(1 - \cos x)}$ $= \frac{(1 - \cos x)(1 + \cos x)}{\cos x(1 - \cos x)} = \frac{1}{\cos x} + 1$ (ii) $\frac{1}{\cos x} + 1 + 2 = 0$ $\rightarrow \cos x = -\frac{1}{3}$ $\rightarrow x = 109.5^\circ$ or 250.5°	M1 M1 M1 M1 A1 A1√	[3] [3]	Use of $\tan x = \sin x \div \cos x$ Use of $\sin^2 x = 1 - \cos^2 x$ Realising the need to use difference of 2 squares. Answer given. Uses part (i) with $\cos x$ as subject. co. √ for $360^\circ - 1^{\text{st}}$ answer.
5	$\vec{AC} = -6\mathbf{i} + 10\mathbf{k}$ $\vec{BC} = -8\mathbf{j} + 10\mathbf{k}$ $\vec{AC} \cdot \vec{BC} = 100$ $\vec{AC} \cdot \vec{BC} = \sqrt{136}\sqrt{164} \cos ACB$ Angle $ACB = 48.0^\circ$	B1 B1 M1 M1 M1 A1	[6]	co (or \vec{CA}) co (or \vec{CB}) Must be scalar – available for any pair For modulus – available for any vector All linked correctly – for ACB only co

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6	(a) $a + 4d = 18$ $\frac{5}{2}(2a + 4d) = 75$ Solution $\rightarrow a = 12, d = 1\frac{1}{2}$	B1 B1 M1 A1	[4]	co or $75 = 5/2(a + 18) \rightarrow a = 12$ etc co Solution of sim equations co for both
	(b) $a = 16$ and $ar^3 = \frac{27}{4}$ $r = \frac{3}{4}$ Sum to infinity = 64	B1 M1 A1	[3]	Needs both of these Correct formula and $ r < 1$
7	$x \mapsto 3 - 2 \tan(\frac{1}{2}x)$ (i) Range of $f \leq 3$	B1	[1]	co. Allow $<$
	(ii) $f(\frac{2}{3}\pi) = 3 - 2\sqrt{3}$	B1	[1]	co
	(iii)	B2, 1, 0 Indep.	[2]	Starting at $y = 3$ Shape correct – no turning points. Tending tangentially towards $x = \pi$
	(iv) $y = 3 - 2 \tan\left(\frac{x}{2}\right)$ $\rightarrow f^{-1}(x) = 2 \tan^{-1}\left(\frac{3-x}{2}\right)$	M1 M1 A1	[3]	Attempt at making x the subject. Order of operations all ok. co – but with x , not y .
8	(i) $2x + 2y + \frac{\pi x}{2} = 60$ $\rightarrow y = 30 - x - \frac{\pi x}{4}$	M1 A1	[2]	Linking 60 with sum of at least 4 sides and use of radians co
	(ii) $A = xy + \frac{\pi x^2}{4}$ $= x(30 - x - \frac{\pi x}{4}) + \frac{\pi x^2}{4}$ $= 30x - x^2$	M1 A1	[2]	Subs “y” into area eqn and use $\frac{1}{2}r^2\theta$ co.
	(iii) $\frac{dA}{dx} = 30 - 2x$ $= 0$ when $x = 15$ cm	M1 A1	[2]	Knowing to differentiate Sets differential to 0 + solution. co.
	(iv) Max.	M1 A1	[2]	Any valid method. co.

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9	(i) $RS^2 = 10^2 - 6^2$ $\rightarrow RS = 8 \text{ cm.}$	M1 A1	[2]	Use of Pythagoras (or other) Answer given.
	(ii) $\sin \theta = 8/10$ oe \rightarrow angle $RPQ = 0.9273$ radians	M1 A1	[2]	Use of trig – even if with degrees. co in radians. (Accept 0.927)
	(iii) Region = trapezium – 2 sectors Area of trapezium = 40 cm^2 Large sector = $\frac{1}{2} \times 8^2 \times 0.9273$ Small sector angle = $(\pi - 0.9273)$ Small sector = $\frac{1}{2} \times 2^2 \times 2.214$ $\rightarrow 5.90 \text{ cm}^2$	B1 M1 M1 A1	 [4]	co Use of $\frac{1}{2}r^2\theta$. Use of $\frac{1}{2}r^2\theta$ with angle = $\pi -$ (ii) co
10	$y = 4x - x^2 + 3$			
	(i) $\frac{dy}{dx} = 4 - 2x$ At $x = 3$, $m = -2$ Gradient of normal = $\frac{1}{2}$ Eqn of normal $y - 6 = \frac{1}{2}(x - 3)$ $\rightarrow 2y = x + 9$	B1 M1 M1 A1	 [4]	co Use of $m_1m_2 = -1$ Use of $y - k = m(x - h)$ or $y = mx + c$ (where m is gradient of normal)
	(ii) Meets axes at $(0, \frac{9}{2})$ and $(-9, 0)$ Mid-point is $(\frac{-9}{2}, \frac{9}{4})$	M1 A1	 [2]	Sets x and y to 0 + midpoint formula. co.
(iii) $2y = x + 9$, $y = 4x - x^2 + 3$ $\rightarrow 2x^2 - 7x + 3 = 0$ oe $\rightarrow (\frac{1}{2}, \frac{3}{4})$	M1 A1 M1 A1	 [4]	Eliminates x completely. Correct eqn. Solution of quadratic. co	

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11	$y = \frac{9}{2-x}$ <p>(i) $\frac{dy}{dx} = -9(2-x)^{-2} \times -1$</p> $\frac{9}{(2-x)^2} \neq 0. \text{ No turning points.}$ <p>(ii) $V = \pi \int \frac{81}{(2-x)^2} dx$</p> $\int y^2 dx = -81(2-x)^{-1} \div (-1)$ <p>Use of limits 0 to 1</p> $\rightarrow \frac{81\pi}{2} \text{ (or 127)}$ <p>(iii) $\frac{9}{2-x} = x + k$</p> $\rightarrow x^2 - 2x + kx - 2k + 9 = 0$ <p>Uses $b - 4ac$</p> $\rightarrow k^2 + 4k - 32$ <p>\rightarrow end-points of 4 and -8</p> <p>Range for 2 points of intersection</p> $\rightarrow k < -8, k > 4.$	<p>B1 B1</p> <p>B1√</p> <p>[3]</p> <p>B1 B1</p> <p>M1</p> <p>A1</p> <p>[4]</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[4]</p>	<p>Without the “$\times -1$” Indep. With the “$\times -1$”. Indep.</p> <p>√ provided of form $k \div (2-x)^2$.</p> <p>Answer without the “$\div -1$” including π For “$\div -1$”.</p> <p>Uses both limits in an integral of y^2 – if “0” ignored, M0.</p> <p>co (If π omitted – max 3/4)</p> <p>Elimination of y</p> <p>Uses discriminant</p> <p>End-values correct.</p> <p>Accept \leq, \geq.</p>
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