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- 1 Use law for the logarithm of a power of at least once *M1
 Obtain correct linear equation, e.g. $5x \ln 2 = (2x + 1) \ln 3$ A1
 Solve a linear equation for x M1 dep *M
 Obtain $x = 0.866$ A1 [4]
- 2 Attempt calculation of at least 3 ordinates M1
 Obtain 9, 7, 1, 17 A1
 Use trapezium rule with $h = 1$ M1
 Obtain $\frac{1}{2}(9 + 14 + 2 + 17)$ or equivalent and hence 21 A1 [4]
- 3 Either Obtain correct (unsimplified) version of x^2 or x^4 term in $(1 - 2x^2)^{-2}$ M1
 Obtain $1 + 4x^2$ A1
 Obtain ... $+12x^4$ A1
 Obtain correct (unsimplified) version of x^2 or x^4 term in $(1 + 6x^2)^{\frac{2}{3}}$ M1
 Obtain $1 + 4x^2 - 4x^4$ A1
 Combine expansions to obtain $k = 16$ with no error seen A1
- Or Obtain correct (unsimplified) version of x^2 or x^4 term in $(1 + 6x^2)^{\frac{2}{3}}$ M1
 Obtain $1 + 4x^2$ A1
 Obtain ... $-4x^4$ A1
 Obtain correct (unsimplified) version of x^2 or x^4 term in $(1 - 2x^2)^{-2}$ M1
 Obtain $1 + 4x^2 + 12x^4$ A1
 Combine expansions to obtain $k = 16$ with no error seen A1 [6]
- 4 Differentiate to obtain form $a \sin 2x + b \cos x$ M1
 Obtain correct $-6 \sin 2x + 7 \cos x$ A1
 Use identity $\sin 2x = 2 \sin x \cos x$ B1
 Solve equation of form $c \sin x \cos x + d \cos x = 0$ to find at least one value of x M1
 Obtain 0.623 A1
 Obtain 2.52 A1
 Obtain 1.57 or $\frac{1}{2} \pi$ from equation of form $c \sin x \cos x + d \cos x = 0$ A1
 Treat answers in degrees as MR – 1 situation [7]

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- 5 (a) Use identity $\tan^2 2x = \sec^2 2x - 1$ B1
 Obtain integral of form $ax + b \tan 2x$ M1
 Obtain correct $3x + \frac{1}{2} \tan 2x$, condoning absence of $+c$ A1 [3]
- (b) State $\sin x \cos \frac{1}{2} \pi + \cos x \sin \frac{1}{6} \pi$ B1
 Simplify integrand to $\cos \frac{1}{6} \pi + \frac{\cos x \sin \frac{1}{6} \pi}{\sin x}$ or equivalent B1
 Integrate to obtain at least term of form $a \ln(\sin x)$ *M1
 Apply limits and simplify to obtain two terms M1 dep *M
 Obtain $\frac{1}{8} \pi \sqrt{3} - \frac{1}{2} \ln\left(\frac{1}{\sqrt{2}}\right)$ or equivalent A1 [5]
- 6 (i) Obtain $\pm \begin{pmatrix} 2 \\ -3 \\ -4 \end{pmatrix}$ as direction vector of l_1 B1
 State that two direction vectors are not parallel B1
 Express general point of l_1 or l_2 in component form, e.g. $(2\lambda, 1 - 3\lambda, 5 - 4\lambda)$
 or $(7 + \mu, 1 + 2\mu, 1 + 5\mu)$ B1
 Equate at least two pairs of components and solve for λ or for μ M1
 Obtain correct answers for λ and μ A1
 Verify that all three component equations are not satisfied (with no errors seen) A1 [6]
- (ii) Carry out correct process for evaluating scalar product of $\begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ M1
 Use correct process for finding modulus and evaluating inverse cosine M1
 Obtain 79.5° or 1.39 radians A1 [3]
- 7 Separate variables and factorise to obtain $\frac{dy}{(3y+1)(y+3)} = 4x dx$ or equivalent B1
