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Question	Answer	Marks	Guidance
	For $\left(\frac{2}{x} - 3x\right)^5$ term in x is 10 or 5C ₃ or 5C2 × $\left(\frac{2}{x}\right)^2$ × $(-3x)^3$ or	B2,1	3 elements required. -1 for each error with or without <i>x</i> 's. Can be seen in an expansion.
	$\left(\frac{2}{x}\right)^5 \frac{5.4.3}{3!} \left(-\frac{3}{2}x^2\right)^3$ or $\left(-3x\right)^5 \frac{5.4}{2!} \left(\frac{2}{3x^2}\right)^2$		
	-1080 identified	B1	Allow $-1080x$ Allow if expansion stops at this term. Allow from expanding brackets.
		3	

Question	Answer	Marks	Guidance
2	Midpoint of <i>AB</i> is (5, 1)	B1	Can be seen in working, accept $\left(\frac{10}{2}, \frac{2}{2}\right)$.
	$m_{AB} = -\frac{1}{2}$ oe	B1	
	C to (5, 1) has gradient 2	*M1	Use of $m_1 \times m_2 = -1$.
	Forming equation of line $(y = 2x - 9)$	DM1	Using their perpendicular gradient and their midpoint to form the equation.
	C(0, -9) or $y = -9$	A1	
		5	

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Question	Answer	Marks	Guidance
3(i)	$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t} = 7 \times -0.05$	M1	Multiply numerical gradient at $x = 2$ by ± 0.05 .
	-0.35 (units/s) or Decreasing at a rate of (+) 0.35	A1	Ignore notation and omission of units
		2	
3(ii)	$(y) = \frac{x^4}{4} + \frac{4}{x} (+c)$ oe	B1	Accept unsimplified
	Uses (2, 9) in an integral to find c.	M1	The power of at least one term increase by 1.
	$c = 3 \text{ or } (y =) \frac{x^4}{4} + \frac{4}{x} + 3 \text{ oe}$	A1	A0 if candidate continues to a final equation that is a straight line.
		3	

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Answer	Marks	Guidance
$a^2 + 2ab + b^2, a^2 - 2ab + b^2$	B1	Correct expansions.
$\sin^2 x + \cos^2 x = 1$ used $\rightarrow (a+b)^2 + (a-b)^2 = 1$	M1	Appropriate use of $\sin^2 x + \cos^2 x = 1$ with $(a+b)^2$ and $(a-b)^2$
$a^2 + b^2 = \frac{1}{2}$	A1	No evidence of $\pm 2ab$, scores 2/3
Alternative method for question 4(i)		
$2a = (s+c) \& 2b = (s-c) \text{ or } a = \frac{1}{2}(s+c) \& b = \frac{1}{2}(s-c)$	B1	
$a^{2}+b^{2} = \frac{1}{4}(s+c)^{2} + \frac{1}{4}(s-c)^{2} = \frac{1}{2}(s^{2}+c^{2})$	M1	Appropriate use of $\sin^2 x + \cos^2 x = 1$
$a^2 + b^2 = \frac{1}{2}$	A1	Method using only $(\sin x - b)^2$ and $(a - \cos x)^2$ scores 0/3.
	3	SC B1 for assuming θ is acute giving $a = \frac{1}{\sqrt{5}} + b$ or $2\sqrt{5} - b$
	$\sin^{2}x + \cos^{2}x = 1 \text{ used} \rightarrow (a+b)^{2} + (a-b)^{2} = 1$ $a^{2} + b^{2} = \frac{1}{2}$ Alternative method for question 4(i) $2a = (s+c) \& 2b = (s-c) \text{ or } a = \frac{1}{2}(s+c) \& b = \frac{1}{2}(s-c)$ $a^{2}+b^{2} = \frac{1}{4}(s+c)^{2} + \frac{1}{4}(s-c)^{2} = \frac{1}{2}(s^{2}+c^{2})$	$\frac{a^{2} + 2ab + b^{2}}{\sin^{2}x + \cos^{2}x = 1 \text{ used}} \rightarrow (a + b)^{2} + (a - b)^{2} = 1$ M1 $\frac{a^{2} + b^{2} = \frac{1}{2}}{A1}$ Alternative method for question 4(i) $2a = (s+c) \& 2b = (s-c) \text{ or } a = \frac{1}{2}(s+c) \& b = \frac{1}{2}(s-c)$ B1 $a^{2} + b^{2} = \frac{1}{4}(s+c)^{2} + \frac{1}{4}(s-c)^{2} = \frac{1}{2}(s^{2}+c^{2})$ M1

