| Question | Answer | Marks |
| :---: | :---: | :---: |
| 1 | Use law of the logarithm of a power or a quotient | M1 |
|  | Remove logarithms and obtain a correct equation in $x$. e.g. $x^{2}+1=\mathrm{e} x^{2}$ | A1 |
|  | Obtain answer 0.763 and no other | A1 |
|  | Total: | 3 |
| 2 | EITHER: <br> State or imply non-modular inequality $(x-3)^{2}<(3 x-4)^{2}$, or corresponding equation | (B1 |
|  | Make reasonable attempt at solving a three term quadratic | M1 |
|  | Obtain critical value $x=\frac{7}{4}$ | A1 |
|  | State final answer $x>\frac{7}{4}$ only | A1) |
|  | OR1: <br> State the relevant critical inequality $3-x<3 x-4$, or corresponding equation | (B1 |
|  | Solve for $x$ | M1 |
|  | Obtain critical value $x=\frac{7}{4}$ | A1 |
|  | State final answer $x>\frac{7}{4}$ only | A1) |
|  | OR2: <br> Make recognizable sketches of $y=\|x-3\|$ and $y=3 x-4$ on a single diagram | (B1 |
|  | Find $x$-coordinate of the intersection | M1 |
|  | $\text { Obtain } x=\frac{7}{4}$ | A1 |
|  | State final answer $x>\frac{7}{4}$ only | A1) |
|  | Total: | 4 |


| Question | Answer |  | Marks |
| :---: | :---: | :---: | :---: |
| 3(i) | Use correct formulae to express the equation in terms of $\cos \theta$ and $\sin \theta$ |  | M1 |
|  | Use Pythagoras and express the equation in terms of $\cos \theta$ only |  | M1 |
|  | Obtain correct 3-term equation, e.g. $2 \cos ^{4} \theta+\cos ^{2} \theta-2=0$ |  | A1 |
|  |  | Total: | 3 |
| 3(ii) | Solve a 3-term quadratic in $\cos ^{2} \theta$ for $\cos \theta$ |  | M1 |
|  | Obtain answer $\theta=152.1^{\circ}$ only |  | A1 |
|  |  | Total: | 2 |
| 4(i) | State $\frac{\mathrm{d} y}{\mathrm{~d} t}=4+\frac{2}{2 t-1}$ |  | B1 |
|  | Use $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{\mathrm{d} y}{\mathrm{~d} t} \div \frac{\mathrm{d} x}{\mathrm{~d} t}$ |  | M1 |
|  | Obtain answer $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{8 t-2}{2 t(2 t-1)}$, or equivalent e.g. $\frac{2}{t}+\frac{2}{4 t^{2}-2 t}$ |  | A1 |
|  |  | Total: | 3 |
| 4(ii) | Use correct method to find the gradient of the normal at $t=1$ |  | M1 |
|  | Use a correct method to form an equation for the normal at $t=1$ |  | M1 |
|  | Obtain final answer $x+3 y-14=0$, or horizontal equivalent |  | A1 |
|  |  | Total: | 3 |


| Question | Answer | Marks |
| :---: | :---: | :---: |
| 5(i) | State $\frac{\mathrm{d} y}{\mathrm{~d} t}=-\frac{2 y}{(1+t)^{2}}$, or equivalent | B1 |
|  | Separate variables correctly and attempt integration of one side | M1 |
|  | Obtain term $\ln y$, or equivalent | A1 |
|  | Obtain term $\frac{2}{(1+t)}$, or equivalent | A1 |
|  | Use $y=100$ and $t=0$ to evaluate a constant, or as limits in an expression containing terms of the form $a \ln y$ and $\frac{b}{1+t}$ | M1 |
|  | Obtain correct solution in any form, e.g. $\ln y=\frac{2}{1+t}-2+\ln 100$ | A1 |
|  | Total: | 6 |
| 5(ii) | State that the mass of $B$ approaches $\frac{100}{\mathrm{e}^{2}}$, or exact equivalent | B1 |
|  | State or imply that the mass of $A$ tends to zero | B1 |
|  | Total: | 2 |


