| Question | Answer | Marks | Guidance |
| :---: | :--- | ---: | :--- |
| 1 | State or imply ordinates $3,2,0,4$ | $\mathbf{B 1}$ | These and no more <br> Accept in unsimplified form $\left\|2^{0}-4\right\|$ etc. |
|  | Use correct formula, or equivalent, with $h=1$ and four ordinates | $\mathbf{M 1}$ |  |
|  | Obtain answer 5.5 | $\mathbf{A 1}$ |  |
|  |  | $\mathbf{3}$ |  |


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| :---: | :---: | :---: | :---: |
| 2 | Use law for the logarithm of a product, quotient or power | M1 | Condone $\ln \frac{x}{x-1}$ for M 1 |
|  | Obtain a correct equation free of logarithms | A1 | e.g. $(2 x-3)(x-1)=x^{2}$ or $x^{2}-5 x+3=0$ |
|  | Solve a 3-term quadratic obtaining at least one root | M1 | Must see working if using an incorrect quadratic $\left(\frac{5 \pm \sqrt{13}}{2}\right)$ |
|  | Obtain answer $x=4.30$ only | A1 | Q asks for 2 dp . Do not ISW. Overspecified answers score A0 Overspecified and no working can score M1A0 |
|  |  | 4 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 3 | State or imply $3 y^{2}+6 x y \frac{\mathrm{~d} y}{\mathrm{~d} x}$ as derivative of $3 x y^{2}$ | B1 |  |
|  | State or imply $3 y^{2} \frac{\mathrm{~d} y}{\mathrm{~d} x}$ as derivative of $y^{3}$ | B1 |  |
|  | Equate derivative of LHS to zero, substitute $(1,3)$ and find the gradient | M1 | $\left(\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{x^{2}+y^{2}}{y^{2}-2 x y}\right)$ For incorrect derivative need to see the substitution |
|  | Obtain final answer $\frac{10}{3}$ or equivalent | A1 | 3.33 or better. Allow $\frac{30}{9}$ ISW after correct answer seen |
|  |  | 4 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 4 | Use correct trig formula and obtain an equation in $\tan \theta$ | M1 | Allow with $45^{\circ}$ e.g. $\frac{1}{\tan \theta}-\frac{1}{\frac{\tan \theta+\tan 45^{\circ}}{1-\tan \theta \tan 45^{\circ}}}=3$ |
|  | Obtain a correct horizontal equation in any form | A1 | e.g. $1+\tan \theta-\tan \theta(1-\tan \theta)=3 \tan \theta(1+\tan \theta)$ |
|  | Reduce to $2 \tan ^{2} \theta+3 \tan \theta-1=0$ | A1 | or 3-term equivalent |
|  | Solve 3-term quadratic and find a value of $\theta$ | M1 | Must see working if using an incorrect quadratic |
|  | Obtain answer $15.7^{\circ}$ | A1 | One correct solution (degrees to at least 3 sf ) |
|  | Obtain answer 119.(3) ${ }^{\circ}$ | A1 | Second correct solution and no others in range (degrees to at least 3 sf ) <br> Mark 0.274, 2.082 as MR: A0A1 |
|  |  | 6 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 5(i) | Use chain rule | M1 | $\begin{aligned} & k \cos \theta \sin ^{-3} \theta\left(=-k \operatorname{cosec}^{2} \theta \cot \theta\right) \\ & \text { Allow M1 for }-2 \cos \theta \sin ^{-1} \theta \end{aligned}$ |
|  | Obtain correct answer in any form | A1 | $\text { e.g. }-2 \operatorname{cosec}^{2} \theta \cot \theta, \frac{-2 \cos \theta}{\sin ^{3} \theta} \text { Accept } \frac{-2 \sin \theta \cos \theta}{\sin ^{4} \theta}$ |
