Dogo 4	Mark Scheme		9709_s15_ms_12
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1	f'(x) = 5 - 2x ² and (3, 5) f(x) = 5x - $\frac{2x^3}{3}$ (+c) Uses (3, 5) → c = 8	B1 M1 A1 [3]	For integral Uses the point in an integral co
2	Radius of semicircle = $\frac{1}{2}AB = r\sin\theta$ Area of semicircle = $\frac{1}{2}\pi r^2 \sin^2\theta = A_1$ Shaded area = semicircle - segment = $A_1 - \frac{1}{2}r^22\theta + \frac{1}{2}r^2\sin2\theta$	B1 B1√ [™] B1B1 [4]	aef Uses $\frac{1}{2}\pi r^2$ with $r = f(\theta)$ B1 (-sector), B1 for + (triangle)
3 (i)	$(2-x)^{6}$ Coeff of x ² is 240 Coeff of x ³ is - 20 × 8 = -160	B1 B2,1 [3]	co B1 for +160
(ii)	$(3x+1)(2-x)^6$ Product needs exactly 2 terms $\rightarrow 720 - 160 = 560$	M1 A1√ [≜] [2]	$3 \times$ their 240 + their -160 \checkmark for candidate's answers.
4	u = 2x(y - x) and x + 3y = 12, $u = 2x\left(\frac{12 - x}{3} - x\right)$ $= 8x - \frac{8x^2}{3}$ $\frac{du}{dx} = 8 - \frac{16x}{3}$	M1 A1	Expresses u in terms of x
	$= 0$ when $x = 1\frac{1}{2}$	M1 A1	Differentiate candidate's quadratic, sets to $0 +$ attempt to find <i>x</i> , or other valid method
	$ \rightarrow (y = 3\frac{1}{2}) \rightarrow u = 6 $	A1 [5]	Complete method that leads to <i>u</i> Co
5 (i)	$\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}.$ Divides top and bottom by $\cos\theta$ $\rightarrow \frac{t-1}{t+1}$	B1 [1]	Answer given.
(ii)	$\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} = \frac{1}{6}\tan\theta$ $\rightarrow \frac{t-1}{t+1} = \frac{t}{6}$ $\rightarrow t^2 - 5t + 6 = 0$ $\rightarrow t = 2 \text{ or } t = 3$ $\rightarrow \theta = 63.4^\circ \text{ or } 71.6^\circ$	B1 M1 A1 A1 [4]	Using the identity. Forms a 3 term quadratic with terms all on same side. co co

$Mark Scheme$ Cambridge International AS/A Level – $h = 60(1 - \cos kt)$ Max h when $\cos = -1 \rightarrow 120$ $h = 0 \text{ and } t = 30, \text{ or } h = 120 \text{ and } t = 15$ $\rightarrow \cos 30k = 1 \text{ or } \cos 15k = -1$ $\rightarrow 30k = 2\pi \text{ or } 15k = \pi$ $\rightarrow k = \frac{2\pi}{30} = \frac{\pi}{15}$ 90 = 60(1 - \cos kt) $\rightarrow \cos kt = \frac{-30}{60} = -0.5$ $\rightarrow kt = \frac{2\pi}{3} \text{ or } \rightarrow kt = \frac{4\pi}{3}$ $\rightarrow \text{Either } t = 10 \text{ or } 20 \text{ or both}$ $\rightarrow t = 10 \text{ minutes}$ $A(4, 6), B(10, 2).$ $M = (7, 4)$ $m \text{ of } AB = -\frac{2}{3}$	B1 [1] M1 [1] M1 [2] B1 [2] B1 [3] B1 [3]	Syllabus Paper 2015 9709 12 Co Substituting a correct pair of values into the equation. co ag co – but there must be evidence of correct subtraction. co
Max h when $\cos = -1 \rightarrow 120$ h = 0 and $t = 30$, or $h = 120$ and $t = 15\rightarrow \cos 30k = 1 or \cos 15k = -1\rightarrow 30k = 2\pi or 15k = \pi\rightarrow k = \frac{2\pi}{30} = \frac{\pi}{15}90 = 60(1 - \cos kt)\rightarrow \cos kt = \frac{-30}{60} = -0.5\rightarrow kt = \frac{2\pi}{3} or \rightarrow kt = \frac{4\pi}{3}\rightarrow \text{Either } t = 10 \text{ or } 20 \text{ or both}\rightarrow t = 10 \text{ minutes}A(4, 6), B(10, 2).M = (7, 4)$	[1] M1 A1 [2] B1 B1 [3] B1	Substituting a correct pair of values into the equation. co ag co – but there must be evidence of correct subtraction.