| Question | Answer | Marks |
| :---: | :---: | :---: |
| 6(i) | EITHER: <br> Substitute $x=2-\mathrm{i}$ (or $x=2+\mathrm{i}$ ) in the equation and attempt expansions of $x^{2}$ and $x^{3}$ | (M1 |
|  | Equate real and/or imaginary parts to zero | M1 |
|  | Obtain $a=-2$ | A1 |
|  | Obtain $b=10$ | A1) |
|  | OR1: <br> Substitute $x=2-\mathrm{i}$ in the equation and attempt expansions of $x^{2}$ and $x^{3}$ | (M1 |
|  | Substitute $x=2+\mathrm{i}$ in the equation and add/subtract the two equations | M1 |
|  | Obtain $a=-2$ | A1 |
|  | Obtain $b=10$ | A1) |
|  | OR2: <br> Factorise to obtain $(x-2+\mathrm{i})(x-2-\mathrm{i})(x-p)\left(=\left(x^{2}-4 x+5\right)(x-p)\right)$ | (M1 |
|  | Compare coefficients | M1 |
|  | Obtain $a=-2$ | A1 |
|  | Obtain $b=10$ | A1) |
|  | OR3: <br> Obtain the quadratic factor $\left(x^{2}-4 x+5\right)$ | (M1 |
|  | Use algebraic division to obtain a real linear factor of the form $x-p$ and set the remainder equal to zero | M1 |
|  | Obtain $a=-2$ | A1 |
|  | Obtain $b=10$ | A1) |
|  | OR4: <br> Use $\alpha \beta=5$ and $\alpha+\beta=4$ in $\alpha \beta+\beta \gamma+\gamma \alpha=-3$ | (M1 |
|  | Solve for $\gamma$ and use in $\alpha \beta \gamma=-b$ and/or $\alpha+\beta+\gamma=-a$ | M1 |
|  | Obtain $a=-2$ | A1 |
|  | Obtain $b=10$ | A1) |


| Question | Answer | Marks |
| :---: | :---: | :---: |
|  | OR5: <br> Factorise as $(x--(2-i))\left(x^{2}+e x+g\right)$ and compare coefficients to form an equation in $a$ and $b$ | (M1 |
|  | Equate real and/or imaginary parts to zero | M1 |
|  | Obtain $a=-2$ | A1 |
|  | Obtain $b=10$ | A1) |
|  | Total: | 4 |
| 6(ii) | Show a circle with centre $2-\mathrm{i}$ in a relatively correct position | B1 |
|  | Show a circle with radius 1 and centre not at the origin | B1 |
|  | Show the perpendicular bisector of the line segment joining 0 to -i | B1 |
|  | Shade the correct region | B1 |
|  | Total: | 4 |
| 7(i) | Use quotient or chain rule | M1 |
|  | Obtain given answer correctly | A1 |
|  | Total: | 2 |
| 7(ii) | EITHER: <br> Multiply numerator and denominator of LHS by $1+\sin \theta$ | (M1 |
|  | Use Pythagoras and express LHS in terms of $\sec \theta$ and $\tan \theta$ | M1 |
|  | Complete the proof | A1) |
|  | OR1: <br> Express RHS in terms of $\cos \theta$ and $\sin \theta$ | (M1 |
|  | Use Pythagoras and express RHS in terms of $\sin \theta$ | M1 |
|  | Complete the proof | A1) |
|  | OR2: <br> Express LHS in terms of $\sec \theta$ and $\tan \theta$ | (M1 |
|  | Multiply numerator and denominator by $\sec \theta+\tan \theta$ and use Pythagoras | M1 |
|  | Complete the proof | A1) |
|  | Total: | 3 |


