

Page 4	Mark Scheme	Syllabus	Paper
	Cambridge International A Level – May/June 2015	9709	33
1	Use law for the logarithm of a product, quotient or power	M1	
	Obtain a correct equation free of logarithms, e.g. $\frac{x+4}{x^2} = 4$	A1	
	Solve a 3-term quadratic obtaining at least one root	M1	
	Obtain final answer $x = 1.13$ only	A1	4
2	<i>EITHER:</i> State or imply non-modular inequality $(x-2)^2 > (2x-3)^2$, or corresponding equation	B1	
	Solve a 3-term quadratic, as in Q1.	M1	
	Obtain critical value $x = \frac{5}{3}$	A1	
	State final answer $x < \frac{5}{3}$ only	A1	
	<i>OR1:</i> State the relevant critical linear inequality $(2-x) > (2x-3)$, or corresponding equation	B1	
	Solve inequality or equation for x	M1	
	Obtain critical value $x = \frac{5}{3}$	A1	
	State final answer $x < \frac{5}{3}$ only	A1	
	<i>OR2:</i> Make recognisable sketches of $y = 2x - 3$ and $y = x - 2 $ on a single diagram	B1	
	Find x -coordinate of the intersection	M1	
	Obtain $x = \frac{5}{3}$	A1	
	State final answer $x < \frac{5}{3}$ only	A1	4
3	Use correct $\tan 2A$ and $\cot A$ formulae to form an equation in $\tan x$	M1	
	Obtain a correct equation in any form	A1	
	Reduce equation to the form $\tan^2 x + 6 \tan x - 3 = 0$, or equivalent	A1	
	Solve a three term quadratic in $\tan x$ for x , as in Q1.	M1	
	Obtain answer, e.g. 24.9° (24.896)	A1	
	Obtain second answer, e.g. 98.8 (98.794) and no others in the given interval [Ignore outside the given interval. Treat answers in radians as a misread.]	A1	6
	Radian answers 0.43452, 1.7243		
4	Use correct quotient or product rule	M1	
	Obtain correct derivative in any form	A1	
	Equate derivative to zero and obtain a horizontal equation	M1	
	Carry out complete method for solving an equation of the form $ae^{3x} = b$, or $ae^{5x} = be^{2x}$	M1	
	Obtain $x = \ln 2$, or exact equivalent	A1	
	Obtain $y = \frac{1}{3}$, or exact equivalent	A1	6

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5	(i) State $\frac{dx}{dt} = -4a \cos^3 t \sin t$, or $\frac{dy}{dt} = 4a \sin^3 t \cos t$ Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ Obtain correct expression for $\frac{dy}{dx}$ in a simplified form		B1 M1 A1	3
	(ii) Form the equation of the tangent Obtain a correct equation in any form Obtain the given answer		M1 A1 A1	3
	(iii) State the x -coordinate of P or the y -coordinate of Q in any form Obtain the given result correctly		B1 B1	2
6	(i) Integrate and reach $\pm x \sin x \mp \int \sin x \, dx$ Obtain integral $x \sin x + \cos x$ Substitute limits correctly, must be seen since AG, and equate result to 0.5 Obtain the given form of the equation		M1* A1 M1(dep*) A1	4
	(ii) <i>EITHER</i> : Consider the sign of a relevant expression at $a = 1$ and at another relevant value, e.g. $a = 1.5 \leq \frac{\pi}{2}$ <i>OR</i> : Using limits correctly, consider the sign of $[x \sin x + \cos x]_0^a - 0.5$, or compare the value of $[x \sin x + \cos x]_0^a$ with 0.5, for $a = 1$ AND for another relevant value, e.g. $a = 1.5 \leq \frac{\pi}{2}$.		M1 M1	
	Complete the argument, so change of sign, or above and below stated, both with correct calculated values		A1	2
	(iii) Use the iterative formula correctly at least once Obtain final answer 1.2461 Show sufficient iterations to 6 d.p. to justify 1.2461 to 4 d.p., or show there is a sign change in the interval (1.24605, 1.24615)		M1 A1 A1	3
7	(i) Separate variables correctly and integrate one side Obtain term $2\sqrt{M}$, or equivalent Obtain term $50k \sin(0.02t)$, or equivalent Evaluate a constant of integration, or use limits $M = 100, t = 0$ in a solution with terms of the form $a\sqrt{M}$ and $b \sin(0.02t)$ Obtain correct solution in any form, e.g. $2\sqrt{M} = 50k \sin(0.02t) + 20$		B1 B1 B1 M1*	5
	(ii) Use values $M = 196, t = 50$ and calculate k Obtain answer $k = 0.190$		M1(dep*) A1	2
	(iii) State an expression for M in terms of t , e.g. $M = (4.75 \sin(0.02t) + 10)^2$ State that the least possible number of micro-organisms is 28 or 27.5 or 27.6 (27.5625)		M1(dep*) A1	2