h = 0 and t = 30, or h = 120 and t = 15 $\rightarrow \cos 30k = 1 \text{ or } \cos 15k = -1$ $\rightarrow 30k = 2\pi \text{ or } 15k = \pi$ $\rightarrow k = \frac{2\pi}{30} = \frac{\pi}{15}$ $90 = 60(1 - \cos kt)$ $\rightarrow \cos kt = \frac{-30}{60} = -0.5$ $\rightarrow kt = \frac{2\pi}{3} \text{ or } \rightarrow kt = \frac{4\pi}{3}$ $\rightarrow \text{ Either } t = 10 \text{ or } 20 \text{ or both}$ $\rightarrow t = 10 \text{ minutes}$ A(4, 6), B(10, 2). M = (7, 4)	[1] M1 A1 [2] B1 B1 [3] B1	Substituting a correct pair of values into the equation. co ag co – but there must be evidence of correct subtraction.
$\rightarrow \cos 30k = 1 \text{or } \cos 15k = -1$ $\rightarrow 30k = 2\pi \text{or } 15k = \pi$ $\rightarrow k = \frac{2\pi}{30} = \frac{\pi}{15}$ $90 = 60(1 - \cos kt)$ $\rightarrow \cos kt = \frac{-30}{60} = -0.5$ $\rightarrow kt = \frac{2\pi}{3} \text{or } \rightarrow kt = \frac{4\pi}{3}$ $\rightarrow \text{Either } t = 10 \text{ or } 20 \text{ or both}$ $\rightarrow t = 10 \text{ minutes}$ A(4, 6), B(10, 2). M = (7, 4)	A1 [2] B1 [3] B1 [3]	into the equation. co ag co – but there must be evidence of correct subtraction.
$90 = 60(1 - \cos kt)$ $\rightarrow \cos kt = \frac{-30}{60} = -0.5$ $\rightarrow kt = \frac{2\pi}{3} \text{ or } \rightarrow kt = \frac{4\pi}{3}$ $\rightarrow \text{Either } t = 10 \text{ or } 20 \text{ or both}$ $\rightarrow t = 10 \text{ minutes}$ $A(4, 6), B(10, 2).$ $M = (7, 4)$	[2] B1 B1 [3] B1	co – but there must be evidence of correct subtraction.
$\rightarrow \cos kt = \frac{-30}{60} = -0.5$ $\rightarrow kt = \frac{2\pi}{3} \text{ or } \rightarrow kt = \frac{4\pi}{3}$ $\rightarrow \text{Either } t = 10 \text{ or } 20 \text{ or both}$ $\rightarrow t = 10 \text{ minutes}$ A(4, 6), B(10, 2). M = (7, 4)	B1 B1 [3] B1	correct subtraction.
$\rightarrow t = 10 \text{ minutes}$ A(4, 6), B(10, 2). M = (7, 4)	B1 [3] B1	со
M = (7, 4)		со
		со
5	B1	со
<i>m</i> of perpendicular = $\frac{3}{2}$ $\rightarrow y - 4 = \frac{3}{2}(x - 7)$	M1 A1 [4]	Use of $m_1m_2 = -1$ & their midpoint in the equation of a line. co
Eqn of line parallel to <i>AB</i> through (3, 11) $\rightarrow y - 11 = -\frac{2}{3}(x - 3)$ Sim eqns $\rightarrow C(9, 7)$	M1 DM1A1 [3]	Needs to use <i>m</i> of <i>AB</i> Must be using their correct lines. Co
1st, 2nd, <i>n</i> th are 56, 53 and -22		
	M1 A1 M1 A1 [4]	Uses correct u_n formula. co Needs positive integer n Co
1^{st} , 2^{nd} , 3^{rd} are $2k + 6$, $2k$ and $k + 2$.		
