May/June 2019

9709_s19_ms_11

| Question | Answer | Marks | Guidance |
|----------|---|------------|---|
| 1(i) | Ind term = $(2x)^3 \times \left(\frac{k}{x}\right)^3 \times {}_6C_3$ | B2,1,0 | Term must be isolated |
| | $= 540 \rightarrow k = 1\frac{1}{2}$ | B 1 | |
| | | 3 | |
| 1(ii) | Term, in x^2 is $(2x)^4 \times \left(\frac{k}{x}\right)^2 \times {}_6C_2$ | B1 | All correct – even if <i>k</i> incorrect. |
| | $15 \times 16 \times k^2 = 540 \text{ (or } 540 x^2 \text{)}$ | B 1 | FT For $240k^2$ or $240k^2x^2$ |
| | | 2 | |

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|----------|--|-------|---|
| 2(i) | Eliminates x or $y \to y^2 - 4y + c - 3 = 0$ or $x^2 + (2c - 16)x + c^2 - 48 = 0$ | M1 | Eliminates <i>x</i> or <i>y</i> completely to a quadratic |
| | Uses $b^2 = 4ac \rightarrow 4c - 28 = 0$ | M1 | Uses discriminant = 0. (c the only variable) |
| | | | Any valid method (may be seen in part (i)) |
| | <i>c</i> = 7 | A1 | |
| | Alternative method for question 2(i) | | |
| | $\frac{dy}{dx} = \frac{1}{2\sqrt{(x+3)}} = \frac{1}{4}$ | M1 | |
| | Solving | M1 | |
| | <i>c</i> = 7 | A1 | |
| | | 3 | |
| 2(ii) | Uses $c = 7$, $y^2 - 4y + 4 = 0$ | M1 | Ignore (1,-2), c=-9 |
| | (1, 2) | A1 | |
| | | 2 | |

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|----------|---------------------------------|-------|--|
| 3 | Uses $A = \frac{1}{2}r^2\theta$ | M1 | Uses area formula. |
| | | | |
| | $\theta = \frac{2A}{r^2}$ | A1 | |
| | $P = r + r + r\theta$ | B1 | |
| | $P = 2r + \frac{2A}{r}$ | A1 | Correct simplified expression for <i>P</i> . |
| | | 4 | |

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| Question | Answer | Marks | Guidance |
|----------|--|-------|---|
| 4(i) | Gradient of $AB = -\frac{1}{2} \rightarrow$ Gradient of $BC = 2$ | M1 | Use of $m_1.m_2 = -1$ for correct lines |
| | Forms equation in $h \frac{3h-2}{h} = 2$ | M1 | Uses normal line equation or gradients for <i>h</i> . |
| | h = 2 | A1 | |
| | Alternative method for question 4(i) | | |
| | Vectors AB.BC=0 | M1 | Use of vectors AB and BC |
| | Solving | M1 | |
| | <i>h</i> = 2 | A1 | |
| | Alternative method for question 4(i) | · | |
| | Use of Pythagoras to find 3 lengths | M1 | |
| | Solving | M1 | |
| | h = 2 | A1 | |
| | | 3 | |
| 4(ii) | y coordinate of D is 6, $(3 \times \text{'their' h})$ $\frac{6-0}{x-4} = 2 \rightarrow x = 7 \rightarrow D(7, 6)$ | B1 | FT |
| | Vectors: AD.AB=0 | M1 A1 | Must use $y = 6$ Realises the <i>y</i> values of <i>C</i> and <i>D</i> are equal. Uses gradient or line equation to find <i>x</i> . |
| | | 3 | |

