Question	Answer	Marks	Guidance
1	State or imply ordinates 3, 2, 0, 4	B1	These and no more Accept in unsimplified form $ 2^0 - 4 $ etc.
	Use correct formula, or equivalent, with $h = 1$ and four ordinates	M1	
	Obtain answer 5.5	A1	
		3	

Question	Answer	Marks	Guidance
2	Use law for the logarithm of a product, quotient or power	M1	Condone $\ln \frac{x}{x-1}$ for M1
	Obtain a correct equation free of logarithms	A1	e.g. $(2x-3)(x-1) = x^2$ or $x^2 - 5x + 3 = 0$
	Solve a 3-term quadratic obtaining at least one root	M1	Must see working if using an incorrect quadratic $\left(\frac{5\pm\sqrt{13}}{2}\right)$
	Obtain answer $x = 4.30$ only	A1	Q asks for 2 dp. Do not ISW. Overspecified answers score A0 Overspecified and no working can score M1A0
		4	

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Question	Answer	Marks	Guidance
3	State or imply $3y^2 + 6xy \frac{dy}{dx}$ as derivative of $3xy^2$	B1	
	State or imply $3y^2 \frac{dy}{dx}$ as derivative of y^3	B1	
	Equate derivative of LHS to zero, substitute (1, 3) and find the gradient	M1	$\left(\frac{dy}{dx} = \frac{x^2 + y^2}{y^2 - 2xy}\right)$ For incorrect derivative need to see the substitution
	Obtain final answer $\frac{10}{3}$ or equivalent	A1	3.33 or better. Allow $\frac{30}{9}$ ISW after correct answer seen
		4	

Question	Answer	Marks	Guidance
4	Use correct trig formula and obtain an equation in tan θ	M1	Allow with 45° e.g. $\frac{1}{\tan \theta} - \frac{1}{\frac{\tan \theta + \tan 45^{\circ}}{1 - \tan \theta \tan 45^{\circ}}} = 3$
	Obtain a correct horizontal equation in any form	A1	e.g. $1 + \tan \theta - \tan \theta (1 - \tan \theta) = 3 \tan \theta (1 + \tan \theta)$
	Reduce to $2\tan^2\theta + 3\tan\theta - 1 = 0$	A1	or 3-term equivalent
	Solve 3-term quadratic and find a value of θ	M1	Must see working if using an incorrect quadratic
	Obtain answer 15.7°	A1	One correct solution (degrees to at least 3 sf)
	Obtain answer 119.(3)°	A1	Second correct solution and no others in range (degrees to at least 3 sf) Mark 0.274, 2.082 as MR: A0A1
		6	

			<u> </u>
Question	Answer	Marks	Guidance
5(i)	Use chain rule	M1	$k\cos\theta\sin^{-3}\theta\Big(=-k\csc^{2}\theta\cot\theta\Big)$
			Allow M1 for $-2\cos\theta\sin^{-1}\theta$
	Obtain correct answer in any form	A1	e.g. $-2\csc^2\theta\cot\theta$, $\frac{-2\cos\theta}{\sin^3\theta}$ Accept $\frac{-2\sin\theta\cos\theta}{\sin^4\theta}$
		2	
5(ii)	Separate variables correctly and integrate at least one side	B1	$\int x \mathrm{d}x = \int -\mathrm{cosec}^2 \theta \cot \theta \mathrm{d}\theta$
	Obtain term $\frac{1}{2}x^2$	B1	
	Obtain term of the form $\frac{k}{\sin^2\theta}$	M1*	or equivalent
	Obtain term $\frac{1}{2\sin^2\theta}$	A1	or equivalent
	Use $x = 4$, $\theta = \frac{1}{6}\pi$ to evaluate a constant, or as limits, in a solution	DM1	Dependent on the preceding M1
	with terms ax^2 and $\frac{b}{\sin^2\theta}$, where $ab \neq 0$		
	Obtain solution $x = \sqrt{(\csc^2 \theta + 12)}$	A1	or equivalent
		6	

