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1	$f : x \mapsto 10 - 3x, g : x \mapsto \frac{10}{3 - 2x},$ $ff(x) = 10 - 3(10 - 3x)$ $gf(2) = \frac{10}{3 - 2(10 - 3(2))} (= -2)$ $x = 2$	<p>B1</p> <p>B1</p> <p>B1</p> <p>[3]</p>	<p>Correct unsimplified expression</p> <p>Correct unsimplified expression with 2 in for x</p>
2	$f'(x) = \frac{8}{(5 - 2x)^2}$ $f(x) = \frac{8(5 - 2x)^{-1}}{-1} \div -2 (+c)$ <p>Uses $x = 2, y = 7,$</p> $c = 3$	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>Correct without (\div by -2)</p> <p>An attempt at integration (\div by -2)</p> <p>Substitution of correct values into an integral to find c</p>
3	$\overline{OA} = 2\mathbf{i} - 5\mathbf{j} - 2\mathbf{k} \text{ and } \overline{OB} = 4\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}.$ $\overline{AB} = 2\mathbf{i} + \mathbf{j} + 4\mathbf{k} \text{ or } \overline{AC} = 4\mathbf{i} + 2\mathbf{j} + 8\mathbf{k}$ $\overline{OC} = \overline{OA} + \overline{AC} = 6\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$ <p>OR</p> $\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} x - 4 \\ y + 4 \\ z - 2 \end{pmatrix},$ $\overline{OC} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \\ 6 \end{pmatrix}$ <p>OR</p> $\overline{OB} - \overline{OA} = \overline{OC} - \overline{OB}$ $\therefore \overline{OC} = 2\overline{OB} - \overline{OA}$ $= \begin{pmatrix} 8 \\ -8 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ -5 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \\ 6 \end{pmatrix}$ <p>Unit vector = (Their \overline{OC}) \div (Mod their \overline{OC})</p> $= (6\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}) \div 9$	<p>B1</p> <p>M1</p> <p>B1</p> <p>M1</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>correct method for \overline{OC}</p> <p>Divides by their mod of their \overline{OC}</p> <p>Correct unsimplified expression</p>

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<p>4 (i)</p> $\left(x - \frac{2}{x}\right)^6$ <p>Term is ${}_6C_3 \times (-2)^3 = (-160)$ -160</p> <p>(ii)</p> $\left(2 + \frac{3}{x^2}\right)\left(x - \frac{2}{x}\right)^6$ <p>Term in $x^2 = {}_6C_2(-2)^2 x^2$ = 60 (x^2)</p> <p>Term independent of x: = 2 × (their -160) + 3 × (their 60) -140</p>	<p>B1 B1</p> <p>[2]</p> <p>B1 B1</p> <p>M1 A1</p> <p>[4]</p>	<p>±160 seen anywhere</p> <p>±60 seen anywhere</p> <p>Using 2 products correctly</p>
<p>5 (i)</p> $\tan\left(\frac{\pi}{3}\right) = \frac{AC}{2x} \text{ or } \cos\left(\frac{\pi}{3}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{2x}{AB}$ <p>→ $AC = 2\sqrt{3}x$ or $AB = 4x$</p> $AM = \sqrt{13x^2}, \sqrt{13}x, 3.61x$ <p>(ii)</p> $\tan(\hat{MAC}) = \frac{x}{\text{Their } AC}$ $\theta = \frac{1}{6}\pi - \tan^{-1}\frac{1}{2\sqrt{3}} \text{ AG}$	<p>B1</p> <p>M1A1</p> <p>[3]</p> <p>M1</p> <p>A1</p> <p>[2]</p>	<p>Either trig ratio</p> <p>Complete method.</p> <p>“Their AC” must be $f(x)$, (\hat{MAC}) $\neq \theta$.</p> <p>Justifies $\frac{\pi}{6}$ and links MAC & θ</p>
