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- 1 Use law of the logarithm of a product, power or quotient M1\*  
 Obtain a correct linear equation, e.g.  $(3x - 1)\ln 4 = \ln 3 + x \ln 5$  A1  
 Solve a linear equation for  $x$  DM1\*  
 Obtain answer  $x = 0.975$  A1 [4]
- 2 State a correct un-simplified version of the  $x$  or  $x^2$  or  $x^3$  term M1  
 State correct first two terms  $1 + x$  A1  
 Obtain the next two terms  $\frac{3}{2}x^2 + \frac{5}{2}x^3$  A1 A1 [4]  
 [Symbolic binomial coefficients, e.g.  $\binom{-\frac{1}{2}}{3}$  are not sufficient for the M mark.]
- 3 Integrate by parts and reach  $ax^2 \cos 2x + b \int x \cos 2x \, dx$  M1\*  
 Obtain  $-\frac{1}{2}x^2 \cos 2x + \int x \cos 2x$ , or equivalent A1  
 Complete the integration and obtain  $-\frac{1}{2}x^2 \cos 2x + \frac{1}{2}x \sin 2x + \frac{1}{4} \cos 2x$ , or equivalent A1  
 Use limits correctly having integrated twice DM1\*  
 Obtain answer  $\frac{1}{8}(\pi^2 - 4)$ , or exact equivalent, with no errors seen A1 [5]
- 4 State or imply derivative of  $(\ln x)^2$  is  $\frac{2 \ln x}{x}$  B1  
 Use correct quotient or product rule M1  
 Obtain correct derivative in any form, e.g.  $\frac{2 \ln x}{x^2} - \frac{(\ln x)^2}{x^2}$  A1  
 Equate derivative (or its numerator) to zero and solve for  $\ln x$  M1  
 Obtain the point  $(1, 0)$  with no errors seen A1  
 Obtain the point  $(e^2, 4e^{-2})$  A1 [6]
- 5 (i) EITHER: Express  $\cos 4\theta$  in terms of  $\cos 2\theta$  and/or  $\sin 2\theta$  B1  
 Use correct double angle formulae to express LHS in terms of  $\sin \theta$  and/or  $\cos \theta$  M1  
 Obtain a correct expression in terms of  $\sin \theta$  alone A1  
 Reduce correctly to the given form A1  
 OR: Use correct double angle formula to express RHS in terms of  $\cos 2\theta$  M1  
 Express  $\cos^2 2\theta$  in terms of  $\cos 4\theta$  B1  
 Obtain a correct expression in terms of  $\cos 4\theta$  and  $\cos 2\theta$  A1  
 Reduce correctly to the given form A1 [4]
- (ii) Use the identity and carry out a method for finding a root M1  
 Obtain answer  $68.5^\circ$  A1  
 Obtain a second answer, e.g.  $291.5^\circ$  A1<sup>h</sup>  
 Obtain the remaining answers, e.g.  $111.5^\circ$  and  $248.5^\circ$ , and no others in the given interval A1<sup>h</sup>  
 [Ignore answers outside the given interval. Treat answers in radians as a misread.] [4]

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- 6 (i) Separate variables correctly and attempt integration of at least one side **B1**  
 Obtain term  $\ln x$  **B1**  
 Obtain term of the form  $k \ln(3 + \cos 2\theta)$ , or equivalent **M1**  
 Obtain term  $-\frac{1}{2} \ln(3 + \cos 2\theta)$ , or equivalent **A1**  
 Use  $x = 3$ ,  $\theta = \frac{1}{4}\pi$  to evaluate a constant or as limits in a solution  
 with terms  $a \ln x$  and  $b \ln(3 + \cos 2\theta)$ , where  $ab \neq 0$  **M1**  
 State correct solution in any form, e.g.  $\ln x = -\frac{1}{2} \ln(3 + \cos 2\theta) + \frac{3}{2} \ln 3$  **A1**  
 Rearrange in a correct form, e.g.  $x = \sqrt{\left(\frac{27}{3 + \cos 2\theta}\right)}$  **A1** [7]
- (ii) State answer  $x = 3\sqrt{3}/2$ , or exact equivalent (accept decimal answer in [2.59, 2.60]) **B1** [1]
- 7 (i) State or imply the form  $A + \frac{B}{2x+1} + \frac{C}{x+2}$  **B1**  
 State or obtain  $A = 2$  **B1**  
 Use a correct method for finding a constant **M1**  
 Obtain one of  $B = 1$ ,  $C = -2$  **A1**  
 Obtain the other value **A1** [5]
- (ii) Integrate and obtain terms  $2x + \frac{1}{2} \ln(2x+1) - 2 \ln(x+2)$  **B3<sup>+</sup>**  
 Substitute correct limits correctly in an integral with terms  $a \ln(2x+1)$   
 and  $b \ln(x+2)$ , where  $ab \neq 0$  **M1**  
 Obtain the given answer after full and correct working **A1** [5]
- 8 (i) Use correct quotient or chain rule **M1**  
 Obtain correct derivative in any form **A1**  
 Obtain the given answer correctly **A1** [3]
- (ii) State a correct equation, e.g.  $-e^{-a} = -\operatorname{cosec} a \cot a$  **B1**  
 Rearrange it correctly in the given form **B1** [2]
- (iii) Calculate values of a relevant expression or pair of expressions at  $x = 1$  and  $x = 1.5$  **M1**  
 Complete the argument correctly with correct calculated values **A1** [2]
- (iv) Use the iterative formula correctly at least once **M1**  
 Obtain final answer 1.317 **A1**  
 Show sufficient iterations to 5 d.p. to justify 1.317 to 3 d.p., or show there is a sign  
 change in the interval (1.3165, 1.3175) **A1** [3]

