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- 1 *EITHER*: State or imply non-modular inequality $(2(x-2))^2 > (3x+1)^2$, or corresponding quadratic equation, or pair of linear equations $2(x-2) = \pm(3x+1)$ **B1**
 Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x **M1**
 Obtain critical values $x = -5$ and $x = \frac{3}{5}$ **A1**
 State final answer $-5 < x < \frac{3}{5}$ **A1**
- OR*: Obtain critical value $x = -5$ from a graphical method, or by inspection, or by solving a linear equation or inequality **(B1)**
 Obtain critical value $x = \frac{3}{5}$ similarly **B2**
 State final answer $-5 < x < \frac{3}{5}$ **(B1)**
 [Do not condone \leq for $<$.] **[4]**
- 2 (i) State or imply $y \ln 3 = (2-x) \ln 4$ **B1**
 State that this is of the form $ay = bx + c$ and thus a straight line, or equivalent **B1**
 State gradient is $-\frac{\ln 4}{\ln 3}$, or exact equivalent **B1**
[3]
- (ii) Substitute $y = 2x$ and solve for x , using a log law correctly at least once **M1**
 Obtain answer $x = \ln 4 / \ln 6$, or exact equivalent **A1**
[2]
- 3 (i) State answer $R = 3$ **B1**
 Use trig formula to find **M1**
 Obtain $\alpha = 41.81^\circ$ with no errors seen **A1**
[3]
- (ii) Evaluate $\cos^{-1}(0.4)$ to at least 1 d.p. (66.42° to 2 d.p.) **B1^h**
 Carry out an appropriate method to find a value of x in the given range **M1**
 Obtain answer 216.5° only **A1**
 [Ignore answers outside the given interval.] **[3]**
- 4 (i) State $\frac{dx}{dt} = 1 - \sin t$ **B1**
 Use chain rule to find the derivative of y **M1**
 Obtain $\frac{dy}{dt} = \frac{\cos t}{1 + \sin t}$, or equivalent **A1**
 Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ **M1**
 Obtain the given answer correctly **A1**
[5]
- (ii) State or imply $t = \cos^{-1}(\frac{1}{3})$ **B1**
 Obtain answers $x = 1.56$ and $x = -0.898$ **B1 + B1**
[3]

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- 5 Separate variables and make reasonable attempt at integration of either integral **M1**
 Obtain term $\frac{1}{2}e^{2y}$ **B1**
 Use Pythagoras **M1**
 Obtain terms $\tan x - x$ **A1**
 Evaluate a constant or use $x = 0, y = 0$ as limits in a solution containing terms
 $ae^{\pm 2y}$ and $b \tan x, (ab \neq 0)$ **M1**
 Obtain correct solution in any form, e.g. $\frac{1}{2}e^{2y} = \tan x - x + \frac{1}{2}$ **A1**
 Set $x = \frac{1}{4}\pi$ and use correct method to solve an equation of the form $e^{\pm 2y} = a$ or $e^{\pm y} = a$, where
 $a > 0$ **M1**
 Obtain answer $y = 0.179$ **A1**
[8]
- 6 (i) Use the product rule **M1**
 Obtain correct derivative in any form **A1**
 Equate 2-term derivative to zero and obtain the given answer correctly **A1**
[3]
- (ii) Use calculations to consider the sign of a relevant expression at $p = 2$ and $p = 2.5$, or
 compare values of relevant expressions at $p = 2$ and $p = 2.5$ **M1**
 Complete the argument correctly with correct calculated values **A1**
[2]
- (iii) Use the iterative formula correctly at least once **M1**
 Obtain final answer 2.15 **A1**
 Show sufficient iterations to 4 d.p. to justify 2.15 to 2 d.p., or show there is a sign change
 in the interval (2.145, 2.155) **A1**
[3]
- 7 (i) State or imply $du = 2x dx$, or equivalent **B1**
 Substitute for x and dx throughout **M1**
 Reduce to the given form and justify the change in limits **A1**
[3]
- (ii) Convert integrand to a sum of integrable terms and attempt integration **M1**
 Obtain integral $\frac{1}{2} \ln u + \frac{1}{u} - \frac{1}{4u^2}$, or equivalent **A1 + A1**
 (deduct A1 for each error or omission)
 Substitute limits in an integral containing two terms of the form $a \ln u$ and bu^{-2} **M1**
 Obtain answer $\frac{1}{2} \ln 2 - \frac{5}{16}$, exact simplified equivalent **A1**
[5]