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- 8 (i) *EITHER*: Substitute for u in $\frac{i}{u}$ and multiply numerator and denominator by $1 + i$ M1
Obtain final answer $-\frac{1}{2} + \frac{1}{2}i$, or equivalent A1
OR: Substitute for u , obtain two equations in x and y and solve for x or for y M1
Obtain final answer $-\frac{1}{2} + \frac{1}{2}i$, or equivalent A1 **2**
- (ii) Show a point representing u in a relatively correct position B1
Show the bisector of the line segment joining u to the origin B1
Show a circle with centre at the point representing i B1
Show a circle with radius 2 B1 **4**
- (iii) State argument $-\frac{1}{2}\pi$, or equivalent, e.g. 270° B1
State or imply the intersection in the first quadrant represents $2 + i$ B1
State argument 0.464 , (0.4636) or equivalent, e.g. 26.6° (26.5625) B1 **3**
- 9 (i) State or imply a correct normal vector to either plane, e.g. $\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$, or $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ B1
Carry out correct process for evaluating the scalar product of two normal vectors M1
Using the correct process for the moduli, divide the scalar product of the two normals by the product of their moduli and evaluate the inverse cosine of the result M1
Obtain answer 85.9° or 1.50 radians A1 **4**

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(ii)	<i>EITHER:</i> Carry out a complete strategy for finding a point on l	M1	
	Obtain such a point, e.g. (0, 2, 1)	A1	
	<i>EITHER:</i> State two equations for a direction vector $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ for l ,		
	e.g. $a + 3b - 2c = 0$		
	and $2a + b + 3c = 0$	B1	
	Solve for one ratio, e.g. $a : b$	M1	
	Obtain $a : b : c = 11 : -7 : -5$	A1	
	State a correct answer, e.g. $\mathbf{r} = 2\mathbf{j} + \mathbf{k} + \lambda(11\mathbf{i} - 7\mathbf{j} - 5\mathbf{k})$	A1 [√]	
	<i>OR1:</i> Obtain a second point on l , e.g. $\left(\frac{22}{7}, 0, -\frac{3}{7}\right)$	B1	
	Subtract position vectors and obtain a direction vector for l	M1	
	Obtain $22\mathbf{i} - 14\mathbf{j} - 10\mathbf{k}$, or equivalent	A1	
	State a correct answer, e.g. $\mathbf{r} = 2\mathbf{j} + \mathbf{k} + \lambda(22\mathbf{i} - 14\mathbf{j} - 10\mathbf{k})$	A1 [√]	
	<i>OR2:</i> Attempt to find the vector product of the two normal vectors	M1	
	Obtain two correct components	A1	
	Obtain $11\mathbf{i} - 7\mathbf{j} - 5\mathbf{k}$, or equivalent	A1	
	State a correct answer, e.g. $\mathbf{r} = 2\mathbf{j} + \mathbf{k} + \lambda(11\mathbf{i} - 7\mathbf{j} - 5\mathbf{k})$	A1 [√]	
	<i>OR3:</i> Express one variable in terms of a second	M1	
	Obtain a correct simplified expression, e.g. $x = (22 - 11y)/7$	A1	
	Express the same variable in terms of the third	M1	
	Obtain a correct simplified expression, e.g. $x = (11 - 11z)/5$	A1	
	Form a vector equation for the line M1		
	State a correct answer, e.g. $\mathbf{r} = 2\mathbf{j} + \mathbf{k} + \lambda\left(\mathbf{i} - \frac{7}{11}\mathbf{j} - \frac{5}{11}\mathbf{k}\right)$	A1 [√]	
	<i>OR4:</i> Express one variable in terms of a second	M1	
	Obtain a correct simplified expression, e.g. $y = (22 - 7x)/11$	A1	
	Express the third variable in terms of the second	M1	
	Obtain a correct simplified expression, e.g. $z = (11 - 5x)/11$	A1	
	Form a vector equation for the line	M1	
	State a correct answer, e.g. $\mathbf{r} = 2\mathbf{j} + \mathbf{k} + \lambda\left(\mathbf{i} - \frac{7}{11}\mathbf{j} - \frac{5}{11}\mathbf{k}\right)$	A1 [√]	6
	[The [√] marks are dependent on all M marks being earned.]		
10	(i) State or imply $f(x) \equiv \frac{A}{2x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$	B1	
	Use a relevant method to determine a constant	M1	
	Obtain one of the values $A = 2, B = -1, C = 3$	A1	
	Obtain the remaining values A1 +	A1	5
	[Apply an analogous scheme to the form $\frac{A}{2x-1} + \frac{Dx+E}{(x+2)^2}$; the values being $A = 2,$		
	$D = -1, E = 1.$]		

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- (ii) Integrate and obtain terms $\frac{1}{2} \cdot 2 \ln(2x-1) - \ln(x+2) - \frac{3}{x+2}$ B1[✓] + B1[✓] + B1[✓]

Use limits correctly, namely substitution must be seen in at least two of the partial fractions to obtain M1 Integrate all 3 partial fractions and substitute in all three partial fractions for A1 since AG. M1

Obtain the given answer following full and exact working A1

[The t marks are dependent on A, B, C etc.] 5

[SR: If B, C or E omitted, give B1M1 in part (i) and B1[✓]B1[✓]M1 in part (ii).]

[NB: Candidates who follow the A, D, E scheme in part (i) and then integrate $\frac{-x+1}{(x+2)^2}$

by parts should obtain $\frac{1}{2} \cdot 2 \ln(2x-1) - \ln(x+2) + \frac{x-1}{x+2}$ (the third term is equivalent

to $-\frac{3}{x+2} + 1$.)]