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| Question | Answer | Marks | Guidance |
|----------|--|-------|--|
| 5(i) | $-2(x-3)^{2}+15 \ (a=-3, b=15)$ | B1 B1 | Or seen as $a = -3$, $b = 15$ B1 for each value |
| | | 2 | |
| 5(ii) | $(f(x) \leq) 15$ | B1 | FT for (\leq) their "b" Don't accept (3,15) alone |
| | | 1 | |
| 5(iii) | $gf(x) = 2(-2x^2 + 12x - 3) + 5 = -4x^2 + 24x - 6 + 5$ | B1 | |
| | $gf(x) + 1 = 0 \to -4x^2 + 24x = 0$ | M1 | |
| | x = 0 or 6 | A1 | Forms and attempts to solve a quadratic Both answers given. |
| | | 3 | |

| Question | Answer | Marks | Guidance |
|----------|--|-------|---------------------------------------|
| 6(i) | LHS = $\left(\frac{1}{c} - \frac{s}{c}\right)^2 = \frac{(1-s)(1-s)}{c^2} = \frac{(1-s)(1-s)}{1-s^2}$ | B1 | Expresses tan in terms of sin and cos |
| | | B1 | correctly $1-s^2$ as the denominator |
| | $=\frac{(1-s)(1-s)}{(1-s)(1+s)}$ | M1 | Factors and correct cancelling www |
| | $\frac{1-\sin x}{1+\sin x} \qquad \qquad \text{AG}$ | A1 | |
| | | 4 | |

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| Question | Answer | Marks | Guidance |
|----------|---|-------|---|
| 6(ii) | Uses part (i) to obtain $\frac{1-\sin 2x}{1+\sin 2x} = \frac{1}{3} \rightarrow \sin 2x = \frac{1}{2}$ | M1 | Realises use of $2x$ and makes $\sin 2x$ the subject |
| | $x = \frac{\pi}{12}$ | A1 | Allow decimal (0.262) |
| | (or) $x = \frac{5\pi}{12}$ | A1 | FT for $\frac{1}{2}\pi$ – 1st answer. Allow decimal (1.31) $\frac{\pi}{12}$ and $\frac{5\pi}{12}$ only, and no others in range. SC sin $x=\frac{1}{2} \rightarrow \frac{\pi}{6} \frac{5\pi}{6}$ B1 |
| | | 3 | |

| Question | Answer | Marks | Guidance |
|----------|--|--------|--|
| 7(i) | $\overline{AM} = 1.5\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ $\overline{GM} = 6.5\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}$ | B3,2,1 | Loses 1 mark for each error. |
| | | 3 | |
| 7(ii) | \overrightarrow{AM} . $\overrightarrow{GM} = 9.75 - 16 - 25 = -31.25$ | M1 | Use of $x_1x_2 + y_1y_2 + z_1z_2$ on AM and GM |
| | \overrightarrow{AM} . $\overrightarrow{GM} = \sqrt{(1.5^2 + 4^2 + 5^2)} \times \sqrt{(6.5^2 + 4^2 + 5^2)} \cos GMA$ | M1 M1 | M1 for product of 2 modulii M1 all correctly connected |
| | Equating \rightarrow Angle <i>GMA</i> = 121° | A1 | |
| | | 4 | |

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| Question | Answer | Marks | Guidance |
|----------|---|-------|---|
| 8(a) | $ar^2 = 48, ar^3 = 32, r = \frac{2}{3}$ or $a = 108$ | M1 | Solution of the 2 eqns to give <i>r</i> (or <i>a</i>). A1 (both) |
| | $r = \frac{2}{3}$ and $a = 108$ | A1 | |
| | $S\infty = \frac{108}{\frac{1}{3}} = 324$ | A1 | FT Needs correct formula and r between -1 and 1 . |
| | | 3 | |
| 8(b) | Scheme A $a = 2.50, d = 0.16$ S _n = 12(5 + 23×0.16) | M1 | Correct use of either AP S_n formula. |
| | $S_n = 104$ tonnes. | A1 | |
| | Scheme B $a = 2.50, r = 1.06$ | B1 | Correct value of <i>r</i> used in GP. |
| | $=\frac{2.5(1.06^{24}-1)}{1.06-1}$ | M1 | Correct use of either S _n formula. |
| | $S_n = 127$ tonnes. | A1 | |
| | | 5 | |

| Question | Answer | Marks | Guidance |
|----------|--|-------|---|
| 9(i) | $-1 \le f(x) \le 5$ or $[-1, 5]$ (may use y or f instead of $f(x)$) | B1 B1 | $-1 \le f(x) \le 5$ or $-1 \le x \le 5$ or $(-1,5)$ or $[5,-1]$ B1 only |
| | | 2 | |