Question	Answer	Marks	Guidance
4(ii)	$\tan x = \frac{\sin x}{\cos x} \longrightarrow \frac{a+b}{a-b} = 2$	M1	Use of $tanx = \frac{sinx}{cosx}$ to form an equation in a and b only
	a = 3b	A1	
		2	

Question	Answer	Marks	Guidance
5	Perimeter of $AOC = 2r + r\theta$	B1	
	Angle $COB = \pi - \theta$	B1	Could be on the diagram. Condone $180 - \theta$.
	Perimeter of $BOC = 2r + r(\pi - \theta)$	B1	FT on angle <i>COB</i> if of form $(k\pi - \theta)$, $k > 0$.
	$(2r +) \pi r - r\theta = 2((2r) + r\theta)$ $(2 + \pi - \theta = 4 + 2\theta \longrightarrow \theta = \frac{\pi - 2}{3})$	M1	Sets up equation using $r(k\pi - \theta)$ and $\times 2$ on correct side. Condone any omissions of OA, OB and/or OC.
	$\theta = 0.38$	A1	Equivalent answer in degrees scores A0.
		5	

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Question	Answer	Marks	Guidance
6(i)	3, -3	B1	Accept ± 3
	-1/2	B1	
	21/2	B1	
		3	Condone misuse of inequality signs.
6(ii)			Only mark the curve from $0 \rightarrow 2\pi$. If the <i>x</i> axis is not labelled assume that $0 \rightarrow 2\pi$ is the range shown. Labels on axes are not required.
	2 complete oscillations of a cosine curve starting with a maximum at $(0,a)$, $a>0$	B1	
	Fully correct curve which must appear to level off at 0 and/or 2π .	B1	
	Line starting on positive y axis and finishing below the x axis at 2π . Must be straight.	B1	
		3	
6(iii)	4	B1	
		1	

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Question	Answer	Marks	Guidance
7(i)	$(f^{-1}(x)) = \frac{x+2}{3}$ oe	B1	
	$y = \frac{2x+3}{x-1} \to (x-1)y = 2x+3 \to x(y-2) = y+3$	M1	Correct method to obtain $x =$, (or $y =$, if interchanged) but condone +/- sign errors
	$(g^{-1}(x) \text{ or } y) = \frac{x+3}{x-2} \text{ oe } \left(eg\frac{5}{x-2}+1\right)$	A1	Must be in terms of <i>x</i>
	$x \neq 2$ only	B1	FT for value of <i>x</i> from their denominator $= 0$
		4	
7(ii)	$(fg(x)=)\frac{3(2x+3)}{x-1} - 2(=\frac{7}{3})$	B1	
	18x + 27 = 13x - 13 or 3(4x + 11) = 7(x - 1) (5x = -40)	M1	Correct method from their $fg = \frac{7}{3}$ leading to a
			linear equation and collect like terms. Condone omission of $2(x-1)$.
	Alternative method for question 7(ii)		
	$(f^{-1}(\frac{7}{3})) = \frac{13}{9}$	B1	
	$\frac{2x+3}{x-1} = \frac{13}{9} \longrightarrow 9(2x+3) = 13(x-1) (\longrightarrow 5x = -40)$	M1	Correct method from $g(x) =$ their $\frac{13}{9}$ leading to a
			linear equation and collect like terms.
	x = -8	A1	
		3	