Either $\frac{2k}{2k+6} = \frac{k+2}{2k}$ or uses <i>a</i> , <i>r</i> and eliminates $\rightarrow 2k^2 - 10k - 12 = 0$	M1 DM1 A1	Correct method for equation in <i>k</i> . Forms quad. or cubic equation with no brackets or fractions. Co
	Sim eqns $\rightarrow C(9, 7)$ 1st, 2nd, <i>n</i> th are 56, 53 and -22 a = 56, d = -3 -22 = 56 + (n - 1)(-3) $\rightarrow n = 27$ $S_{27} = \frac{27}{2}(112 + 26(-3))$ $\rightarrow 459$ 1 st , 2 nd , 3 rd are 2k + 6, 2k and k + 2. Either $\frac{2k}{2k+6} = \frac{k+2}{2k}$	Sim eqns $\rightarrow C(9, 7)$ DM1A1 [3] 1st, 2nd, <i>n</i> th are 56, 53 and -22 a = 56, d = -3 -22 = 56 + (n - 1)(-3) $\rightarrow n = 27$ $S_{27} = \frac{27}{2}(112 + 26(-3))$ $\rightarrow 459$ Ist, 2 nd , 3 rd are 2k + 6, 2k and k + 2. Either $\frac{2k}{2k+6} = \frac{k+2}{2k}$ or uses a, r and eliminates M1 M1

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(ii)	$S_{\infty} = \frac{a}{1-r}$ with $r = \frac{2k}{2k+6}$ or $\frac{k+2}{2k} (=\frac{2}{3})$ $\rightarrow 54$	M1 A1 [2]	Needs attempt at a and r and S_{∞} Co
9	$\overrightarrow{OA} = 2\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$ and $\overrightarrow{OB} = 3\mathbf{i} + \mathbf{j} + 4\mathbf{k}$		
(i)	\overrightarrow{OA} . $\overrightarrow{OB} = 6 + 4 + 16 = 26$	M1	Must be numerical at some stage
	$\left \overrightarrow{OA}\right = \sqrt{36}$, $\left \overrightarrow{OB}\right = \sqrt{26}$	M1	Product of 2 moduli
	$\cos AOB = \frac{26}{6\sqrt{26}}$	M1	All linked correctly
	$\rightarrow 31.8^{\circ}$	A1 [4]	со
(ii)	$\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix}$ $\overrightarrow{OC} = \begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix} + 2\overrightarrow{AB} \text{ or } \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} + \overrightarrow{AB}$	B1 M1	Correct link
	$\overrightarrow{OC} = \begin{pmatrix} 4\\ -2\\ 4 \end{pmatrix}$		
	Unit vector \div modulus $\rightarrow \frac{1}{6} \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix}$	M1 A1 [4]	÷ by modulus. co
(iii)	$\left \overrightarrow{OC}\right = 6, \left \overrightarrow{OA}\right = 6$	B1 [1]	со

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		F	
10	$y = \frac{4}{2x-1}.$ $\int \frac{16}{(2x-1)^2} dx = \frac{-16}{2x-1} \div 2$	B1	Correct without the ÷2
(i)	$\int \frac{16}{(2x-1)^2} dx = \frac{-16}{2x-1} \div 2$	B1	For the ÷2 even if first B1 is lost
	$Vol = \pi \left[\frac{-8}{2x-1} \right]$ with limits 1 and 2	M1 A1	Use of limits in a changed expression. co
	$\rightarrow \frac{16\pi}{3}$	[4]	
(ii)	$m = \frac{1}{2}m$ of tangent = -2	M1	Use of $m_1m_2 = -1$
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-4}{(2x-1)^2} \times 2$	B1 B1	Correct without the ×2 For the ×2 even if first B1 is lost
	Equating their $\frac{dy}{dx}$ to -2 $\rightarrow x = \frac{3}{2}$ or $-\frac{1}{2}$	DM1	
	$\rightarrow x = \frac{3}{2} \text{ or } -\frac{1}{2}$ $(y = 2 \text{ or } -2)$ $\rightarrow c = \frac{5}{2} \text{ or } -\frac{7}{2}$	A1	со
		A1	со
		[6]	
11	$f: x \mapsto 2x^2 - 6x + 5$		
(i)	$2x^2 - 6x + 5 - p = 0$ has no real roots	M1	Sets to 0 with p on LHS.
	Uses $b^2 - 4ac \rightarrow 36 - 8(5 - p)$	DM1	Uses discriminant.
	Sets to $0 \rightarrow p < \frac{1}{2}$	A1 [3]	$co - must be "<", not "\leq".$
(ii)	$2x^{2} - 6x + 5 = 2\left(x - \frac{3}{2}\right)^{2} + \frac{1}{2}$	3 × B1 [3]	со
(iii)	Range of g $\frac{1}{2} \le g(x) \le 13$	B1√ [≜] B1 [2]	\checkmark^{h} on (ii) co from sub of $x = 4$
	h: $x \mapsto 2x^2 - 6x + 5$ for $k \le x \le 4$		
(iv)	Smallest $k = \frac{3}{2}$	B1√ [^] [1]	⊌ [≜] on (ii)
(v)	$h(x) = 2\left(x - \frac{3}{2}\right)^2 + \frac{1}{2}$	M1	Using comp square form to try and get x as subject or y if transposed.
	Order of operations $\pm \frac{1}{2}$, $\div 2$, $\sqrt{2}$, $\pm \frac{3}{2}$	DM1	Order must be correct
	\rightarrow Inverse = $\frac{3}{2} + \sqrt{\left(\frac{x}{2} - \frac{1}{4}\right)}$	A1 [3]	co (without ±)