 State or imply the form $\frac{A}{3y+1} + \frac{B}{y+3}$ and use a relevant method to find A or B M1
 Obtain $A = \frac{3}{8}$ and $B = -\frac{1}{8}$ A1
 Integrate to obtain form $k_1 \ln(3y+1) + k_2 \ln(y+3)$ M1
 Obtain correct $\frac{1}{8} \ln(3y+1) - \frac{1}{8} \ln(y+3) = 2x^2$ or equivalent A1
 Substitute $x = 0$ and $y = 1$ in equation of form $k_1 \ln(3y+1) + k_2 \ln(y+3) = k_3 x^2 + c$
 to find a value of c M1
 Obtain $c = 0$ A1
 Use correct process to obtain equation without natural logarithm present M1
 Obtain $y = \frac{3e^{16x^2} - 1}{3 - e^{16x^2}}$ or equivalent A1 [9]

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| 8 | (i) <u>Either</u> Expand $(2-i)^2$ to obtain $3-4i$ or unsimplified equivalent | B1 | | | |
| | | Multiply by $\frac{3+4i}{3+4i}$ and simplify to $x+iy$ form or equivalent | M1 | | |
| | | Confirm given answer $2+4i$ | A1 | | |
| | <u>Or</u> Expand $(2-i)^2$ to obtain $3-4i$ or unsimplified equivalent | B1 | | | |
| | | Obtain two equations in x and y and solve for x or y | M1 | | |
| | | Confirm given answer $2+4i$ | A1 | [3] | |
| | (ii) Identify $4+4$ or $-4+4i$ as point at either end or state $p=2$ or state $p=-6$ | B1 | | | |
| | | Use appropriate method to find both critical values of p | M1 | | |
| | | State $-6 \leq p \leq 2$ | A1 | [3] | |
| | (iii) Identify equation as of form $ z-a =a$ or equivalent | M1 | | | |
| | | Form correct equation for a not involving modulus, e.g. $(a-2)^2+4^2=a^2$ | A1 | | |
| | | State $ z-5 =5$ | A1 | [3] | |
| 9 | (i) Use product rule to find first derivative | M1 | | | |
| | | Obtain $2xe^{2-x} - x^2e^{2-x}$ | A1 | | |
| | | Confirm $x=2$ at M | A1 | [3] | |
| | (ii) Attempt integration by parts and reach $\pm x^2e^{2-x} \pm \int 2xe^{2-x} dx$ | *M1 | | | |
| | | Obtain $-x^2e^{2-x} + \int 2xe^{2-x} dx$ | A1 | | |
| | | Attempt integration by parts and reach $\pm x^2e^{2-x} \pm 2xe^{2-x} \pm 2e^{2-x}$ | *M1 | | |
| | | Obtain $-x^2e^{2-x} - 2xe^{2-x} - 2e^{2-x}$ | A1 | | |
| | | Use limits 0 and 2 having integrated twice | M1 dep *M | | |
| | | Obtain $2e^2 - 10$ | A1 | [6] | |
| | 10 | (i) Obtain $\frac{dx}{dt} = \frac{2}{t+2}$ and $\frac{dy}{dt} = 3t^2 + 2$ | B1 | | |
| | | | Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ | M1 | |
| | | | Obtain $\frac{dy}{dx} = \frac{1}{2}(3t^2+2)(t+2)$ | A1 | |
| Identify value of t at the origin as -1 | | B1 | | | |
| Substitute to obtain $\frac{5}{2}$ as gradient at the origin | | A1 | | [5] | |
| (ii) (a) Equate derivative to $\frac{1}{2}$ and confirm $p = \frac{1}{3p^2+2} - 2$ | | B1 | | [1] | |
| | | (b) Use the iterative formula correctly at least once | M1 | | |
| | | | Obtain value $p = -1.924$ or better $(-1.92367\dots)$ | A1 | |
| | | Show sufficient iterations to justify accuracy or show a sign change in appropriate interval | A1 | | |
| | | Obtain coordinates $(-5.15, -7.97)$ | A1 | | [4] |