|  |  | 2 |  |
| 5(ii) | Separate variables correctly and integrate at least one side | B1 | $\int x \mathrm{~d} x=\int-\operatorname{cosec}^{2} \theta \cot \theta \mathrm{~d} \theta$ |
|  | Obtain term $\frac{1}{2} x^{2}$ | B1 |  |
|  | Obtain term of the form $\frac{k}{\sin ^{2} \theta}$ | M1* | or equivalent |
|  | Obtain term $\frac{1}{2 \sin ^{2} \theta}$ | A1 | or equivalent |
|  | Use $x=4, \theta=\frac{1}{6} \pi$ to evaluate a constant, or as limits, in a solution with terms $a x^{2}$ and $\frac{b}{\sin ^{2} \theta}$, where $a b \neq 0$ | DM1 | Dependent on the preceding M1 |
|  | Obtain solution $x=\sqrt{\left(\operatorname{cosec}^{2} \theta+12\right)}$ | A1 | or equivalent |
|  |  | 6 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 6(i) | State correct expansion of $\sin (2 x+x)$ | B1 |  |
|  | Use trig formulae and Pythagoras to express $\sin 3 x$ in terms of $\sin x$ | M1 |  |
|  | Obtain a correct expression in any form | A1 | e.g. $2 \sin x\left(1-\sin ^{2} x\right)+\sin x\left(1-2 \sin ^{2} x\right)$ |
|  | Obtain $\sin 3 x \equiv 3 \sin x-4 \sin ^{3} x$ correctly AG | A1 | Accept $=$ for $\equiv$ |
|  |  | 4 |  |
| 6(ii) | Use identity, integrate and obtain $-\frac{3}{4} \cos x+\frac{1}{12} \cos 3 x$ | B1 B1 | One mark for each term correct |
|  | Use limits correctly in an integral of the form $a \cos x+b \cos 3 x$, where $a b \neq 0$ | M1 | $\left(-\frac{3}{8}-\frac{1}{12}+\frac{3}{4}-\frac{1}{12}=-\frac{11}{24}+\frac{2}{3}\right)$ |
|  | Obtain answer $\frac{5}{24}$ | A1 | Must be exact. Accept simplified equivalent e.g. $\frac{15}{72}$ Answer only with no working is $0 / 4$ |
|  |  | 4 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 7(i) | State at least one correct derivative | B1 | $-2 \sin \frac{1}{2} x, \frac{1}{(4-x)^{2}}$ |
|  | Equate product of derivatives to - 1 | M1 | or equivalent |
|  | Obtain a correct equation, e.g. $2 \sin \frac{1}{2} x=(4-x)^{2}$ | A1 |  |
|  | Rearrange correctly to obtain $a=4-\sqrt{2 \sin \frac{a}{2}}$ <br> AG | A1 |  |
|  |  | 4 |  |
| 7(ii) | Calculate values of a relevant expression or pair of expressions at $a=2$ and $a=3$ | M1 | $\text { e.g. } \begin{array}{lc} a=2 & 2<2.7027 . . \\ a=3 & 3>2.587 . . \end{array}\binom{0.703}{-0.412}\binom{2.317}{-0.995}$ <br> Values correct to at least 2 dp |
|  | Complete the argument correctly with correct calculated values | A1 |  |
|  |  | 2 |  |
| 7(iii) | Use the iterative formula $a_{n+1}=4-\sqrt{\left(2 \sin \frac{1}{2} a_{n}\right)}$ correctly at least once | M1 |  |
|  | Obtain final answer 2.611 | A1 |  |
|  | Show sufficient iterations to 5 d.p. to justify 2.611 to 3 d.p., or show there is a sign change in the interval $(2.6105,2.6115)$ | A1 | $\begin{aligned} & 2,2.70272,2.60285,2.61152,2.61070,2.61077 \\ & 2.5,2.62233,2.60969,2.61087,2.61076 \\ & 3,2.58756,2.61301,2.61056,2.61079 \end{aligned}$ <br> Condone truncation. Accept more than 5 dp |