| Question | Answer | Marks |
| :---: | :---: | :---: |
| 7(iii) | Use the identity and obtain integral $2 \tan \theta+2 \sec \theta-\theta$ | B2 |
|  | Use correct limits correctly in an integral containing terms $a \tan \theta$ and $b \sec \theta$ | M1 |
|  | Obtain answer $2 \sqrt{2}-\frac{1}{4} \pi$ | A1 |
|  | Total: | 4 |
| 8(i) | State or imply the form $\frac{A}{3 x+2}+\frac{B x+C}{x^{2}+5}$ | B1 |
|  | Use a relevant method to determine a constant | M1 |
|  | Obtain one of the values $A=2, B=1, C=-3$ | A1 |
|  | Obtain a second value | A1 |
|  | Obtain the third value | A1 |
|  | Total: | 5 |
| 8(ii) | Use correct method to find the first two terms of the expansion of $(3 x+2)^{-1},\left(1+\frac{3}{2} x\right)^{-1}$, $\left(5+x^{2}\right)^{-1}$ or $\left(1+\frac{1}{5} x^{2}\right)^{-1}$ <br> [Symbolic coefficients, e.g. $\binom{-1}{2}$ are not sufficient] | M1 |
|  | Obtain correct unsimplified expansions up to the term in $x^{2}$ of each partial fraction. The FT is on $A, B, C$. from part (i) | $\begin{array}{r} \text { A1FT + } \\ \text { A1FT } \end{array}$ |
|  | Multiply out up to the term in $x^{2}$ by $B x+C$, where $B C \neq 0$ | M1 |
|  | Obtain final answer $\frac{2}{5}-\frac{13}{10} x+\frac{237}{100} x^{2}$, or equivalent | A1 |
|  | Total: | 5 |
| 9 (i) | EITHER: <br> Find $\overrightarrow{A P}$ for a general point $P$ on $l$ with parameter $\lambda$, e.g. $(8+3 \lambda,-3-\lambda, 4+2 \lambda)$ | (B1 |
|  | Equate scalar product of $\overrightarrow{A P}$ and direction vector of $l$ to zero and solve for $\lambda$ | M1 |
|  | Obtain $\lambda=-\frac{5}{2}$ and foot of perpendicular $\frac{3}{2} \mathbf{i}+\frac{3}{2} \mathbf{j}+3 \mathbf{k}$ | A1 |
|  | Carry out a complete method for finding the position vector of the reflection of $A$ in $l$ | M1 |
|  | Obtain answer $2 \mathbf{i}+\mathbf{j}+2 \mathbf{k}$ | A1) |


| Question | Answer | Marks |
| :---: | :---: | :---: |
|  | OR: <br> Find $\overrightarrow{A P}$ for a general point $P$ on $l$ with parameter $\lambda$, e.g. $(8+3 \lambda,-3-\lambda, 4+2 \lambda)$ | (B1 |
|  | Differentiate $\|A P\|^{2}$ and solve for $\lambda$ at minimum | M1 |
|  | Obtain $\lambda=-\frac{5}{2}$ and foot of perpendicular $\frac{3}{2} \mathbf{i}+\frac{3}{2} \mathbf{j}+3 \mathbf{k}$ | A1 |
|  | Carry out a complete method for finding the position vector of the reflection of $A$ in $l$ | M1 |
|  | Obtain answer $2 \mathbf{i}+\mathbf{j}+2 \mathbf{k}$ | A1) |
|  | Total: | 5 |
| 9(ii) | EITHER: <br> Use scalar product to obtain an equation in $a, b$ and $c$, e.g. $3 a-b+2 c=0$ | (B1 |
|  | Form a second relevant equation, e.g. $9 a-b+8 c=0$ and solve for one ratio, e.g. $a: b$ | M1 |
|  | Obtain final answer $a: b: c=1: 1:-1$ and state plane equation $x+y-z=0$ | A1) |
|  | OR1: <br> Attempt to calculate vector product of two relevant vectors, e.g. ( $3 \mathbf{i}-\mathbf{j}+2 \mathbf{k}) \times(9 \mathbf{i}-\mathbf{j}+8 \mathbf{k})$ | (M1 |
|  | Obtain two correct components | A1 |
|  | Obtain correct answer, e.g. $-6 \mathbf{i}-6 \mathbf{j}+6 \mathbf{k}$, and state plane equation $-x-y+z=0$ | A1) |
|  | OR2: <br> Using a relevant point and relevant vectors, attempt to form a 2 -parameter equation for the plane, e.g. $\mathbf{r}=6 \mathbf{i}+6 \mathbf{k}+s(3 \mathbf{i}-\mathbf{j}+2 \mathbf{k})+t(9 \mathbf{i}-\mathbf{j}+8 \mathbf{k})$ | (M1 |
|  | State 3 correct equations in $x, y, z, s$ and $t$ | A1 |
|  | Eliminate $s$ and $t$ and state plane equation $x+y-z=0$, or equivalent | A1) |
|  | OR3: Using a relevant point and relevant vectors, attempt to form a determinant equation for the plane, e.g. $\left\|\begin{array}{ccc}x-3 & y-1 & z-4 \\ 3 & -1 & 2 \\ 9 & -1 & 8\end{array}\right\|=0$ | (M1 |
|  | Expand a correct determinant and obtain two correct cofactors | A1 |
|  | Obtain answer $-6 x-6 y+6 z=0$, or equivalent | A1) |
|  | Total: | 3 |