<p>6 (i)</p> $PT = r \tan \alpha$ $QT = OT - OQ = \frac{r}{\cos \alpha} - r$ <p>or $\sqrt{r^2 + r^2 \tan^2 \alpha} - r$</p> <p>Perimeter = sum of the 3 parts including $r\alpha$</p> <p>(ii)</p> <p>Area of triangle = $\frac{1}{2} \times 10 \times 10 \tan \frac{\pi}{3}$</p> <p>Area of sector = $\frac{1}{2} \times 10^2 \times \frac{1}{3}\pi$</p> <p>Shaded region has area 34 (2sf)</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>[3]</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>Correct formula used, $50\sqrt{3}, 86.6$</p> <p>Correct formula used, $\frac{50\pi}{3}, 52.36$</p>

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<p>7 (i)</p> $\frac{1 + \cos \theta}{1 - \cos \theta} - \frac{1 - \cos \theta}{1 + \cos \theta} \equiv \frac{4}{\sin \theta \tan \theta}$ $\text{LHS} = \frac{1 + 2c + c^2 - (1 - 2c + c^2)}{(1 - c)(1 + c)}$ $= \frac{4c}{1 - c^2}$ $= \frac{4c}{s^2}$ $= \frac{4}{ts} \text{ AG}$ <p>(ii)</p> $\sin \theta \left(\frac{1 + \cos \theta}{1 - \cos \theta} - \frac{1 - \cos \theta}{1 + \cos \theta} \right) = 3.$ $\rightarrow s \times \frac{4}{ts} = 3 \left(\rightarrow t = \frac{4}{3} \right)$ $\theta = 53.1^\circ \text{ and } 233.1^\circ$		<p>M1</p> <p>A1 A1</p> <p>A1</p> <p>[4]</p> <p>M1</p> <p>A1 A1 \checkmark^h</p> <p>[3]</p>	<p>Attempt at combining fractions.</p> <p>A1 for numerator. A1 denominator</p> <p>Essential step for award of A1</p> <p>Uses part (i) to eliminate “s” correctly.</p> <p>\checkmark^h for $180^\circ + 1^{\text{st}}$ answer.</p>
<p>8</p> <p>(i)</p> <p>m of AB is $-\frac{1}{2}$ oe. Eqn of AB is $y = -\frac{1}{2}x + 7$ Let $x = 3k, y = k$ $k = 2.8$ oe</p> <p>OR</p> $\frac{7 - k}{0 - 3k} = \frac{3 - k}{8 - 3k}$ $\rightarrow 20k = 56 \rightarrow k = 2.8$ <p>OR</p> $\frac{7 - k}{0 - 3k} = \frac{7 - 3}{0 - 8}$ $\rightarrow 20k = 56 \rightarrow k = 2.8$		<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1A1</p> <p>DM1A1</p> <p>M1A1</p> <p>DM1A1</p> <p>[4]</p>	<p>Using A, B or C to get an equation Using C or A, B in the equation</p> <p>Using A, B & C to equate gradients</p> <p>Simplifies to a linear or 3 term quadratic = 0.</p> <p>Using A, B and C to equate gradients</p> <p>Simplifies to a linear or 3 term quadratic = 0.</p>

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(ii)	<p>M(4, 5) Perpendicular gradient = 2. Perp bisector has eqn $y - 5 = 2(x - 4)$</p> <p>Let $x = 3k, y = k$ $k = \frac{3}{5}$ oe</p> <p>OR</p> <p>$(0 - 3k)^2 + (7 - k)^2 = (8 - 3k)^2 + (3 - k)^2$ $-14k + 49 = 73 - 54k \rightarrow 40k = 24 \rightarrow k = 0.6$</p>	<p>B1 M1 M1</p> <p>A1</p> <p>M1A1</p> <p>DM1A1 [4]</p>	<p>anywhere in (ii) Use of $m_1 m_2 = -1$ soi Forming eqn using their M and their “perpendicular m”</p> <p>Use of Pythagoras.</p> <p>Simplifies to a linear or 3 term quadratic = 0.</p>