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Question	Answer	Marks	Guidance
8(i)	$6 \times 3 + 2 \times k + 6 \times 3 = 0$ (18 - 2k + 18 = 0)	M1	Use of scalar product = 0. Could be \overrightarrow{AO} . \overrightarrow{OB} , \overrightarrow{AO} . \overrightarrow{BO} or \overrightarrow{OA} . \overrightarrow{BO}
	k = 18	A1	
	Alternative method for question 8(i)		
	$76 + 18 + k^2 = 18 + (k+2)^2$	M1	Use of Pythagoras with appropriate lengths.
	k = 18	A1	
		2	
8(ii)	$36 + 4 + 36 = 9 + k^2 + 9$	M1	Use of modulus leading to an equation and solve to $k=$ or $k^2 =$
	$k = \pm \sqrt{58}$ or ± 7.62	A1	Accept exact or decimal answers. Allow decimals to greater accuracy.
		2	

Question	Answer	Marks	Guidance	
8(iii)	$\overline{AB} = \begin{pmatrix} -3\\6\\3 \end{pmatrix} \rightarrow \overline{AC} = \begin{pmatrix} -2\\4\\2 \end{pmatrix} \text{ then } \overline{OA} + \overline{AC}$	M1	Complete method using $\overrightarrow{AC} = \pm \frac{2}{3} \overrightarrow{AB}$ And then $\overrightarrow{OA} + their \overrightarrow{AC}$	
	$\overrightarrow{OC} = \begin{pmatrix} 4\\2\\-4 \end{pmatrix}$	A1		
	$\div \sqrt{\left(their 4\right)^2 + \left(their 2\right)^2 + \left(their - 4\right)^2}$	M1	Divides by modulus of their \overrightarrow{OC}	
	$= \frac{1}{6} \begin{pmatrix} 4 \\ 2 \\ -4 \end{pmatrix} \text{ or } \frac{1}{6} (4i + 2j - 4k)$	A1		
	Alternative method for question 8(iii)			
	Let $\overrightarrow{OC} = \begin{pmatrix} p \\ q \\ r \end{pmatrix} \rightarrow \overrightarrow{AC} = \begin{pmatrix} p-6 \\ q+2 \\ r+6 \end{pmatrix} \& \overrightarrow{CB} = \begin{pmatrix} 3-p \\ 4-q \\ -3-r \end{pmatrix}$	M1	Correct method. Equates coefficients leading to values for <i>p</i> , <i>q</i> , <i>r</i>	
	p-6=2(3-p); q+2=2(4-q); r+6=2(-3-r) $\rightarrow p=4, q=2 \& r=-4$	A1		
	$\div \sqrt{\left(their 4\right)^2 + \left(their 2\right)^2 + \left(their - 4\right)^2}$	M1	Divides by modulus of their \overrightarrow{OC}	
	$= \frac{1}{6} \begin{pmatrix} 4 \\ 2 \\ -4 \end{pmatrix} \text{ or } \frac{1}{6} (4i + 2j - 4k)$	A1		

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Question	Answer	Marks	Guidance
8(iii)	Alternative method for question 8(iii)		
	$\overline{CB} = \overline{OB} - \overline{OC} \therefore 2\left(\overline{OB} - \overline{OC}\right) = \overline{OC} - \overline{OA}$ $\rightarrow 2 \ \overline{OB} + \overline{OA} = 3 \ \overline{OC} \therefore 3 \ \overline{OC} = \begin{pmatrix} 12\\6\\-12 \end{pmatrix}$	M1	Correct method. Gets to a numerical expression for $k \overrightarrow{OC}$ from $\overrightarrow{OA} \& \overrightarrow{OB}$.
	$\overrightarrow{OC} = \begin{pmatrix} 4\\2\\-4 \end{pmatrix}$	A1	
	$\div \sqrt{\left(their 4\right)^2 + \left(their 2\right)^2 + \left(their - 4\right)^2}$	M1	Divides by modulus of their \overrightarrow{OC}
	$= \frac{1}{6} \begin{pmatrix} 4\\2\\-4 \end{pmatrix} \text{ or } \frac{1}{6} (4i+2j-4k)$	A1	
		4	