|  |  | 3 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 8(i) | State or imply the form $\frac{A}{2+x}+\frac{B}{3-x}+\frac{C}{(3-x)^{2}}$ | B1 |  |
|  | Use a correct method to obtain a constant | M1 |  |
|  | Obtain one of $A=2, B=2, C=-7$ | A1 |  |
|  | Obtain a second value | A1 |  |
|  | Obtain the third value | A1 | [Mark the form $\frac{A}{2+x}+\frac{D x+E}{(3-x)^{2}}$, where $A=2, D=-2$ and $E=-1, \mathrm{~B} 1 \mathrm{M} 1 \mathrm{~A} 1 \mathrm{~A} 1 \mathrm{~A} 1$. |
|  |  | 5 |  |
| 8(ii) | Use a correct method to find the first two terms of the expansion of $(2+x)^{-1},(3-x)^{-1}$ or $(3-x)^{-2}$, or equivalent, e.g. $\left(1+\frac{1}{2} x\right)^{-1}$ | M1 |  |
|  | Obtain correct unsimplified expansions up to the term in $x^{2}$ of each partial fraction | $\begin{aligned} & \text { A1 } \\ & \text { A1 } \\ & \text { A1 } \end{aligned}$ | FT on $A, B$ and $C$ $1-\frac{x}{2}+\frac{x^{2}}{4} \frac{2}{3}\left(1+\frac{x}{3}+\frac{x^{2}}{9}\right)-\frac{7}{9}\left(1+\frac{2 x}{3}+\frac{3 x^{2}}{9}\right)$ |
|  | Obtain final answer $\frac{8}{9}-\frac{43}{54} x+\frac{7}{108} x^{2}$ | A1 |  |
|  |  |  | For the $A, D, E$ form of fractions give M1A1ftA1ft for the expanded partial fractions, then, if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer. |
|  |  | 5 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 9(i) | Obtain a vector parallel to the plane, e.g. $\overrightarrow{C B}=2 \mathbf{i}+\mathbf{j}$ | B1 |  |
|  | Use scalar product to obtain an equation in $a, b, c$, | M1 | e.g. $2 a+b=0, a+5 c=0, a+b-5 c=0$ |
|  | Obtain two correct equations in $a, b, c$ | A1 |  |
|  | Solve to obtain $a: b: c$, | M1 | or equivalent |
|  | Obtain $a: b: c=5:-10:-1$, | A1 | or equivalent |
|  | Obtain equation $5 x-10 y-z=-25$, | A1 | or equivalent |
|  | Alternative method 1 |  |  |
|  | Obtain a vector parallel to the plane, e.g. $\overrightarrow{C D}=\mathbf{i}+5 \mathbf{k}$ | B1 | $\overrightarrow{B D}=-\mathbf{i}-\mathbf{j}+5 \mathbf{k}$ |
|  | Obtain a second such vector and calculate their vector product, e.g. $(2 \mathbf{i}+\mathbf{j}) \times(\mathbf{i}+5 \mathbf{k})$ | M1 |  |
|  | Obtain two correct components | A1 |  |
|  | Obtain correct answer, e.g. $5 \mathbf{i}-10 \mathbf{j}-\mathbf{k}$ | A1 |  |
|  | Substitute to find $d$ | M1 |  |
|  | Obtain equation $5 x-10 y-z=-25$, | A1 | or equivalent |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 9(i) | Alternative method 2 |  |  |
|  | Obtain a vector parallel to the plane, e.g. $\overrightarrow{D B}=\mathbf{i}+\mathbf{j}-5 \mathbf{k}$ | B1 |  |
|  | Obtain a second such vector and form correctly a 2-parameter equation for the plane | M1 |  |
|  | State a correct equation, e.g. $\mathbf{r}=3 \mathbf{i}+4 \mathbf{j}+\lambda(\mathbf{i}+5 \mathbf{k})+\mu(\mathbf{i}+\mathbf{j}-5 \mathbf{k})$ | A1 |  |
|  | State three equations in $x, y, z, \lambda$ and $\mu$ | A1 |  |
|  | Eliminate $\lambda$ and $\mu$ | M1 |  |
|  | Obtain equation $5 x-10 y-z=-25$ | A1 | or equivalent |