9	<p>(i) (a) $a + (n - 1)d = 10 + 29 \times 2$ $= 68$</p> <p>(b) $\frac{1}{2}n(20 + 2(n - 1)) = 2000$ or 0 $\rightarrow 2n^2 + 18n - 4000 = 0$ oe (n=) 41</p> <p>(ii) $r = 1.1$, oe Uses $S_{30} = \frac{10(1.1^{30} - 1)}{1.1 - 1}$ (= 1645) Percentage lost = $\frac{2000 - 1645}{2000} \times 100$ $= 17.75$</p>	<p>M1 A1 [2]</p> <p>M1 A1 A1 [3]</p> <p>B1</p> <p>M1</p> <p>DM1</p> <p>A1 [4]</p>	<p>Use of nth term of an AP with $a = \pm 10, d = \pm 2, n = 30$ or 29 Condone – $68 \rightarrow 68$</p> <p>Use of S_n formula for an AP with $a = \pm 10, d = \pm 2$ and equated to either 0 or 2000. Correct 3 term quadratic = 0.</p> <p>e.g. $\frac{11}{10}, 110\%$</p> <p>Use of S_n formula for a GP, $a = \pm 10, n = 30$.</p> <p>Fully correct method for % left with “their 1645” allow 17.7 or 17.8.</p>
10	<p>(i) $y = \frac{8}{x} + 2x$ $\frac{dy}{dx} = -8x^{-2} + 2$ $\frac{d^2y}{dx^2} = 16x^{-3}$ $\int y^2 dx = -64x^{-1}$ oe $+ 32x$ oe $+ \frac{4x^3}{3}$ oe (+c)</p>	<p>B1</p> <p>B1</p> <p>3 × B1 [5]</p>	<p>unsimplified ok</p> <p>unsimplified ok</p> <p>B1 for each term – unsimplified ok</p>

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(ii)	sets $\frac{dy}{dx}$ to 0 $\rightarrow x = \pm 2$ $\rightarrow M(2, 8)$ Other turning point is $(-2, -8)$ If $x = -2$, $\frac{d^2y}{dx^2} < 0$ \therefore Maximum	M1 A1 A1 M1 A1 [5]	Sets to 0 and attempts to solve Any pair of correct values A1 Second pair of values A1 Using their $\frac{d^2y}{dx^2}$ if kx^{-3} and $x < 0$
(iii)	Vol = $\pi \times$ [part (i)] from 1 to 2 $\frac{220\pi}{3}, 73.3\pi, 230$	M1 A1 [2]	Evidence of using limits 1&2 in their integral of y^2 (ignore π)
11	(i) $f: x \mapsto 6x - x^2 - 5$ $6x - x^2 - 5 \leq 3$ $\rightarrow x^2 - 6x + 8 \geq 0$ $\rightarrow x = 2, x = 4$ $x \leq 2, x \geq 4$ condone $<$ and/or $>$ (ii) Equate $mx + c$ and $6x - x^2 - 5$ Use of “ $b^2 - 4ac$ ” $4c = m^2 - 12m + 16$. AG OR $\frac{dy}{dx} = 6 - 2x = m \rightarrow x = \left(\frac{6-m}{2}\right)$ $m\left(\frac{6-m}{2}\right) + c = 6\left(\frac{6-m}{2}\right) - \left(\frac{6-m}{2}\right)^2 - 5$ $4c = m^2 - 12m + 16$. AG (iii) $6x - x^2 - 5 = 4 - (x - 3)^2$ (iv) $k = 3$. (v) $g^{-1}(x) = \sqrt{4-x} + 3$	M1 A1 A1 [3] M1 DM1 A1 M1 M1 A1 [3] B1 B1 [2] B1 [∧] [1] M1 A1 [2]	$\pm(6x - x^2 - 8) =, \leq, \geq 0$ and attempts to solve Needs both values whether $=2, <2, >2$ Accept all recognisable notation. Equates, sets to 0. Use of discriminant with values of a, b, c independent of x . $= (0)$ must appear before last line. Equates $\frac{dy}{dx}$ to m and rearrange Equates $mx + c$ and $6x - x^2 - 5$ and substitutes for x 4 B1 $-(x - 3)^2$ B1 [∧] for “ b ”. Correct order of operations. $\pm\sqrt{4-x} + 3$ M1A0 $\sqrt{x-4} + 3$ M1A0 $\sqrt{4-y} + 3$ M1A0