Question	Answer	Marks	Guidance
6(i)	State correct expansion of $sin(2x + x)$	B1	
	Use trig formulae and Pythagoras to express $\sin 3x$ in terms of $\sin x$	M1	
	Obtain a correct expression in any form	A1	e.g. $2\sin x (1 - \sin^2 x) + \sin x (1 - 2\sin^2 x)$
	Obtain $\sin 3x \equiv 3\sin x - 4\sin^3 x$ correctly AG	A1	Accept = for \equiv
		4	
6(ii)	Use identity, integrate and obtain $-\frac{3}{4}\cos x + \frac{1}{12}\cos 3x$	B1 B1	One mark for each term correct
	Use limits correctly in an integral of the form $a \cos x + b \cos 3x$, where $ab \neq 0$	M1	$\left(-\frac{3}{8} - \frac{1}{12} + \frac{3}{4} - \frac{1}{12} = -\frac{11}{24} + \frac{2}{3}\right)$
	Obtain answer $\frac{5}{24}$	A1	Must be exact. Accept simplified equivalent e.g. $\frac{15}{72}$ Answer only with no working is 0/4
		4	

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Question	Answer	Marks	Guidance		
7(i)	State at least one correct derivative	B1	$-2\sin\frac{1}{2}x, \frac{1}{(4-x)^2}$		
	Equate product of derivatives to – 1	M1	or equivalent		
	Obtain a correct equation, e.g. $2\sin\frac{1}{2}x = (4-x)^2$	A1			
	Rearrange correctly to obtain $a = 4 - \sqrt{2\sin\frac{a}{2}}$ AG	A1			
		4			
7(ii)	Calculate values of a relevant expression or pair of expressions at $a = 2$ and $a = 3$	M1	$\begin{array}{c} a = 2 2 < 2.7027 \\ e.g. \\ a = 3 3 > 2.587 \\ \end{array} \begin{pmatrix} 0.703 \\ -0.412 \end{pmatrix} \begin{pmatrix} 2.317 \\ -0.995 \end{pmatrix}$ Values correct to at least 2 dp		
	Complete the argument correctly with correct calculated values	A1			
		2			
7(iii)	Use the iterative formula $a_{n+1} = 4 - \sqrt{(2\sin\frac{1}{2}a_n)}$ correctly at least once	M1			
	Obtain final answer 2.611	A1			
	Show sufficient iterations to 5 d.p. to justify 2.611 to 3 d.p., or show there is a sign change in the interval (2.6105, 2.6115)	A1	2, 2.70272, 2.60285, 2.61152, 2.61070, 2.61077 2.5, 2.62233, 2.60969, 2.61087, 2.61076 3, 2.58756, 2.61301, 2.61056, 2.61079 Condone truncation. Accept more than 5 dp		
		3			

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Question	Answer	Marks	Guidance
8(i)	State or imply the form $\frac{A}{2+x} + \frac{B}{3-x} + \frac{C}{(3-x)^2}$	B1	
	Use a correct method to obtain a constant	M1	
	Obtain one of $A = 2, B = 2, C = -7$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	[Mark the form $\frac{A}{2+x} + \frac{Dx+E}{(3-x)^2}$, where $A = 2, D = -2$ and
			E = -1, B1M1A1A1A1.]
		5	
8(ii)	Use a correct method to find the first two terms of the expansion of $(2+x)^{-1}$, $(3-x)^{-1}$ or $(3-x)^{-2}$, or equivalent, e.g. $\left(1+\frac{1}{2}x\right)^{-1}$	M1	
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction		FT on A, B and C $1 - \frac{x}{2} + \frac{x^2}{4} = \frac{2}{3} \left(1 + \frac{x}{3} + \frac{x^2}{9} \right) - \frac{7}{9} \left(1 + \frac{2x}{3} + \frac{3x^2}{9} \right)$
	Obtain final answer $\frac{8}{9} - \frac{43}{54}x + \frac{7}{108}x^2$	A1	
			For the <i>A</i> , <i>D</i> , <i>E</i> form of fractions give M1A1 ft A1 ft for the expanded partial fractions, then, if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer.
		5	

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Question	Answer	Marks	Guidance
9(i)	Obtain a vector parallel to the plane, e.g. $\overrightarrow{CB} = 2\mathbf{i} + \mathbf{j}$	B1	
	Use scalar product to obtain an equation in a, b, c ,	M1	e.g. $2a + b = 0$, $a + 5c = 0$, $a + b - 5c = 0$
	Obtain two correct equations in <i>a</i> , <i>b</i> , <i>c</i>	A1	
	Solve to obtain $a : b : c$,	M1	or equivalent
	Obtain $a: b: c = 5: -10: -1$,	A1	or equivalent
	Obtain equation $5x - 10y - z = -25$,	A1	or equivalent
	Alternative method 1		
	Obtain a vector parallel to the plane, e.g. $\overrightarrow{CD} = \mathbf{i} + 5\mathbf{k}$	B1	$\overrightarrow{BD} = -\mathbf{i} - \mathbf{j} + 5\mathbf{k}$
	Obtain a second such vector and calculate their vector product, e.g. $(2i + j) \times (i + 5k)$	M1	
	Obtain two correct components	A1	
	Obtain correct answer, e.g. $5\mathbf{i} - 10\mathbf{j} - \mathbf{k}$	A1	
	Substitute to find <i>d</i>	M1	
	Obtain equation $5x - 10y - z = -25$,	A1	or equivalent

