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- 1 (i) *EITHER*: State or imply non-modular equation $(2(x-1))^2 = (3x)^2$, or pair of linear equations $2(x-1) = \pm 3x$ **B1**
 Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations **M1**
 Obtain answers $x = -2$ and $x = \frac{2}{5}$ **A1**
- OR*: Obtain answer $x = -2$ by inspection or by solving a linear equation **(B1)**
 Obtain answer $x = \frac{2}{5}$ similarly **(B2)**
[3]
- (ii) Use correct method for solving an equation of the form $5^x = a$ or $5^{x+1} = a$, where $a > 0$ **M1**
 Obtain answer $x = -0.569$ only **A1**
[2]
- 2 Integrate by parts and reach $axe^{-2x} + b \int e^{-2x} dx$ **M1**
 Obtain $-\frac{1}{2}xe^{-2x} + \frac{1}{2} \int e^{-2x} dx$, or equivalent **A1**
 Complete the integration correctly, obtaining $-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$, or equivalent **A1**
 Use limits $x = 0$ and $x = \frac{1}{2}$ correctly, having integrated twice **M1**
 Obtain answer $\frac{1}{4} - \frac{1}{2}e^{-1}$, or exact equivalent **A1**
[5]
- 3 Correctly restate the equation in terms of $\sin \theta$ and $\cos \theta$ **B1**
 Using Pythagoras obtain a horizontal equation in $\cos \theta$ **M1**
 Reduce the equation to a correct quadratic in $\cos \theta$, e.g. $3\cos^2 \theta - \cos \theta - 2 = 0$ **A1**
 Solve a 3-term quadratic for $\cos \theta$ **M1**
 Obtain answer $\theta = 131.8^\circ$ only **A1**
[5]
- [Ignore answers outside the given interval.]
- 4 Separate variables and attempt integration of at least one side **M1***
 Obtain term $\ln y$ **A1**
 Obtain terms $\ln x - x^2$ **A1**
 Use $x = 1$ and $y = 2$ to evaluate a constant, or as limits **DM1***
 Obtain correct solution in any form, e.g. $\ln y = \ln x - x^2 + \ln 2 + 1$ **A1**
 Obtain correct expression for y , free of logarithms, i.e. $y = 2x \exp(1 - x^2)$ **A1**
[6]

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- 5 Use product rule **M1**
 Obtain correct derivative in any form, e.g. $\cos x \cos 2x - 2 \sin x \sin 2x$ **A1**
 Equate derivative to zero and use double angle formulae **M1**
 Remove factor of $\cos x$ and reduce equation to one in a single trig function **M1**
 Obtain $6 \sin^2 x = 1$, $6 \cos^2 x = 5$ or $5 \tan^2 x = 1$ **A1**
 Solve and obtain $x = 0.421$ **A1**
[6]
- [Alternative: Use double angle formula M1. Use chain rule to differentiate M1. Obtain correct derivative
 e.g. $\cos \theta - 6 \sin^2 \theta \cos \theta$ **A1**, then as above.]
- 6 (i) Make recognizable sketch of a relevant graph **B1**
 Sketch the other relevant graph and justify the given statement **B1**
[2]
- (ii) State $x = \frac{1}{2} \ln(25/x)$ **B1**
 Rearrange this in the form $5e^{-x} = \sqrt{x}$ **B1**
[2]
- (iii) Use the iterative formula correctly at least once **M1**
 Obtain final answer 1.43 **A1**
 Show sufficient iterations to 4 d.p. to justify 1.43 to 2 d.p., or show there is a sign change in the interval (1.425, 1.435) **A1**
[3]
- 7 (i) State or imply $6xy + 3x^2 \frac{dy}{dx}$ as derivative of $3x^2y$ **B1**
 State $3y^2 \frac{dy}{dx}$ as derivative of y^3 **B1**
 Equate attempted derivative of the LHS to zero and solve for $\frac{dy}{dx}$ **M1**
 Obtain the given answer **A1**
[4]
- (ii) Equate numerator to zero **M1***
 Obtain $x = 2y$, or equivalent **A1**
 Obtain an equation in x or y **DM1***
 Obtain the point $(-2, -1)$ **A1**
 State the point $(0, 1.44)$ **B1**
[5]

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- 8 (i) State or imply the form $\frac{A}{x+1} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$ **B1**
- Use a correct method to determine a constant **M1**
- Obtain one of the values $A = 1, B = 3, C = 12$ **A1**
- Obtain a second value **A1**
- Obtain a third value **A1**
- [5]

[Mark the form $\frac{A}{x+1} + \frac{Dx+E}{(x-3)^2}$, where $A = 1, D = 3, E = 3$, B1M1A1A1A1 as above.]

