May/June 2018

9709_s18_ms_11

Question	Answer	Marks	Guidance
1(i)	$(1-2x)^5 = 1 - 10x + 40x^2$ (no penalty for extra terms)	B2,1	Loses a mark for each incorrect term. Treat $-32x^5 + 80x^4 - 80x^3$ as MR -1
		2	
1(ii)	$\rightarrow (1 + ax + 2x^2)(1 - 10x + 40x^2)$		
	$3 \text{ terms in } x^2 \rightarrow 40 - 10a + 2$	M1 A1FT	Selects 3 terms in x^2 . FT from (i)
	Equate with $12 \rightarrow a = 3$	A1	САО
		3	

Question	Answer	Marks	Guidance
2	$y = 2x + \frac{5}{x} \rightarrow \frac{dy}{dx} = 2 - \frac{5}{x^2} = -3$ (may be implied) when $x = 1$.	M1 A1	Reasonable attempt at differentiation CAO (-3)
	$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t} \to -0.06$	M1 A1	Ignore notation, but needs to multiply $\frac{dy}{dx}$ by 0.02.
		4	

May/June 2018

Question	Answer	Marks	Guidance
3	$\frac{dy}{dx} = \frac{12}{(2x+1)^2} \to y = \frac{-12}{2x+1} \div 2 \ (+c)$	B1 B1	Correct without " \div 2". For " \div 2". Ignore " c ".
	Uses (1, 1) $\rightarrow c = 3 \ (\rightarrow y = \frac{-6}{2x+1} + 3)$	M1 A1	Finding " <i>c</i> " following integration. CAO
	Sets y to 0 and attempts to solve for $x \to x = \frac{1}{2} \to ((\frac{1}{2}, 0))$	DM1 A1	Sets y to 0. $x = \frac{1}{2}$ is sufficient for A1.
		6	

Question	Answer	Marks	Guidance
4(i)	$(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) \equiv \sin^3\theta + \cos^3\theta.$		Accept abbreviations s and c
	$LHS = \sin\theta + \cos\theta - \sin^2\theta\cos\theta - \sin\theta\cos^2\theta$	M1	Expansion
	$= \sin\theta(1 - \cos^2\theta) + \cos\theta(1 - \sin^2\theta) \text{ or } (s + c - c(1 - c^2) - s(1 - s^2))$	M1A1	Uses identity twice. Everything correct. AG
	Uses $\sin^2\theta + \cos^2\theta = 1 \rightarrow \sin^3\theta + \cos^3\theta$ (RHS)		or from RHS: M1 for use of trig ID twice
	Or		
	LHS = $(\sin\theta + \cos\theta)(\sin^2\theta + \cos^2\theta - \sin\theta\cos\theta)$	M1	M1 for factorisation
	$= \sin^{3}\theta + \sin\theta\cos^{2}\theta - \sin^{2}\theta\cos\theta + \cos\theta\sin^{2}\theta + \cos^{3}\theta - \sin\theta\cos^{2}\theta = \sin^{3}\theta + \cos^{3}\theta$	M1A1	
		3	

9709/11

Cambridge International AS/A Level – Mark Scheme **PUBLISHED**

Question	Answer	Marks	Guidance
4(ii)	$(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) = 3\cos^3\theta \rightarrow \sin^3\theta = 2\cos^3\theta$	M1	
	$\rightarrow \tan^3 \theta = 2 \rightarrow \theta = 51.6^\circ \text{ or } 231.6^\circ \text{ (only)}$	A1A1FT	Uses $\tan^3 = \sin^3 \div \cos^3$. A1 CAO. A1FT, 180 + their acute angle. $tan^3\theta = 0$ gets M0
		3	

Question	Answer	Marks	Guidance
5(i)	Eqn of AC $y = -\frac{1}{2}x + 4$ (gradient must be $\Delta y / \Delta x$)	M1A1	Uses gradient and a given point for equa. CAO
	Gradient of $OB = 2 \rightarrow y = 2x$ (If y missing only penalise once)	M1 A1	Use of $m_1m_2 = -1$, answers only ok.
		4	

