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Question	Answer	Marks	Guidance
1(i)	$\left[\left(x-2\right)^2\right] \left[+4\right]$	B1 DB1	2nd B1 dependent on 2 inside bracket
		2	
1(ii)	$(x-2)^2 < 5 \rightarrow -\sqrt{5} < x-2$ and/or $x-2 < \sqrt{5}$	M1	Allow e.g. $x-2 < \pm \sqrt{5}$ , $x-2 = \pm \sqrt{5}$ and decimal equivalents for $\sqrt{5}$ For M1, ft from <i>their</i> (i). Also allow $\sqrt{13}$ instead of $\sqrt{5}$ for clear slip
	$2 - \sqrt{5} < x < 2 + \sqrt{5}$	A1A1	A1 for each inequality – allow two separate statements but there must be 2 inequalities for <i>x</i> . Non-hence methods, if completely correct, score SC 1/3. Condone $\leq$
		[3]	

Question	Answer	Marks	Guidance
2(i)	$\frac{-5}{x} + \frac{5}{8x^3} - \frac{1}{32x^5} \text{ (or } -5x^{-1} + \frac{5}{8}x^{-3} - \frac{1}{32}x^{-5} \text{ )}$	B1B1B1	B1 for each correct term SCB1 for both $\frac{+5}{x} & \frac{+1}{32x^5}$
		3	
2(ii)	$1 \times 20 + 4 \times their(-5) = 0$	M1A1	Must be from exactly 2 terms SCB1 for $20 + 20 = 40$
		2	

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Question	Answer	Marks	Guidance
3(i)	Angle $EAD$ = Angle $ACD = \frac{3\pi}{10}$ or 54° or 0.942 soi	B1	
	or Angle $DAC = \frac{\pi}{5}$ or 36° or 0.628 soi		
	$AD = 8\sin(\frac{3\pi}{10}) \text{ or } 8\cos(\frac{\pi}{5})$	M1	Angles used must be correct
	(AD =) 6.47	A1	
	Alternative method for question 3(i)		
	$AB = \frac{8}{\tan\left(\frac{\pi}{5}\right)} \text{ or } AB = \frac{8\sin\left(\frac{3\pi}{10}\right)}{\sin\left(\frac{\pi}{5}\right)} \text{ or } 11.(01)$	B1	Angles used must be correct
	$AD = 11.0(1)\sin\frac{\pi}{5}$ oe	M1	
	(AD =) 6.47	A1	
		3	
3(ii)	Area sector = $\frac{1}{2} (theirAD)^2 \times their\left(\frac{\pi}{2} - \frac{\pi}{5}\right)$	M1	19.7(4)
	Area $\Delta ADC = \frac{1}{2} \times 8 \times theirAD \times \sin\frac{\pi}{5}$ or $\frac{1}{2} \times 8\cos\left(\frac{3\pi}{10}\right) \times 8\sin\left(\frac{3\pi}{10}\right)$	M1	Or e.g. $\frac{1}{2}$ their $AD \times \sqrt{8^2 - their AD^2}$ . 15.2(2)
	(Shaded area =) 35.0 or 34.9	A1	
		3	

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Question	Answer	Marks	Guidance
4(i)	Max( <i>a</i> ) is 8	B1	Allow $a = 8$ or $a \leq 8$
	Min( <b>b</b> ) is 24	B1	Allow $b = 24$ or $b \ge 24$
		2	SCB1 for 8 and 24 seen
4(ii)	$gf(x) = \frac{96}{x-1} - 4$ or $gf(x) = \frac{100 - 4x}{x-1}$	B1	$2\left(\frac{48}{x-1}\right) - 4$ is insufficient Apply ISW
		1	
4(iii)	$y = \frac{96}{x-1} - 4 \rightarrow y + 4 = \frac{96}{x-1} \rightarrow x - 1 = \frac{96}{y+4}$	M1	<b>FT</b> from <i>their</i> ( <b>ii</b> ) provided ( <b>ii</b> ) involves algebraic fraction. Allow sign errors
	$(gf)^{-1}(x) = \frac{96}{x+4} + 1$	A1	OR $\frac{100+x}{x+4}$ . Must be a function of <i>x</i> . Apply ISW
		2	

Question	Answer	Marks	Guidance
5(i)	$\frac{x}{2} \Big[ 2 + (x-1)(-/+0.02) \Big] \text{ or } 1.01x - 0.01x^2 \text{ or } 0.99x + 0.01x^2 \text{ oe}$	B1	Allow – or + 0.02. Allow $n$ used
		1	

