Question	Answer		Marks	Guidance
1	*EITHER*:	State or imply non-modular equation $3^2\left(2^x-1\right)^2=\left(2^x\right)^2$, or pair of equations $3\left(2^x-1\right)=\pm 2^x$	**M1**	$8\left(2^x\right)^2-18\left(2^x\right)+9=0$
		Obtain $2^x=\frac{3}{2}$ and $2^x=\frac{3}{4}$ or equivalent	**A1**	
	OR:	Obtain $2^x=\frac{3}{2}$ by solving an equation	**B1**	
		Obtain $2^x=\frac{3}{4}$ by solving an equation	**B1**	
	Use correct method for solving an equation of the form $2^x=a$, where $a>0$		**M1**	
	Obtain **final** answers $x = 0.585$ and $x = -0.415$ only		**A1**	The question requires 3 s.f. Do not ISW if they go on to reject one value
			4	

Question	Answer	Marks	Guidance
2	Use correct tan ($A \pm B$) formula and obtain an equation in tan θ	**M1**	$\frac{1}{\tan\theta}+\frac{1-\tan\theta\tan 45}{\tan\theta+\tan 45}=2$ Allow M1 with tan 45° $=\frac{1}{\tan\theta}+\frac{1-\tan\theta}{\tan\theta+1}$
	Obtain a correct equation in any form	**A1**	With values substituted
	Reduce to $3\tan^2\theta = 1$, or equivalent	**A1**	
	Obtain answer $x = 30°$	**A1**	One correct solution
	Obtain answer $x = 150°$	**A1**	Second correct solution and no others in range
	OR: use correct $\sin(A \pm B)$ and $\cos(A \pm B)$ to form equation in $\sin\theta$ and $\cos\theta$ M1A1		
	Reduce to $\tan^2\theta=\frac{1}{3}, \sin^2\theta=\frac{1}{4}, \cos^2\theta=\frac{3}{4}$ or $\cot^2\theta=3$ etc. A1		
		5	

Question	Answer	Marks	Guidance
3(i)	Fully justify the given statement	**B1**	Some indication of use of gradient of curve = gradient of tangent (*PT*) and no errors seen /no incorrect statements
		1	
3(ii)	Separate variables and attempt integration of at least one side Obtain terms ln y and $\frac{1}{2}x$	**B1 B1**	Must be working from $\int \frac{1}{y}\,dy = \int k\,dx$ B marks are not available for fortuitously correct answers
	Use $x = 4$, $y = 3$ to evaluate a constant or as limits in a solution with terms $a \ln y$ and bx, where $ab \neq 0$	**M1**	
	Obtain correct solution in any form	**A1**	$\ln y = \frac{1}{2}x + \ln 3 - 2$
	Obtain answer $y = 3e^{\frac{1}{2}x-2}$, or equivalent	**A1**	Accept $y = e^{\frac{1}{2}x+\ln 3-2}$, $y = e^{\frac{x-1.80}{2}}$, $y = 3\sqrt{e^{x-4}}$ $\lvert y\rvert = \ldots$ scores A0
		5	

Question	Answer	Marks	Guidance
4(i)	Use correct double angle formulae and express LHS in terms of $\cos x$ and $\sin x$	M1	$\dfrac{2\sin x - 2\sin x\cos x}{1-\left(2\cos^2 x - 1\right)}$
	Obtain a correct expression	A1	
	Complete method to get correct denominator e.g. by factorising to remove a factor of $1-\cos x$	M1	
	Obtain the given RHS correctly *OR (working R to L)*:	A1	
	$\dfrac{\sin x}{1+\cos x}\times\dfrac{1-\cos x}{1-\cos x}=\dfrac{\sin x-\sin x\cos x}{1-\cos^2 x}$ $=\dfrac{2\sin x-2\sin x\cos x}{2-2\cos^2 x}$ M1A1		Given answer so check working carefully
	$=\dfrac{2\sin x-\sin 2x}{1-\cos 2x}$ M1A1		
		4	
4(ii)	State integral of the form $a\ln(1+\cos x)$	M1*	If they use the substitution $u=1+\cos x$ allow M1A1 for $-\ln u$
	Obtain integral $-\ln(1+\cos x)$	A1	
	Substitute correct limits in correct order	M1(dep)*	
	Obtain answer $\ln\left(\dfrac{3}{2}\right)$, or equivalent	A1	
		4	

Question	Answer	Marks	Guidance
5(i)	State or imply $3y^2\dfrac{dy}{dx}$ as derivative of y^3	B1	
	State or imply $6xy+3x^2\dfrac{dy}{dx}$ as derivative of $3x^2y$ *OR* State or imply $2x(x+3y)+x^2\left(1+3\dfrac{dy}{dx}\right)$ as derivative of $x^2(x+3y)$	B1	$3x^2+6xy+3x^2\dfrac{dy}{dx}-3y^2\dfrac{dy}{dx}=0$
	Equate derivative of the LHS to zero and solve for $\dfrac{dy}{dx}$	M1	Given answer so check working carefully
	Obtain the given answer	A1	
		4	
5(ii)	Equate derivative to – 1 and solve for y	M1*	
	Use their $y = -2x$ or equivalent to obtain an equation in x or y	M1(dep*)	
	Obtain answer $(1, -2)$	A1	
	Obtain answer $(\sqrt[3]{3}, 0)$	B1	Must be exact e.g. $e^{\frac{1}{3}\ln 3}$ but ISW if decimals after exact value seen
		4	