9709/11

May/June 2018

97	09	s18	ms	11

Question	Answer	Marks	Guidance
5(ii)	Simultaneous equations \rightarrow ((1.6, 3.2))	M1	Equate and solve for M1 and reach ≥ 1 solution
	This is mid-point of OB . $\rightarrow B$ (3.2, 6.4)	M1 A1	Uses mid-point. CAO
	or		
	Let coordinates of $B(h, k)$ $OA = AB \rightarrow h^2 = 8k - k^2$ $OC = BC \rightarrow k^2 = 16h - h^2 \rightarrow (3.2, 6.4)$		M1 for both equations, M1 for solving with $y = 2x$
	or		
	gradients $\left(\frac{k-4}{h} \times \frac{k}{h-8} = -1\right)$		M1 for gradient product as -1 , M1 solving with $y = 2x$
	or		
	Pythagoras: $h^2 + (k-4)^2 + (h-8)^2 + k^2 = 4^2 + 8^2$		M1 for complete equation, M1 solving with $y = 2x$
		3	

Question	Answer	Marks	Guidance
6(i)	$(\tan\theta = \frac{AT}{r}) \rightarrow AT = r \tan\theta \text{ or } OT = \frac{r}{\cos\theta}$ SOI	B1	CAO
	$\rightarrow A = \frac{1}{2} r^2 \tan \theta \qquad -\frac{1}{2} r^2 \theta$		B1 for $\frac{1}{2}r^2 \tan\theta$. B1 for " $-\frac{1}{2}r^2\theta$ " If Pythagoras used may see area of triangle as $\frac{1}{2}r\sqrt{r^2 + r^2 \tan^2\theta}$ or $\frac{1}{2}r\left(\frac{r}{\cos\theta}\right)sin\theta$
		3	

May/June 2018

Question	Answer	Marks	Guidance
6(ii)	$\tan\theta = \frac{AT}{3} \rightarrow AT = 7.716$	M1	Correct use of trigonometry and radians in rt angle triangle
	Arc length = $r\theta$ = 3.6	B 1	Accept 3×1.2
	OT by Pythagoras or $\cos 1.2 = \frac{3}{OT}$ (= 8.279)	M1	Correct method for <i>OT</i>
	Perimeter = AT + arc + OT - radius = 16.6	A1	CAO, www
		4	

Question	Answer	Marks	Guidance
7	$\overrightarrow{OA} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} \text{ and } \overrightarrow{OC} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$		
7(i)	$\overrightarrow{AC} = \begin{pmatrix} 2\\ 4\\ -4 \end{pmatrix}$	B1	B1 for \overline{AC} .
		1	

May/June 2018

Question	Answer	Marks	Guidance
7(ii)	$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM} = \begin{pmatrix} 2\\-1\\0 \end{pmatrix} \text{ or } \frac{1}{2} \begin{bmatrix} 1\\-3\\2 \end{bmatrix} + \begin{pmatrix} 3\\1\\-2 \end{bmatrix}$	M1	M1 for their $\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$ oe
	Unit vector in direction of $\overrightarrow{OM} = \frac{1}{\sqrt{5}} (\overrightarrow{OM})$	M1 A1	M1 for dividing their \overline{OM} by their modulus
		3	
7(iii)	$\overrightarrow{AB} = \begin{pmatrix} -2\\6\\3 \end{pmatrix}, \text{ Allow } \pm$	B1	
	$ \overrightarrow{AB} =7, \overrightarrow{AC} =6 \begin{pmatrix} -2\\6\\3 \end{pmatrix} \begin{pmatrix} 2\\4\\-4 \end{pmatrix} = -4 + 24 - 12 = 8$	M1 M1	Product of both moduli, Scalar product of \pm their AB and AC
	$7 \times 6 \cos \theta = 8 \rightarrow \theta = 79.(0)^{\circ}$	A1	1.38 radians ok
		4	

9709/11

May/June 2018

Question	Answer	Marks	Guidance
8(a)	$ar = 12$ and $\frac{a}{1-r} = 54$	B1 B1	CAO, OE CAO, OE
	Eliminates <i>a</i> or $r \to 9r^2 - 9r + 2 = 0$ or $a^2 - 54a + 648 = 0$	M1	Elimination leading to a 3-term quadratic in <i>a</i> or <i>r</i>
	$\rightarrow r = \frac{2}{3} \text{ or } \frac{1}{3} \text{ hence to } a \rightarrow a = 18 \text{ or } 36$	A1	Needs both values.
		4	
8(b)	<i>n</i> th term of a progression is $p + qn$		
8(b)(i)	first term = $p + q$. Difference = q or last term = $p + qn$	B1	Need first term and, last term or common difference
	$S_n = \frac{n}{2} (2(p+q) + (n-1)q) \text{ or } \frac{n}{2} (2p+q+nq)$	M1A1	Use of S_n formula with their a and d . ok unsimplified for A1.
		3	
8(b)(ii)	Hence $2(2p+q+4q) = 40$ and $3(2p+q+6q) = 72$	DM1	Uses their S_n formula from (i)
	Solution $\rightarrow p = 5$ and $q = 2$ [Could use S_n with a and $d \rightarrow a = 7, d = 2 \rightarrow p = 5, q = 2.$]	A1	Note: answers 7, 2 instead of 5, 2 gets M1A0 – must attempt to solve for M1
		2	