| Question | Answer | Marks |
| :---: | :---: | :---: |
| 9(iii) | EITHER: <br> Using the correct processes, divide the scalar product of $\overrightarrow{O A}$ and a normal to the plane by the modulus of the normal or make a recognisable attempt to apply the perpendicular formula | (M1 |
|  | Obtain a correct expression in any form, e.g. $\frac{1+2-4}{\sqrt{\left(1^{2}+1^{2}+(-1)^{2}\right)}}$, or equivalent | A1 FT |
|  | Obtain answer $1 / \sqrt{3}$, or exact equivalent | A1) |
|  | OR1: <br> Obtain equation of the parallel plane through $A$, e.g. $x+y-z=-1$ [The f.t. is on the plane found in part (ii).] | (B1 FT |
|  | Use correct method to find its distance from the origin | M1 |
|  | Obtain answer $1 / \sqrt{3}$, or exact equivalent | A1) |
|  | OR2: <br> Form equation for the intersection of the perpendicular through $A$ and the plane [FT on their $\mathbf{n}$ ] | (B1 FT |
|  | Solve for $\lambda$ | M1 |
|  | $\|\lambda \mathbf{n}\|=\frac{1}{\sqrt{3}}$ | A1) |
|  | Total: | 3 |
| 10(i) | Use correct product rule | M1 |
|  | Obtain correct derivative in any form $\left(y^{\prime}=2 x \cos 2 x-2 x^{2} \sin 2 x\right)$ | A1 |
|  | Equate to zero and derive the given equation | A1 |
|  | Total: | 3 |
| 10(ii) | Use the iterative formula correctly at least once e.g. $0.5 \rightarrow 0.55357 \rightarrow 0.53261 \rightarrow 0.54070 \rightarrow 0.53755$ | M1 |
|  | Obtain final answer 0.54 | A1 |
|  | Show sufficient iterations to 4 d.p. to justify 0.54 to 2 d.p., or show there is a sign change in the interval $(0.535,0.545)$ | A1 |
|  | Total: | 3 |


| Question | Answer | Marks |
| :---: | :--- | :---: |
| $10($ iii $)$ | Integrate by parts and reach $a x^{2} \sin 2 x+b \int x \sin 2 x \mathrm{~d} x$ | *M1 |
|  | Obtain $\frac{1}{2} x^{2} \sin 2 x-\int 2 x \cdot \frac{1}{2} \sin 2 x \mathrm{~d} x$ | A1 |
|  | Complete integration and obtain $\frac{1}{2} x^{2} \sin 2 x+\frac{1}{2} x \cos 2 x-\frac{1}{4} \sin 2 x$, or equivalent | A1 |
|  | Substitute limits $x=0, x=\frac{1}{4} \pi$, having integrated twice | DM1 |
|  | Obtain answer $\frac{1}{32}\left(\pi^{2}-8\right)$, or exact equivalent | A1 |