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Question	Answer	Marks	Guidance
9	For C ₁ : $\frac{dy}{dx} = 2x - 4 \rightarrow m = 2$	B1	
	y - 'their 4' = 'their m' (x - 3) or using $y = mx + c$	M1	Use of : $\frac{dy}{dx}$ and (3, their 4) to find the tangent equation.
	y-4 = 2(x-3) or $y = 2x-2$	A1	If using $= mx + c$, getting $c = -2$ is enough.
	$2x - 2 = \sqrt{4x + k} (\to 4x^2 - 12x + 4 - k = 0)$	*M1	Forms an equation in one variable using tangent & C_2
	Use of $b^2 - 4ac = 0$ on a 3 term quadratic set to 0.	*DM1	Uses 'discriminant = 0'
	$144 = 16(4 - k) \longrightarrow k = -5$	A1	
	$4x^2 - 12x + 4 - k = 0 \longrightarrow 4x^2 - 12x + 9 = 0$	DM1	Uses k to form a 3 term quadratic in x
	$x = \frac{3}{2} \left(or \frac{1}{2} \right), y = 1(or - 1).$	A1	Condone 'correct' extra solution.
	Alternative method for question 9		
	For C ₁ : $\frac{dy}{dx} = 2x - 4 \rightarrow m = 2$	B1	
	y - 'their 4' = 'their m' (x - 3) or using $y = mx + c$	M1	Use of : $\frac{dy}{dx}$ and (3, their 4) to find the tangent equation.
	y-4=2(x-3) or $y=2x-2$	A1	If using $= mx + c$, getting $c = -2$ is enough.
	For C ₂ : $\frac{dy}{dx} = A(4x+k)^{-\frac{1}{2}}$	*M1	Finds $\frac{dy}{dx}$ for C_2 in the form $A(4x+k)^{-\frac{1}{2}}$

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Question	Answer	Marks	Guidance	
9	At P: 'their 2' = $A(4x+k)^{-\frac{1}{2}} \to (x = \frac{1-k}{4} \text{ or } 4x + k = 1)$	*DM1	Equating ' <i>their</i> 2' to ' <i>their</i> $\frac{dy}{dx}$ ' and simplify to	
			form a linear equation linking $4x + k$ and a constant.	
	$(2x-2)^2 = 4x + k \rightarrow (2x-2)^2 = 1 \rightarrow (4x^2 - 8x + 3 = 0)$	DM1	Using <i>their</i> $y = 2x - 2$, $y^2 = 4x + k$ and <i>their</i> 4x + k = 1 (but not =0) to form a 3 term quadratic in x .	
	$x = \frac{3}{2} \left(or \frac{1}{2} \right)$ and from $k = -5 \left(or -1 \right)$	A1	Needs correct values for <i>x</i> and <i>k</i> .	
	from $y^2 = 4x + k$, $y = 1(or - 1)$.	A1	Condone 'correct' extra solution.	
	Alternative method for question 9			
	For C ₁ : $\frac{dy}{dx} = 2x - 4 \rightarrow m = 2$	B1		
	y - 'their 4' = 'their m' (x - 3) or using $y = mx + c$	M1	Use of : $\frac{dy}{dx}$ and (3, their 4) to find the tangent equation.	
	y-4 = 2(x-3) or $y = 2x-2$	A1	If using $= mx + c$, getting $c = -2$ is enough.	
	For C ₂ : $\frac{dy}{dx} = A(4x+k)^{-\frac{1}{2}}$	*M1	Finds $\frac{dy}{dx}$ for C_2 in the form $A(4x+k)^{-\frac{1}{2}}$	
	At P: 'their 2' = $A(4x+k)^{-\frac{1}{2}} \to (x = \frac{1-k}{4} \text{ or } 4x + k = 1)$	*DM1	Equating ' <i>their</i> 2' to ' <i>their</i> $\frac{dy}{dx}$, and simplify to form a linear equation linking $4x + k$ and a constant.	
	From $4x + k = 1$ and $y^2 = 4x + k \rightarrow y^2 = 1$	DM1	Using <i>their</i> $4x + k = 1$ (but not =0) and C_2 to form $y^2 = a$ constant	

Question	Answer	Marks	Guidance
9	$y = 1(or - 1)$ and $x = \frac{3}{2}\left(or\frac{1}{2}\right)$	A1	Needs correct values for <i>y</i> and <i>x</i> .
	From $4x + k = 1$, $k = -5$ (or -1)	A1	Condone 'correct' extra solution
		8	