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- 9 (i) *Either* state or imply  $\overline{AB}$  or  $\overline{BC}$  in component form, *or* state position vector of midpoint of  $\overline{AC}$  B1
- Use a correct method for finding the position vector of  $D$  M1  
 Obtain answer  $3\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ , or equivalent A1
- EITHER:* Using the correct process for the moduli, compare lengths of a pair of adjacent sides,  
 e.g.  $AB$  and  $BC$  M1  
 Show that  $ABCD$  has a pair of adjacent sides that are equal A1
- OR:* Calculate scalar product  $\overline{AC} \cdot \overline{BD}$  or equivalent M1  
 Show that  $ABCD$  has perpendicular diagonals A1 [5]
- (ii) *EITHER:* State  $a + 2b + 3c = 0$  or  $2a + b - 2c = 0$  B1  
 Obtain two relevant equations and solve for one ratio, e.g.  $a : b$  M1  
 Obtain  $a : b : c = -7 : 8 : -3$ , or equivalent A1  
 Substitute coordinates of a relevant point in  $-7x + 8y - 3z = d$ , and evaluate M1  
 Obtain answer  $-7x + 8y - 3z = 29$ , or equivalent A1
- OR1:* Attempt to calculate vector product of relevant vectors,  
 e.g.  $(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) \times (2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$  M1  
 Obtain two correct components of the product A1  
 Obtain correct product, e.g.  $-7\mathbf{i} + 8\mathbf{j} - 3\mathbf{k}$  A1  
 Substitute coordinates of a relevant point in  $-7x + 8y - 3z = d$  and evaluate  $d$  M1  
 Obtain answer  $-7x + 8y - 3z = 29$  or equivalent A1
- OR2:* Attempt to form a 2-parameter equation with relevant vectors M1  
 State a correct equation, e.g.  $\mathbf{r} = 2\mathbf{i} + 5\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$  A1  
 State 3 equations in  $x, y, z, \lambda$  and  $\mu$  A1  
 Eliminate  $\lambda$  and  $\mu$  M1  
 Obtain answer  $-7x + 8y - 3z = 29$ , or equivalent A1
- OR3:* Using a relevant point and relevant direction vectors, form a determinant equation for the plane M1
- State a correct equation, e.g. 
$$\begin{vmatrix} x-2 & y-5 & z+1 \\ 1 & 2 & 3 \\ 2 & 1 & -2 \end{vmatrix} = 0$$
 A1
- Attempt to expand the determinant M1  
 Obtain correct values of two cofactors A1  
 Obtain answer  $-7x + 8y - 3z = 29$ , or equivalent A1 [5]

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- 10 (a)** *EITHER*: Use quadratic formula to solve for  $z$  **M1**
- Use  $i^2 = -1$  **M1**
- Obtain a correct answer in any form, simplified as far as  $(-2 \pm i\sqrt{8}) / 2i$  **A1**
- Multiply numerator and denominator by  $i$ , or equivalent **M1**
- Obtain final answers  $\sqrt{2} + i$  and  $-\sqrt{2} + i$  **A1**
- OR*: Substitute  $x + iy$  and equate real and imaginary parts to zero **M1**
- Use  $i^2 = -1$  **M1**
- Obtain  $-2xy + 2x = 0$  and  $x^2 - y^2 + 2y - 3 = 0$ , or equivalent **A1**
- Solve for  $x$  and  $y$  **M1**
- Obtain final answers  $\sqrt{2} + i$  and  $-\sqrt{2} + i$  **A1** [5]
- (b) (i)** *EITHER*: Show the point representing  $4 + 3i$  in relatively correct position **B1**
- Show the perpendicular bisector of the line segment joining this point to the origin **B1**<sup>h</sup> [2]
- OR*: Obtain correct Cartesian equation of the locus in any form, e.g.  
 $8x + 6y = 25$  **B1**
- Show this line **B1**<sup>h</sup>
- [This f.t. is dependent on using a correct method to determine the equation.]
- (ii)** State or imply the relevant point is represented by  $2 + 1.5i$  or is at  $(2, 1.5)$  **B1**
- Obtain modulus 2.5 **B1**<sup>h</sup>
- Obtain argument 0.64 (or  $36.9^\circ$ ) (allow decimals in  $[0.64, 0.65]$  or  $[36.8, 36.9]$ ) **B1**<sup>h</sup> [3]