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|----------|---|-------|---|
| Question | Answer | Marks | Guidance |
| 9(ii) | | *B1 | Start and end at –ve <i>y</i> , symmetrical, centre +ve. |
| | $g(x) = 2 - 3\cos x \text{ for } 0 \le x \le p$ | DB1 | Shape all ok. Curves not lines. One cycle $[0,2\pi]$ Flattens at each end. |
| | | 2 | |

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| Question | Answer | Marks | Guidance |
|----------|---|-------|---|
| 9(iii) | (greatest value of $p =$) π | B1 | |
| | | 1 | |
| 9(iv) | $x = 2 - 3\cos x \rightarrow \cos x = \frac{1}{3}(2 - x)$ | M1 | Attempt at $\cos x$ the subject. Use of \cos^{-1} |
| | $g^{-1}(x) = \cos^{-1}\frac{2-x}{3}$ (may use 'y =') | A1 | Must be a function of x, |
| | | 2 | |

| Question | Answer | Marks | Guidance |
|----------|---|-------|---|
| 10(i) | integrating $\rightarrow \frac{dy}{dx} = x^2 - 5x (+c)$ | B1 | |
| | = 0 when $x = 3$ | M1 | Uses the point to find <i>c</i> after $\int = 0$. |
| | <i>c</i> = 6 | A1 | |
| | integrating again $\rightarrow y = \frac{x^3}{3} - \frac{5x^2}{2} + 6x (+d)$ | B1 | FT Integration again FT if a numerical constant term is present. |
| | use of (3, 6) | M1 | Uses the point to find <i>d</i> after $\int = 0$. |
| | $d = 1\frac{1}{2}$ | A1 | |
| | | 6 | |

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| Question | Answer | Marks | Guidance |
|----------|--|-------|---|
| 10(ii) | $\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 - 5x + 6 = 0 \longrightarrow x = 2$ | B1 | |
| | | 1 | |
| 10(iii) | $x = 3$, $\frac{d^2y}{dx^2} = 1$ and/or +ve Minimum. $x = 2$, $\frac{d^2y}{dx^2} = -1$ and/or -ve Maximum | B1 | WWW |
| | May use shape of ' $+x^3$ ' curve or change in sign of $\frac{dy}{dx}$ | B1 | www SC: $x = 3$, minimum, $x = 2$, maximum, B1 |
| | | 2 | |

| Question | Answer | Marks | Guidance |
|----------|---|-------|--|
| 11(i) | $3 \times -\frac{1}{2} \times (1+4x)^{-\frac{3}{2}}$ | B1 | |
| | $\frac{\mathrm{d}y}{\mathrm{d}x} = 3 \times -\frac{1}{2} \times (1+4x)^{-\frac{3}{2}} \times 4$ | B1 | Must have '× 4' |
| | If $x = 2$, $m = -\frac{2}{9}$, Perpendicular gradient $= \frac{9}{2}$ | M1 | Use of $m_1.m_2 = -1$ |
| | Equation of normal is $y-1=\frac{9}{2}(x-2)$ | M1 | Correct use of line eqn (could use y=0 here) |
| | Put $y = 0$ or on the line before $\rightarrow \frac{16}{9}$ | A1 | AG |
| | | 5 | |

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| Question | Answer | Marks | Guidance |
|----------|--|-------|--|
| 11(ii) | Area under the curve = $\int_{0}^{2} \frac{3}{\sqrt{1+4x}} dx = \frac{3\sqrt{1+4x}}{\frac{1}{2}} \div 4$ | B1 B1 | Correct without '÷4'. For 2nd B1, ÷4'. |
| | Use of limits 0 to $2 \rightarrow 4\frac{1}{2} - 1\frac{1}{2}$ | M1 | Use of correct limits in an integral. |
| | 3 | A1 | |
| | Area of the triangle = $\frac{1}{2} \times 1 \times \frac{2}{9} = \frac{1}{9}$ or attempt to find $\int_{16/9}^{2} \left(\frac{9}{2}x - 8\right) dx$ | M1 | Any correct method. |
| | Shaded area = $3 - \frac{1}{9} = 2\frac{8}{9}$ | A1 | |
| | | 6 | |