Question	Answer	Marks	Guidance
10(a)(i)	$S_{10} = S_{15} - S_{10}$ or $S_{10} = S_{(11 \text{ to } 15)}$	M1	Either statement seen or implied.
	5(2a+9d) oe	B1	
	7.5(2a + 14d) – 5(2a + 9d) or $\frac{5}{2}[(a + 10d) + (a + 14d)]$ oe	A1	
	$d = \frac{a}{3} \mathbf{A}\mathbf{G}$	A1	Correct answer from convincing working
		4	Condone starting with $d = \frac{a}{3}$ and evaluating both summations as 25a.
10(a)(ii)	(a+9d) = 36 + (a+3d)	M1	Correct use of $a + (n-1)d$ twice and addition of ± 36
	<i>a</i> = 18	A1	
		2	Correct answer www scores 2/2

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Question	Answer	Marks	Guidance
10(b)	$S_{\infty} = 9 \times S_4; \ \frac{a}{1-r} = 9 \frac{a(1-r^4)}{1-r} \text{ or } 9(a+ar+ar^2+ar^3)$	B1	May have 12 in place of <i>a</i> .
	$9(1 - r^n) = 1$ where $n = 3,4$ or 5	M1	Correctly deals with <i>a</i> and correctly eliminates $(1 - r)^2$
	$r^4 = \frac{8}{9}$ oe	A1	
	$(5^{\text{th}} \text{ term} =) 10^{2/3} \text{ or } 10.7$	A1	
		4	Final answer of 10.6 suggests premature approximation – award 3/4 www.

Question	Answer	Marks	Guidance
11(i)	$\frac{dy}{dx} = \left[\frac{1}{2}(4x+1)^{-\frac{1}{2}}\right] [\times 4] \left[-\frac{9}{2}(4x+1)^{-\frac{3}{2}}\right] [\times 4]$	B1B1B1	B1 B1 for each, without \times 4. B1 for \times 4 twice.
	$\left(\frac{2}{\sqrt{4x+1}} - \frac{18}{\left(\sqrt{4x+1}\right)^3} \text{ or } \frac{8x-16}{\left(4x+1\right)^{\frac{3}{2}}}\right)$		SC If no other marks awarded award B1 for both powers of $(4x + 1)$ correct.
	$\int y dx = \left[\frac{(4x+1)^{\frac{3}{2}}}{\frac{3}{2}}\right] [\div 4] + \left[\frac{9(4x+1)^{\frac{1}{2}}}{\frac{1}{2}}\right] [\div 4] (+C)$	B1B1B1	B1 B1 for each, without ÷ 4. B1 for ÷4 twice. + C not required.
	$\left(\frac{\left(\sqrt{4x+1}\right)^3}{6} + \frac{9}{2}\left(\sqrt{4x+1}\right)(+C)\right)$		SC If no other marks awarded , B1 for both powers of $(4x + 1)$ correct.
		6	
11(ii)	$\frac{dy}{dx} = 0 \longrightarrow \frac{2}{\sqrt{4x+1}} - \frac{18}{(4x+1)^{\frac{3}{2}}} = 0$	M1	Sets their $\frac{dy}{dx}$ to 0 (and attempts to solve
	$4x + 1 = 9 \text{ or } (4x + 1)^2 = 81$	A1	Must be from correct differential.
	x = 2, y = 6 or M is (2, 6) only.	A1	Both values required. Must be from correct differential.
		3	

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Question	Answer	Marks	Guidance
11(iii)	Realises area is $\int y dx$ and attempt to use their 2 and sight of 0.	*M1	Needs to use their integral and to see ' <i>their 2</i> ' substituted.
	Uses limits 0 to 2 correctly $\rightarrow [4.5 + 13.5] - [\frac{1}{6} + 4.5] (= 13\frac{1}{3})$	DM1	Uses both 0 and ' <i>their 2</i> ' and subtracts. Condone wrong way round.
	$(Area =) 1\frac{1}{3} \text{ or } 1.33$	A1	Must be from a correct differential and integral.
		3	$13\frac{1}{3}$ or $1\frac{1}{3}$ with little or no working scores M1DM0A0.