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- 8 (i) State a correct equation for AB in any form, e.g. $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$, or equivalent **B1**
 Equate at least two pairs of components of AB and l and solve for λ or for μ **M1**
 Obtain correct answer for λ or for μ , e.g. $\lambda = -1$ or $\mu = 2$ **A1**
 Show that not all three equations are not satisfied and that the lines do not intersect **A1**
[4]
- (ii) *EITHER*: Find \overline{AP} (or \overline{PA}) for a general point P on l , e.g. $(1 - \mu)\mathbf{i} + (-3 + 2\mu)\mathbf{j} + (-2 + \mu)\mathbf{k}$ **B1**
 Calculate the scalar product of \overline{AP} and a direction vector for l and equate to zero **M1**
 Solve and obtain $\mu = \frac{3}{2}$ **A1**
 Carry out a method to calculate AP when $\mu = \frac{3}{2}$ **M1**
 Obtain the given answer $\frac{1}{\sqrt{2}}$ correctly **A1**
- OR 1*: Find \overline{AP} (or \overline{PA}) for a general point P on l **(B1)**
 Use correct method to express AP^2 (or AP) in terms of μ **M1**
 Obtain a correct expression in any form, e.g. $(1 - \mu)^2 + (-3 + 2\mu)^2 + (-2 + \mu)^2$ **A1**
- Carry out a complete method for finding its minimum **M1**
 Obtain the given answer correctly **A1)**
- OR 2*: Calling $(2, -2, -1)$ C , state \overline{AC} (or \overline{CA}) in component form, e.g. $\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$ **(B1)**
 Use a scalar product to find the projection of \overline{AC} (or \overline{CA}) on l **M1**
 Obtain correct answer in any form, e.g. $\frac{9}{\sqrt{6}}$ **A1**
 Use Pythagoras to find the perpendicular **M1**
 Obtain the given answer correctly **A1)**
- OR 3*: State \overline{AC} (or \overline{CA}) in component form **(B1)**
 Calculate vector product of \overline{AC} and a direction vector for l , e.g. $(\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}) \times (-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ **M1**
 Obtain correct answer in any form, e.g. $\mathbf{i} + \mathbf{j} - \mathbf{k}$ **A1**
 Divide modulus of the product by that of the direction vector **M1**
 Obtain the given answer correctly **A1)**
[5]
- 9 (i) *EITHER*: Multiply numerator and denominator of $\frac{u}{v}$ by $2 + i$, or equivalent **M1**
 Simplify the numerator to $-5 + 5i$ or denominator to 5 **A1**
 Obtain final answer $-1 + i$ **A1**
- OR*: Obtain two equations in x and y and solve for x or for y **(M1)**
 Obtain $x = -1$ or $y = 1$ **A1**
 Obtain final answer $-1 + i$ **A1)**
[3]
- (ii) Obtain $u + v = 1 + 2i$ **B1**
 In an Argand diagram show points A, B, C representing u, v and $u + v$ respectively **B1**
 State that OB and AC are parallel **B1**
 State that $OB = AC$ **B1**
[4]

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- (iii) Carry out an appropriate method for finding angle AOB , e.g. find $\arg(u/v)$ **M1**
 Show sufficient working to justify the given answer $\frac{3}{4}\pi$ **A1**
 [2]
- 10 (i)** State or imply the form $\frac{A}{x+3} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$ **B1**
 Use a correct method to determine a constant **M1**
 Obtain one of the values $A = -3, B = 1, C = 2$ **A1**
 Obtain a second value **A1**
 Obtain the third value **A1**
 [Mark the form $\frac{A}{x+3} + \frac{Dx+E}{(x-1)^2}$, where $A = -3, D = 1, E = 1$, B1M1A1A1A1 as above.] [5]
- (ii)** Use a correct method to find the first two terms of the expansion of $(x+3)^{-1}, (1+\frac{1}{3}x)^{-1},$
 $(x-1)^{-1}, (1-x)^{-1}, (x-1)^{-2},$ or $(1-x)^{-2}$ **M1**
 Obtain correct unsimplified expressions up to the term in x^2 of each partial fraction **A1^h + A1^h + A1^h**
 Obtain final answer $\frac{10}{3}x + \frac{44}{9}x^2$, or equivalent **A1**
 [5]