Question	Answer	Marks	Guidance
1	Use logarithm subtraction property to produce logarithm of quotient	M1	
	Factorise at least as far as $x(x^2 - 4)$ and $x(x - 2)$ or use correct algebraic long division to obtain a quotient of $x + 2$ and a remainder of 0 from correct working	B1	Allow B1 either before or after application of log property Allow B1 for equivalent using factorisation then use of addition rule Allow B1 for $\frac{(x+2)(x^2-2x)}{(x^2-2x)}$
	Obtain final answer $\ln(x+2)$ using correct process	A1	With no errors seen
		3	

Question	Answer		Guidance
2(i)	State or imply non-modular inequality $(3x-5)^2 < (x+3)^2$ or corresponding equation or pair of different linear equations/inequalities	B1	<b>SC</b> : Allow B1 for $x < 4$ from only one linear inequality
	Attempt solution of 3-term quadratic equation/inequality or of two different linear equations/inequalities	M1	For M1, must get as far as 2 critical values
	Obtain critical values $\frac{1}{2}$ and 4	A1	
	State answer $\frac{1}{2} < x < 4$ or equivalent	A1	If given as 2 separate statements, condone omission of 'and' or $\cap$ but penalise inclusion of 'or' or $\cup$
		4	

Question	Answer	Marks	Guidance
2(ii)	Attempt to find <i>n</i> (not necessarily an integer so far) from $3^{0.1n} = \text{ or } < their$ positive upper value from part (i) or $3^{0.1n+1} = \text{ or } < 3 \times their$ positive upper value from part (i)	M1	0/2 for trial and improvement
	Conclude 12	A1	
		2	

Question	Answer	Marks	Guidance
3	Use product rule to differentiate $x^2 \ln y$	M1	Allow M1 for $2x \ln y + x^2 y^{-1}$ oe
	Obtain $2x \ln y + x^2 \times \frac{1}{y} \times \frac{dy}{dx}$	A1	
	Obtain $+2 + 5\frac{dy}{dx} = 0$	B1	B1 for $+2+5\frac{dy}{dx} = 0$ , maybe implied by later work
	Substitute $x = 3$ and $y = 1$ to find value of their $\frac{dy}{dx}$	*M1	Dependent on at least one $\frac{dy}{dx}$ present
	Obtain $\frac{dy}{dx} = -\frac{2}{14}$	A1	
	Attempt equation of line through (3, 1) with gradient of normal	DM1	Allow one sign error
	Obtain $y = 7x - 20$ or equivalent unsimplified	A1	FT on their perpendicular gradient
		7	

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Question	Answer	Marks	Guidance
4(a)	Use identity $\tan^2 3x = \sec^2 3x - 1$	B1	
	Integrate to obtain form $k_1 \tan 3x + k_2 x$	M1	
	Obtain correct $\frac{1}{3}\tan 3x - x + c$	A1	
		3	
4(b)	Express integrand as $e^{2x} + 4e^{-x}$	B1	
	Integrate to obtain form $k_3 e^{2x} + k_4 e^{-x}$	M1	
	Obtain correct $\frac{1}{2}e^{2x} - 4e^{-x}$	A1	
	Use limits to obtain $\frac{1}{2}e^2 - 4e^{-1} + \frac{7}{2}$ or similarly simplified equivalent	A1	
		4	

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Question	Answer	Marks	Guidance
5(i)	Substitute $x = 2$ and equate to zero	M1	Allow synthetic division for each– must result in an equation from each division
	Substitute $x = -1$ and equate to 27	M1	Allow unsimplified
	Obtain $4a + 2b = -24$ and $a - b = 48$ or equivalents	A1	Allow one error in each equation
	Solve a relevant pair of simultaneous linear equations	M1	Dependent at least one M mark
	Obtain $a = 12, b = -36$	A1	
		5	
5(ii)	Divide by $x-2$ at least as far as the <i>x</i> term to obtain $5x^2 + (their \ a+10)x$	M1	For synthetic division need to see 5 and <i>their</i> $a + 10$ in the bottom line
	Obtain $5x^2 + 22x + 8$	A1	
	Obtain $(x-2)(5x+2)(x+4)$	A1	If solved using a calculator and then forming factors, must be correct for full marks
		3	

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Question	Answer	Marks	Guidance
6(i)	Use quotient rule (or product rule) to differentiate	M1	Penalise missing brackets by withholding the A mark unless recovered later
	Obtain $\frac{dy}{dx} = \frac{3x^2(2-5x) - (-5)(8+x^3)}{(2-5x)^2}$ or equivalent	A1	
	State or imply curve crosses <i>x</i> -axis when $x = -2$	B1	
	Substitute –2 to obtain 1	A1	
		4	
6(ii)	Equate numerator of first derivative to zero and rearrange as far as $kx^3 =$ or equivalent	M1	
	Confirm given result $x = \sqrt{0.6x + 4x^{-1}}$ AG	A1	Condone in this part error(s) in denominator of derivative
		2	
6(iii)	Use iterative process correctly at least once	M1	
	Obtain final answer 1.81	A1	
	Show sufficient iterations to 5 sf to justify answer or show a sign change in the interval [1.805, 1.815]	A1	
		3	

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	I OBLIGHLD 9		
Question	Answer	Marks	Guidance
7(i)	State or imply $\csc 2\theta = \frac{1}{2\sin\theta\cos\theta}$	B1	
	Attempt to express left-hand side in terms of $\sin \theta$ and $\cos \theta$ only	M1	
	Simplify to confirm $\csc^2 \theta$ AG	A1	
		3	
7(ii)	Use identity to express left-hand side in terms of sin 30 or cosec 30	M1	
	Obtain $\frac{2}{\sin 30}$ or 2 cosec 30 and confirm 4 AG	A1	
		2	
7(iii)	Solve quadratic equation of the form $k \operatorname{cosec}^2 \frac{\phi}{2} + \operatorname{cosec} \frac{\phi}{2} - 12 = 0$ or	*M1	Allow sign errors
	$12\sin^2\frac{\phi}{2} - \sin\frac{\phi}{2} - k = 0 \text{ correctly for } \csc\frac{1}{2}\phi \text{ or } \sin\frac{1}{2}\phi \text{ to find two}$ values of $\sin\frac{1}{2}\phi$ or $\csc\frac{1}{2}\phi$ Obtain $\sin\frac{1}{2}\phi = -\frac{1}{4}, \frac{1}{3}$	A1	
	Use correct process to find at least one correct value of $\phi$ from $\sin \frac{1}{2}\phi = \pm \frac{1}{4}, \pm \frac{1}{3}$	DM1	Allow for any rounded or truncated value
	Obtain any two of -331.0, -29.0, 38.9, 321.1	A1	Allow greater accuracy
	Obtain all four values and no others between -360 and 360	A1	Allow greater accuracy
		5	