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Question	Answer	Marks	Guidance
5(ii)	Equate to 13 <b>then</b> <i>either</i> simplify to a 3-term quadratic equation <b>or</b> find at least 1 solution (need not be correct) to an unsimplified quadratic	M1	Expect $n^2 - 101n + 1300$ (=0) or $0.99x + 0.01x^2 = 13$ . Allow x used
	16	A1	Ignore 85.8 or 86
		2	
5(iii)	Use of $\frac{a(1-r^{n})}{1-r}$ with $a = 1, r = 0.92, n = 20$ soi	M1	
	(=) 10.1	A1	
	Use of $(S_{\infty} =) \frac{a}{1-r}$ with $a = 1, r = 0.92$	M1	OR $\frac{(1)(1-0.92^{n})}{1-0.92} = 13 \rightarrow 0.92^{n} = -0.04$ oe
	$S_{\infty} = 12.5$ so never reaches target or $< 13$	A1	Conclusion required – 'Shown' is insufficient No solution so never reaches target or < 13
		4	

Question	Answer	Marks	Guidance
6(i)	$\mathbf{MF} = -4\mathbf{i} + 2\mathbf{j} + 7\mathbf{k}$	B1	
		1	
6(ii)	FN = 2i - j	B1	
		1	
6(iii)	$\mathbf{MN} = -2\mathbf{i} + \mathbf{j} + 7\mathbf{k}$	B1	<b>FT</b> on <i>their</i> ( $MF + FN$ )
		1	

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Question	Answer	Marks	Guidance
6(iv)	MF.MN = 8 + 2 + 49 = 59	*M1	MF.MN or FM.NM but allow if one is reversed (implied by –59)
	$ \mathbf{MF}  \times  \mathbf{MN}  = \sqrt{4^2 + 2^2 + 7^2} \times \sqrt{2^2 + 1^2 + 7^2}$	*DM1	Product of modulus. At least one methodically correct
	$\cos FMN = \frac{+/-59}{\sqrt{69} \times \sqrt{54}}$	DM1	All linked correctly. Note $\sqrt{69} \times \sqrt{54} = 9\sqrt{46}$
	$FMN = 14.9^{\circ} \text{ or } 0.259$	A1	Do not allow if exactly 1 vector is reversed – even if adjusted finally
		4	

Question	Answer	Marks	Guidance
7(i)	D = (5, 1)	B1	
		1	
7(ii)	$(x-5)^2 + (y-1)^2 = 20$ oe	B1	<b>FT</b> on <i>their D</i> . Apply ISW, oe but not to contain square roots
		1	

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Question	Answer	Marks	Guidance
7(iii)	$(x-1)^{2} + (y-3)^{2} = (9-x)^{2} + (y+1)^{2}$ soi	M1	Allow 1 sign slip For M1 allow with $$ signs round both sides but sides must be equated
	$x^{2} - 2x + 1 + y^{2} - 6y + 9 = x^{2} - 18x + 81 + y^{2} + 2y + 1$	A1	
	y = 2x - 9 www <b>AG</b>	A1	
	Alternative method for question 7(iii)		
	grad. of $AB = -\frac{1}{2} \rightarrow \text{grad of perp bisector} = \frac{-1}{-\frac{1}{2}}$	M1	
	Equation of perp. bisector is $y-1=2(x-5)$	A1	
	y = 2x - 9 www <b>AG</b>	A1	
		3	
7(iv)	Eliminate $y$ (or $x$ ) using equations in (ii) and (iii)	*M1	To give an (unsimplified) quadratic equation
	$5x^2 - 50x + 105 (= 0)$ or $5(x-5)^2 = 20$ or $5y^2 - 10y - 75 (= 0)$ or $5(y-1)^2 = 80$	DM1	Simplify to one of the forms shown on the right (allow arithmetic slips)
	x = 3 and 7, or $y = -3$ and 5	A1	
	(3, -3), (7, 5)	A1	Both pairs of $x & y$ correct implies A1A1. SC B2 for no working
		4	

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Question	Answer	Marks	Guidance
8	$f'(-1) = 0 \rightarrow 3 - a + b = 0$ $f'(3) = 0 \rightarrow 27 + 3a + b = 0$	M1	Stationary points at $x = -1$ & $x = 3$ gives sim. equations in <i>a</i> & <i>b</i>
	<i>a</i> = -6	A1	Solve simultaneous equation
	<i>b</i> = –9	A1	
	Hence $f'(x) = 3x^2 - 6x - 9 \rightarrow f(x) = x^3 - 3x^2 - 9x(+c)$	B1	<b>FT</b> correct integration for <i>their a,b</i> (numerical <i>a, b</i> )
	2 = -1 - 3 + 9 + c	M1	Sub $x = -1$ , $y = 2$ into <i>their</i> integrated $f(x)$ . <i>c</i> must be present
	<i>c</i> = -3	A1	<b>FT</b> from <i>their</i> $f(x)$
	$f(3) = k \rightarrow k = 27 - 27 - 27 - 3$	M1	Sub $x = 3$ , $y = k$ into <i>their</i> integrated $f(x)$ (Allow <i>c</i> omitted)
	k = -30	A1	
		8	