- (ii) Use correct method to find the first two terms of the expansion of $(x+1)^{-1}, (x-3)^{-1}, (1-\frac{1}{3}x)^{-1}$,
- $(x-3)^{-2}$, or $(1-\frac{1}{3}x)^{-2}$ **M1**
- Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction **A1[✓] + A1[✓] + A1[✓]**
- Obtain final answer $\frac{4}{3} - \frac{4}{9}x + \frac{4}{3}x^2$, or equivalent **A1**
- [5]

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- 9 (i) *EITHER*: Obtain a vector parallel to the plane, e.g. $\overline{AB} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$ **B1**
- Use scalar product to obtain an equation in a, b, c e.g. $a - 2b - 3c = 0, a + b - c = 0,$
or $3b + 2c = 0$ **M1**
- State two correct equations **A1**
- Solve to obtain ratio $a : b : c$ **M1**
- Obtain $a : b : c = 5 : -2 : 3$ **A1**
- Obtain equation $5x - 2y + 3z = 5,$ or equivalent **A1**
- OR1*: Substitute for two points, e.g. A and $B,$ and obtain $a + 3b + 2c = d$ and
 $2a + b - c = d$ **(B1)**
- Substitute for another point, e.g. $C,$ to obtain a third equation and eliminate one unknown
entirely from all three equations **M1**
- Obtain two correct equations in three unknowns, e.g. in a, b, c **A1**
- Solve to obtain their ratio **M1**
- Obtain $a : b : c = 5 : -2 : 3, a : c : d = 5 : 3 : 5, a : b : d = 5 : -2 : 5,$ or $b : c : d = -2 : 3 : 5$ **A1**
- Obtain equation $5x - 2y + 3z = 5,$ or equivalent **A1)**
- OR2*: Obtain a vector parallel to the plane, e.g. $\overline{AC} = \mathbf{i} + \mathbf{j} - \mathbf{k}$ **(B1)**
- Obtain a second such vector and calculate their vector product, e.g.
 $(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}) \times (\mathbf{i} + \mathbf{j} - \mathbf{k})$ **M1**
- Obtain two correct components of the product **A1**
- Obtain correct answer e.g. $5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ **A1**
- Substitute in $5x - 2y + 3z = d$ to find d **M1**
- Obtain equation $5x - 2y + 3z = 5,$ or equivalent **A1)**
- OR3*: Obtain a vector parallel to the plane, e.g. $\overline{BC} = 3\mathbf{j} + 2\mathbf{k}$ **(B1)**
- Obtain a second such vector and form correctly a 2-parameter equation for the plane **M1**
- Obtain a correct equation, e.g. $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}) + \mu(3\mathbf{j} + 2\mathbf{k})$ **A1**
- State three correct equations in x, y, z, λ, μ **A1**
- Eliminate λ and μ **M1**
- Obtain equation $3x - 2y + 3z = 5,$ or equivalent **A1)**
- [6]
- (ii) Correctly form an equation for the line through D parallel to OA **M1**
- Obtain a correct equation e.g. $\mathbf{r} = -3\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$ **A1**
- Substitute components in the equation of the plane and solve for λ **M1**
- Obtain $\lambda = 2$ and position vector $-\mathbf{i} + 7\mathbf{j} + 6\mathbf{k}$ for P **A1**
- Obtain the given answer correctly **A1**
- [5]
- 10 (a) Square $x + iy$ and equate real and imaginary parts to 7 and $-6\sqrt{2}$ respectively **M1**
- Obtain equations $x^2 - y^2 = 7$ and $2xy = -6\sqrt{2}$ **A1**
- Eliminate one variable and find an equation in the other **M1**
- Obtain $x^4 - 7x^2 - 18 = 0$ or $y^4 + 7y^2 - 18 = 0,$ or 3-term equivalent **A1**
- Obtain answers $\pm(3 - i\sqrt{2})$ **A1**
- [5]

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- (b) (i) Show point representing $1 + 2i$ **B1**
 Show circle with radius 1 and centre $1 + 2i$ **B1**
 Show a half line from the point representing 1 **B1**
 Show line making the correct angle with the real axis **B1**
 [4]
- (ii) State or imply the relevance of the perpendicular from $1 + 2i$ to the line **M1**
 Obtain answer $\sqrt{2} - 1$ (or 0.414) **A1**
 [2]