Question	Answer	Marks	Guidance
6(i)	Use correct method for finding the area of a segment and area of semicircle and form an equation in θ	M1	e.g. $\frac{\pi a^2}{4} = \frac{1}{2}a^2\theta - \frac{1}{2}a^2 \sin\theta$
	State a correct equation in any form	A1	Given answer so check working carefully
	Obtain the given answer correctly	A1	
		3	
6(ii)	Calculate values of a relevant expression or pair of expressions at $\theta = 2.2$ and $\theta = 2.4$	M1	e.g. $f(\theta) = \frac{\pi}{2} + \sin\theta$ $\begin{cases} f(2.2) = 2.37... > 2.2 \\ f(2.4) = 2.24... < 2.4 \end{cases}$ or $f(\theta) = \theta - \frac{\pi}{2} - \sin\theta$ $\begin{cases} f(2.2) = -0.17... < 0 \\ f(2.4) = +0.15... > 0 \end{cases}$
	Complete the argument correctly with correct calculated values	A1	
		2	

Question	Answer	Marks	Guidance
6(iii)	Use $\theta_{n+1} = \frac{1}{2}\pi + \sin\theta_n$ correctly at least once	M1	e.g.
	Obtain final answer 2.31	A1	
	Show sufficient iterations to 4 d.p. to justify 2.31 to 2 d.p. or show there is a sign change in the interval (2.305, 2.315)	A1	
		3	

2.2	2.3	2.4
2.3793	2.3165	2.2463
2.2614	2.3054	2.3512
2.3417	2.3129	2.2814
2.2881	2.3079	2.3288
2.3244		2.2970
2.3000		2.3185
2.3165		2.3041
2.3054		2.3138
2.3129		2.3072

Question	Answer		Marks	Guidance
7(i)	Substitute in uv, expand the product and use $i^2 = -1$		M1	
	Obtain answer $uv = -11 - 5\sqrt{3}i$		A1	
	EITHER:	Substitute in u/v and multiply numerator and denominator by the conjugate of v, or equivalent	M1	
		Obtain numerator $-7 + 7\sqrt{3}i$ or denominator 7	A1	
		Obtain final answer $-1 + \sqrt{3}i$	A1	
	OR:	Substitute in u/v , equate to $x + iy$ and solve for x or for y	M1	$\begin{cases} -3\sqrt{3} = \sqrt{3}x - 2y \\ 1 = 2x + \sqrt{3}y \end{cases}$
	Obtain $x = -1$ or $y = \sqrt{3}$		A1	
	Obtain final answer $-1 + \sqrt{3}\ i$		A1	
			5	

Question	Answer	Marks	Guidance
7(ii)	Show the points *A* and *B* representing *u* and *v* in relatively correct positions	**B1**	
	Carry out a complete method for finding angle *AOB*, e.g. calculate arg(*u*/*v*) If using $\theta = \tan^{-1}\left(-\sqrt{3}\right)$ must refer to $\arg\left(\frac{u}{v}\right)$	**M1**	*OR*: $\tan a = \frac{-1}{3\sqrt{3}}, \tan b = \frac{2}{\sqrt{3}} \Rightarrow \tan\left(a-b\right) = \frac{\frac{-1}{3\sqrt{3}} - \frac{2}{\sqrt{3}}}{1-\frac{2}{9}}$ $= -\sqrt{3}$ $\Rightarrow \theta = \frac{2\pi}{3}$ *OR*: $\cos\theta = \frac{\begin{pmatrix}-3\sqrt{3}\\1\end{pmatrix}\begin{pmatrix}\sqrt{3}\\2\end{pmatrix}}{\sqrt{7}\sqrt{28}} = \frac{-9+2}{14} = \frac{-1}{2}$ $\Rightarrow \theta = \frac{2\pi}{3}$ *OR*: $\cos\theta = \frac{28+7-49}{2\sqrt{28}\sqrt{7}} = -\frac{1}{2} \Rightarrow \theta = \frac{2\pi}{3}$
	Prove the given statement	**A1**	Given answer so check working carefully
		3	