Question	Answer	Marks	Guidance
9(i)	$q \leqslant f(x) \leqslant p + q$	B1B1	B1 each inequality – allow two separate statements Accept < , $(q, p+q)$ , $[q, p+q]$ Condone y or x or f in place of $f(x)$
		2	

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Question	Answer	Marks	Guidance
9(ii)	(a) 2	B1	Allow $\frac{\pi}{4}$ , $\frac{3\pi}{4}$
	<b>(b)</b> 3	B1	Allow $0, \frac{\pi}{2}, \pi$
	(c) 4	B1	Allow $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$
		3	
9(iii)	$3\sin^2 2x + 2 = 4 \rightarrow \sin^2 2x = \frac{2}{3} \text{ soi}$	M1	
	Sin2x = (±)0.816(5). Allow sin 2x = (±) $\sqrt{\frac{2}{3}}$ or 2x = sin <sup>-1</sup> (±) $\sqrt{\frac{2}{3}}$	A1	OR Implied by at least one correct value for $x$ . Allow $\sin^{-1}$ form
	(2x =) at least two of 0.955(3), 2.18(6), 4.09(7), 5.32(8)	A1	Can be implied by corresponding values of <i>x</i> below Allow for at least two of $0.304\pi$ , $0.696\pi$ , $1.30(4)\pi$ , $1.69(6)\pi$ <b>OR</b> at least <u>two</u> of 54.7(4)°, 125.2(6)°, 234.7(4)°, 305.2(6)°
	( <i>x</i> =) 0.478, 1.09, 2.05, 2.66.	A1A1	Allow 0.152 $\pi$ , 0.348 $\pi$ , 0.652 $\pi$ , 0.848 $\pi$ SC A1 for 2 or 3 correct. SC A1 for all of 27.4°, 62.6°, 117.4°, 152.6° Sin2 $x = \pm \frac{2}{3} \rightarrow x = 0.365, 1.21, 1.94, 2.78$ scores SC M1A0A0A1
		5	

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Question	Answer	Marks	Guidance
10(i)	$\left[\frac{1}{2}(3x+4)^{-\frac{1}{2}}\right]$	B1	oe
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \left[\frac{1}{2}(3x+4)^{-\frac{1}{2}}\right] \times 3$	B1	Must have '×3'
	At $x = 4$ , $\frac{dy}{dx} = \frac{3}{8}$ soi	B1	
	Line through (4, <i>their</i> 4) with gradient <i>their</i> $\frac{3}{8}$	M1	If $y \neq 4$ is used then clear evidence of substitution of $x = 4$ is needed
	Equation of tangent is $y-4 = \frac{3}{8}(x-4)$ or $y = \frac{3}{8}x + \frac{5}{2}$	A1	oe
		5	

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Question	Answer	Marks	Guidance
10(ii)	Area under line $=\frac{1}{2}\left(4+\frac{5}{2}\right) \times 4 = 13$	B1	OR $\int_{0}^{4} \frac{3}{8}x + \frac{5}{2} = \left[\frac{3}{16}x^{2} + \frac{5}{2}x\right] = [3+10] = 13$
	Area under curve: $\int (3x+4)^{\frac{1}{2}} = \left[\frac{(3x+4)^{3/2}}{3/2}\right] [\div 3]$	B1B1	Allow if seen as part of the difference of 2 integrals First B1 for integral without [÷3] Second B1 must have [÷3]
	$\frac{128}{9} - \frac{16}{9} = \frac{112}{9} = 12\frac{4}{9}$	M1	Apply limits $0 \rightarrow 4$ to an integrated expression
	Area = $13 - 12\frac{4}{9} = \frac{5}{9}$ (or 0.556)	A1	
	Alternative method for question 10(ii)		
	Area for line = $1/2 \times 4 \times 3/2 = 3$	B1	OR $\int_{5/2}^{4} \frac{1}{3} (8y - 20) = \frac{1}{3} [4y^2 - 20] = \frac{1}{3} [-16 + 25] = 3$
	Area for curve = $\int \frac{1}{3}(y^2 - 4) = \left[\frac{y^3}{9}\right] - \left[\frac{4y}{3}\right]$	B1B1	
	$\left(\frac{64}{9} - \frac{16}{3}\right) - \left(\frac{8}{9} - \frac{8}{3}\right) = \frac{32}{9}$	M1	Apply limits $2 \rightarrow 4$ to an integrated expression for curve
	Area = $\frac{32}{9} - 3 = \frac{5}{9}$ (or 0.556)	A1	
		5	

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Question	Answer	Marks	Guidance
10(iii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}$	B1	
	$\frac{3}{2}(3x+4)^{-\frac{1}{2}} = \frac{1}{2}$	M1	Allow M1 for $\frac{3}{2}(3x+4)^{-\frac{1}{2}} = 2$ .
	$(3x+4)^{\frac{1}{2}} = 3 \rightarrow 3x+4=9 \rightarrow x=\frac{5}{3}$ oe	A1	
		3	