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Question	Answer	Marks	Guidance		
9(i)	Alternative method 2				
	Obtain a vector parallel to the plane, e.g. $\overrightarrow{DB} = \mathbf{i} + \mathbf{j} - 5\mathbf{k}$	B1			
	Obtain a second such vector and form correctly a 2-parameter equation for the plane	M1			
	State a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} + 4\mathbf{j} + \lambda(\mathbf{i} + 5\mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} - 5\mathbf{k})$	A1			
	State three equations in <i>x</i> , <i>y</i> , <i>z</i> , λ and μ	A1			
	Eliminate λ and μ	M1			
	Obtain equation $5x - 10y - z = -25$	A1	or equivalent		
	Alternative method 3				
	Substitute for <i>B</i> and <i>C</i> and obtain $3a + 4b = d$ and $a + 3b = d$	B1			
	Substitute for D to obtain a third equation and eliminate one unknown $(a, b, \text{ or } d)$ entirely	M1			
	Obtain two correct equations in two unknowns, e.g. <i>a</i> , <i>b</i> , <i>c</i>	A1			
	Solve to obtain their ratio, e.g. $a : b : c$	M1			
	Obtain $a: b: c = 5: -10: -1$, a: c: d = 5: -1: -25, or $b: c: d = 10: 1: 25$	A1	or equivalent		
	Obtain equation $5x - 10y - z = -25$	A1	or equivalent		

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Question	Answer	Marks	Guidance		
9(i)	Alternative method 4				
	Substitute for <i>B</i> and <i>C</i> and obtain $3a + 4b = d$ and $a + 3b = d$	B1			
	Solve to obtain <i>a</i> : <i>b</i> : <i>d</i>	M2	or equivalent		
	Obtain $a: b: d = 1: -2: -5$	A1	or equivalent		
	Substitute for C to obtain c	M1			
	Obtain equation $5x - 10y - z = -25$	A1	or equivalent		
		6			
9(ii)	State or imply a normal vector for the plane <i>OABC</i> is k	B1			
	Carry out correct process for evaluating a scalar product of two relevant vectors, e.g. $(5i - 10j - k).(k)$	M1	i.e. correct process using \mathbf{k} and their normal		
	Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	M1	Allow M1M1 for clear use of an incorrect vector that has been stated to be the normal to <i>OABC</i>		
	Obtain answer 84.9° or 1.48 radians	A1			
		4			

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Question	Answer	Marks	Guidance
10(i)	State or imply $r = 2$	B1	Accept $\sqrt{4}$
	State or imply $\theta = \frac{1}{6}\pi$	B1	
	Use a correct method for finding the modulus or the argument of u^4	M1	Allow correct answers from correct <i>u</i> with minimal working shown
	Obtain modulus 16	A1	
	Obtain argument $\frac{2}{3}\pi$	A1	Accept $16e^{i\frac{2\pi}{3}}$
		5	
10(ii)	Substitute u and carry out a correct method for finding u^3	M1	$(u^3 = 8i)$ Follow <i>their</i> u^3 if found in part (i)
	Verify <i>u</i> is a root of the given equation	A1	
	State that the other root is $\sqrt{3} - i$	B1	
	Alternative method	•	
	State that the other root is $\sqrt{3} - i$	B1	
	Form quadratic factor and divide cubic by quadratic	M1	$(z-\sqrt{3}-i)(z-\sqrt{3}+i)(=z^2-2\sqrt{3}z+4)$
	Verify that remainder is zero and hence that u is a root of the given equation	A1	
		3	

Question	Answer	Marks	Guidance
10(iii)	Show the point representing <i>u</i> in a relatively correct position	B1	
	Show a circle with centre <i>u</i> and radius 2	B1	FT on the point representing <i>u</i> . Condone near miss of origin
	Show the line $y = 2$	B1	Im shaded y = 2 • u Re
	Shade the correct region	B1	
	Show that the line and circle intersect on $x = 0$	B1	Condone near miss
		5	