|  | Alternative method 3 |  |  |
|  | Substitute for $B$ and $C$ and obtain $3 a+4 b=d$ and $a+3 b=d$ | B1 |  |
|  | Substitute for $D$ to obtain a third equation and eliminate one unknown ( $a, b$, or $d$ ) entirely | M1 |  |
|  | Obtain two correct equations in two unknowns, e.g. $a, b, c$ | A1 |  |
|  | Solve to obtain their ratio, e.g. $a: b: c$ | M1 |  |
|  | Obtain $a: b: c=5:-10:-1$, $a: c: d=5:-1:-25$, or $b: c: d=10: 1: 25$ | A1 | or equivalent |
|  | Obtain equation $5 x-10 y-z=-25$ | A1 | or equivalent |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 9(i) | Alternative method 4 |  |  |
|  | Substitute for $B$ and $C$ and obtain $3 a+4 b=d$ and $a+3 b=d$ | B1 |  |
|  | Solve to obtain $a: b: d$ | M2 | or equivalent |
|  | Obtain $a: b: d=1:-2:-5$ | A1 | or equivalent |
|  | Substitute for $C$ to obtain $c$ | M1 |  |
|  | Obtain equation $5 x-10 y-z=-25$ | A1 | or equivalent |
|  |  | 6 |  |
| 9(ii) | State or imply a normal vector for the plane $O A B C$ is $\mathbf{k}$ | B1 |  |
|  | Carry out correct process for evaluating a scalar product of two relevant vectors, e.g. ( $5 \mathbf{i}-10 \mathbf{j}-\mathbf{k}$ ).(k) | M1 | i.e. correct process using $\mathbf{k}$ and their normal |
|  | Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result | M1 | Allow M1M1 for clear use of an incorrect vector that has been stated to be the normal to $O A B C$ |
|  | Obtain answer $84.9^{\circ}$ or 1.48 radians | A1 |  |
|  |  | 4 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 10(i) | State or imply $r=2$ | B1 | Accept $\sqrt{4}$ |
|  | State or imply $\theta=\frac{1}{6} \pi$ | B1 |  |
|  | Use a correct method for finding the modulus or the argument of $u^{4}$ | M1 | Allow correct answers from correct $u$ with minimal working shown |
|  | Obtain modulus 16 | A1 |  |
|  | Obtain argument $\frac{2}{3} \pi$ | A1 | Accept $16 \mathrm{e}^{i \frac{2 \pi}{3}}$ |
|  |  | 5 |  |
| 10(ii) | Substitute $u$ and carry out a correct method for finding $u^{3}$ | M1 | $\left(u^{3}=8 i\right)$ Follow their $u^{3}$ if found in part (i) |
|  | Verify $u$ is a root of the given equation | A1 |  |
|  | State that the other root is $\sqrt{3}-\mathrm{i}$ | B1 |  |
|  | Alternative method |  |  |
|  | State that the other root is $\sqrt{3}-\mathrm{i}$ | B1 |  |
|  | Form quadratic factor and divide cubic by quadratic | M1 | $(z-\sqrt{3}-i)(z-\sqrt{3}+i)\left(=z^{2}-2 \sqrt{3} z+4\right)$ |
|  | Verify that remainder is zero and hence that $u$ is a root of the given equation | A1 |  |
|  |  | 3 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 10(iii) | Show the point representing $u$ in a relatively correct position | B1 |  |
|  | Show a circle with centre $u$ and radius 2 | B1 | FT on the point representing $u$. Condone near miss of origin |
|  | Show the line $y=2$ | B1 |  |
|  | Shade the correct region | B1 |  |
|  | Show that the line and circle intersect on $x=0$ | B1 | Condone near miss |
|  |  | 5 |  |