Question	Answer	Marks	Guidance
8(i)	Use correct product or quotient rule	M1	$\frac{dy}{dx} = -\frac{1}{3}(x+1)e^{-\frac{1}{3}x} + e^{-\frac{1}{3}x}$ or $\frac{dy}{dx} = \frac{e^{\frac{1}{3}x} - (x+1)\frac{1}{3}e^{\frac{1}{3}x}}{e^{\frac{2}{3}x}}$
	Obtain complete correct derivative in any form	A1	
	Equate derivative to zero and solve for x	M1	
	Obtain answer $x = 2$ with no errors seen	A1	
		4	
8(ii)	Integrate by parts and reach $a(x+1)e^{-\frac{1}{3}x} + b\int e^{-\frac{1}{3}x}\,dx$	M1*	
	Obtain $-3(x+1)e^{-\frac{1}{3}x} + 3\int e^{-\frac{1}{3}x}\,dx$, or equivalent	A1	$-3xe^{-\frac{1}{3}x} + \int 3e^{-\frac{1}{3}x}dx - 3e^{-\frac{1}{3}x}$
	Complete integration and obtain $-3(x+1)e^{-\frac{1}{3}x} - 9e^{-\frac{1}{3}x}$, or equivalent	A1	
	Use **correct** limits $x = -1$ and $x = 0$ **in the correct order**, having integrated twice	M1(dep*)	
	Obtain answer $9e^{\frac{1}{3}} - 12$, or equivalent	A1	
		5	

Question	Answer	Marks	Guidance
9(i)	Use a correct method to find a constant	**M1**	
	Obtain one of the values $A = -3$, $B = 1$, $C = 2$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		4	
9(ii)	Use a correct method to find the first two terms of the expansion of $(3-x)^{-1}$, $\left(1-\frac{1}{3}x\right)^{-1}$, $\left(2+x^2\right)^{-1}$ or $\left(1+\frac{1}{2}x^2\right)^{-1}$	**M1**	Symbolic binomial coefficients are not sufficient for the M1.
	Obtain correct unsimplified expansions up to the term in x^3 of each partial fraction	**A1Ft + A1Ft**	The ft is on *A*, *B* and *C*. $-1\left(1+\frac{x}{3}+\frac{x^2}{9}+\frac{x^3}{27}...\right) + \frac{x+2}{2}\left(1-\frac{x^2}{2}...\right)$ $-1-\frac{x}{3}-\frac{x^2}{9}-\frac{x^3}{27}+1-\frac{x^2}{2}+\frac{x}{2}-\frac{x^3}{4}$
	Multiply out their expansion, up to the terms in x^3, by $Bx + C$, where $BC \neq 0$	**M1**	
	Obtain **final** answer $\frac{1}{6}x-\frac{11}{18}x^2-\frac{31}{108}x^3$, or equivalent	**A1**	
		5	

Question	Answer		Marks	Guidance
10(i)	Equate at least two pairs of components and solve for s or for t		M1	$\begin{cases} s = \frac{-4}{3} \\ t = \frac{-5}{3} \\ -5 \neq \frac{-1}{3} \end{cases}$ or $\begin{cases} s = -6 \\ t = -11 \\ 7 \neq -7 \end{cases}$ or $\begin{cases} s = \frac{-2}{5} \\ t = \frac{-13}{5} \\ \frac{6}{5} \neq \frac{-8}{5} \end{cases}$
	Obtain correct answer for s or t, e.g. $s = -6$, $t = -11$		A1	
	Verify that all three equations are not satisfied and the lines fail to intersect		A1	
	State that the lines are not parallel		B1	
			4	
10(ii)	*EITHER*:	Use scalar product to obtain a relevant equation in a, b and c, e.g. $2a + 3b - c = 0$	B1	
		Obtain a second equation, e.g. $a + 2b + c = 0$, and solve for one ratio, e.g. $a : b$	M1	
		Obtain $a : b : c$ and state correct answer, e.g. $5\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, or equivalent	A1	
	OR:	Attempt to calculate vector product of relevant vectors, e.g. $(2\mathbf{i} + 3\mathbf{j} - \mathbf{k}) \times (\mathbf{i} + 2\mathbf{j} + \mathbf{k})$	M1	
		Obtain two correct components	A1	
		Obtain correct answer, e.g. $5\mathbf{i} - 3\mathbf{j} + \mathbf{k}$	A1	
			3	

Question	Answer		Marks	Guidance
10(iii)	*EITHER*:	State position vector or coordinates of the mid-point of a line segment joining points on l and m, e.g. $\frac{3}{2}\mathbf{i}+\mathbf{j}+\frac{5}{2}\mathbf{k}$	**B1**	*OR*: Use the result of **(ii)** to form equations of planes containing l and m B1
		Use the result of (**ii**) and the mid-point to find d	**M1**	Use average of distances to find equation of p. M1
		Obtain answer $5x - 3y + z = 7$, or equivalent	**A1**	Obtain answer $5x - 3y + z = 7$, or equivalent A1
	OR:	Using the result of part **(ii)**, form an equation in d by equating perpendicular distances to the plane of a point on l and a point on m	**M1**	
		State a correct equation, e.g. $\left\|\frac{14-d}{\sqrt{35}}\right\| = \left\|\frac{-d}{\sqrt{35}}\right\|$	**A1**	
		Solve for d and obtain answer $5x - 3y + z = 7$, or equivalent	**A1**	
			3	