May/June 2018

Question	Answer	Marks	Guidance
9	$f: x \mapsto \frac{x}{2} - 2, g: x \mapsto 4 + x - \frac{x^2}{2}$		
9(i)	$4 + x - \frac{x^2}{2} = \frac{x}{2} - 2 \to x^2 - x - 12 = 0$	M1	Equates and forms 3 term quadratic
	\rightarrow (4, 0) and (-3, -3.5) Trial and improvement, B3 all correct or B0	A1 A1	A1 For both <i>x</i> values or a correct pair. A1 all.
		3	
9(ii)	f(x) > g(x) for $x > 4, x < -3$	B1, B1	B1 for each part. Loses a mark for \leq or \geq .
		2	
9(iii)	$fg(x) = 2 + \frac{x}{2} - \frac{x^2}{4} - 2 \left(= \frac{x}{2} - \frac{x^2}{4}\right)$	B1	CAO, any correct form
	i.e. $-\frac{1}{4}((x-1)^2 - 1)$ or $\frac{dy}{dx} = \frac{1}{2} - \frac{2x}{4} = 0 \rightarrow x = 1$	M1 A1	Completes the square or uses calculus. First A1 is for $x = 1$ or completed square form
	$\rightarrow y = \frac{1}{4} \rightarrow \text{Range of fg} \leq \frac{1}{4},$	A1	CAO, OE e.g. $y \le \frac{1}{4}$, $[-\infty, \frac{1}{4})$ etc.
		4	
9(iv)	Calculus or completing square on 'h' $\rightarrow x = 1$	M1	May use a sketch or $-\frac{b}{2a}$
	$k = 1$ (accept $k \ge 1$)	A1	Complete method. CAO
		2	

Question	Answer	Marks	Guidance
10	$y = x^3 - 2x^2 + 5x$		
10(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 4x + 5$	B1	CAO
	Using $b^2 - 4ac \rightarrow 16 - 60 \rightarrow$ negative \rightarrow some explanation or completed square and explanation	M1 A1	Uses discriminant on equation (set to 0). CAO
		3	
10(ii)	$m = 3x^2 - 4x + 5$ $\frac{dm}{dx} = 6x - 4 (= 0) \text{ (must identify as } \frac{dm}{dx}\text{)}$	B1FT	FT providing differentiation is equivalent
	$\rightarrow x = \frac{2}{3}, \ m = \frac{11}{3} \text{ or } \frac{dy}{dx} = \frac{11}{3}$	M1 A1	Sets to 0 and solves. A1 for correct <i>m</i> .
	Alt1: $m = 3\left(x - \frac{2}{3}\right)^2 + \frac{11}{3}, \ m = \frac{11}{3}$		Alt1: B1 for completing square, M1A1 for ans
	Alt2: $3x^2 - 4x + 5 - m = 0$, $b^2 - 4ac = 0$, $m = \frac{11}{3}$		Alt2: B1 for coefficients, M1A1 for ans
	$\frac{d^2m}{dx^2} = 6 + ve \rightarrow$ Minimum value or refer to sketch of curve or	M1 A1	M1 correct method. A1 (no errors anywhere)
	check values of <i>m</i> either side of $x = \frac{2}{3}$,		
		5	

9709/11

Cambridge International AS/A Level – Mark Scheme **PUBLISHED**

May/June 2018

Question	Answer	Marks	Guidance
10(iii)	Integrate $\rightarrow \frac{x^4}{4} - \frac{2x^3}{3} + \frac{5x^2}{2}$	B2,1	Loses a mark for each incorrect term
	Uses limits 0 to 6 \rightarrow 270 (may not see use of lower limit)	M1 A1	Use of limits on an integral. CAO Answer only 0/4
		4	