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- 1 *EITHER*: State or imply non-modular inequality $(x + 2a)^2 > (3(x - a))^2$, or corresponding quadratic equation, or pair of linear equations $(x + 2a) = \pm 3(x - a)$ B1
 Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x M1
 Obtain critical values $x = \frac{1}{4}a$ and $x = \frac{5}{2}a$ A1
 State answer $\frac{1}{4}a < x < \frac{5}{2}a$ A1
- OR*: Obtain critical value $x = \frac{5}{2}a$ from a graphical method, or by inspection, or by solving a linear equation or inequality B1
 Obtain critical value $x = \frac{1}{4}a$ similarly B2
 State answer $\frac{1}{4}a < x < \frac{5}{2}a$ B1 4
 [Do not condone \leq for $<$.]
- 2 Remove logarithms and obtain $5 - e^{-2x} = e^{\frac{1}{2}}$, or equivalent B1
 Obtain a correct value for e^{-2x} , e^{2x} , e^{-x} or e^x , e.g. $e^{2x} = 1/(5 - e^{\frac{1}{2}})$ B1
 Use correct method to solve an equation of the form $e^{2x} = a$, $e^{-2x} = a$, $e^x = a$ or $e^{-x} = a$ where $a > 0$. [The M1 is dependent on the correct removal of logarithms.] M1
 Obtain answer $x = -0.605$ only. A1 4
- 3 Use $\cos(A + B)$ formula to obtain an equation in $\cos x$ and $\sin x$ M1
 Use trig formula to obtain an equation in $\tan x$ (or $\cos x$ or $\sin x$) M1
 Obtain $\tan x = \sqrt{3} - 4$, or equivalent (or find $\cos x$ or $\sin x$) A1
 Obtain answer $x = -66.2^\circ$ A1
 Obtain answer $x = 113.8^\circ$ and no others in the given interval A1 5
 [Ignore answers outside the given interval. Treat answers in radians as a misread $(-1.16, 1.99)$.]
 [The other solution methods are *via* $\cos x = \pm 1/\sqrt{1 + (\sqrt{3} - 4)^2}$ and $\sin x = \pm(\sqrt{3} - 4)/\sqrt{1 + (\sqrt{3} - 4)^2}$.]
- 4 (i) State $\frac{dx}{dt} = 1 - \sec^2 t$, or equivalent B1
 Use chain rule M1
 Obtain $\frac{dy}{dt} = -\frac{\sin t}{\cos t}$, or equivalent A1
 Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ M1
 Obtain the given answer correctly. A1 5
- (ii) State or imply $t = \tan^{-1}(\frac{1}{2})$ B1
 Obtain answer $x = -0.0364$ B1 2

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5	(i) Differentiate $f(x)$ and obtain $f'(x) = (x-2)^2 g'(x) + 2(x-2)g(x)$ Conclude that $(x-2)$ is a factor of $f'(x)$		B1 B1
	(ii) <i>EITHER</i> : Substitute $x = 2$, equate to zero and state a correct equation, e.g. $32 + 16a + 24 + 4b + a = 0$ Differentiate polynomial, substitute $x = 2$ and equate to zero or divide by $(x-2)$ and equate constant remainder to zero Obtain a correct equation, e.g. $80 + 32a + 36 + 4b = 0$		B1 M1* A1
	<i>OR1</i> : Identify given polynomial with $(x-2)^2(x^3 + Ax^2 + Bx + C)$ and obtain an equation in a and/or b Obtain a correct equation, e.g. $\frac{1}{4}a - 4(4+a) + 4 = 3$ Obtain a second correct equation, e.g. $-\frac{3}{4}a + 4(4+a) = b$		M1* A1 A1
	<i>OR2</i> : Divide given polynomial by $(x-2)^2$ and obtain an equation in a and b Obtain a correct equation, e.g. $29 + 8a + b = 0$ Obtain a second correct equation, e.g. $176 + 47a + 4b = 0$ Solve for a or for b Obtain $a = -4$ and $b = 3$		M1* A1 A1 M1(dep*) A1
6	(i) Use correct arc formula and form an equation in r and x Obtain a correct equation in any form Rearrange in the given form		M1 A1 A1
	(ii) Consider sign of a relevant expression at $x = 1$ and $x = 1.5$, or compare values of relevant expressions at $x = 1$ and $x = 1.5$ Complete the argument correctly with correct calculated values		M1 A1
	(iii) Use the iterative formula correctly at least once Obtain final answer 1.21 Show sufficient iterations to 4 d.p. to justify 1.21 to 2 d.p., or show there is a sign change in the interval (1.205, 1.215)		M1 A1 A1
7	(a) <i>EITHER</i> : Substitute and expand $(-1 + \sqrt{5}i)^3$ completely Use $i^2 = -1$ correctly at least once Obtain $a = -12$ State that the other complex root is $-1 - \sqrt{5}i$		M1 M1 A1 B1
	<i>OR1</i> : State that the other complex root is $-1 - \sqrt{5}i$ State the quadratic factor $z^2 + 2z + 6$ Divide the cubic by a 3-term quadratic, equate remainder to zero and solve for a or, using a 3-term quadratic, factorise the cubic and determine a Obtain $a = -12$		B1 B1 M1 A1
	<i>OR2</i> : State that the other complex root is $-1 - \sqrt{5}i$ State or show the third root is 2 Use a valid method to determine a Obtain $a = -12$		B1 B1 M1 A1
	<i>OR3</i> : Substitute and use De Moivre to cube $\sqrt{6}\text{cis}(114.1^\circ)$, or equivalent Find the real and imaginary parts of the expression Obtain $a = -12$ State that the other complex root is $-1 - \sqrt{5}i$		M1 M1 A1 B1

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(b)	<i>EITHER:</i> Substitute $w = \cos 2\theta + i \sin 2\theta$ in the given expression		B1
	Use double angle formulae throughout		M1
	Express numerator and denominator in terms of $\cos \theta$ and $\sin \theta$ only		A1
	Obtain given answer correctly		A1
	<i>OR:</i> Substitute $w = e^{2i\theta}$ in the given expression		B1
	Divide numerator and denominator by $e^{i\theta}$, or equivalent		M1
	Express numerator and denominator in terms of $\cos \theta$ and $\sin \theta$ only		A1
	Obtain the given answer correctly		A1
			4
8	(i) Use product rule		M1
	Obtain derivative in any correct form		A1
	Differentiate first derivative using the product rule		M1
	Obtain second derivative in any correct form, e.g. $-\frac{1}{2} \sin \frac{1}{2}x - \frac{1}{4}x \cos \frac{1}{2}x - \frac{1}{2} \sin \frac{1}{2}x$		A1
	Verify the given statement		A1
			5
	(ii) Integrate and reach $kx \sin \frac{1}{2}x + l \int \sin \frac{1}{2}x \, dx$		M1*
	Obtain $2x \sin \frac{1}{2}x - 2 \int \sin \frac{1}{2}x \, dx$, or equivalent		A1
	Obtain indefinite integral $2x \sin \frac{1}{2}x + 4 \cos \frac{1}{2}x$		A1
	Use correct limits $x = 0, x = \pi$ correctly		M1(dep*)
	Obtain answer $2\pi - 4$, or exact equivalent		A1
			5
9	(i) State or imply $\frac{dN}{dt} = kN(1 - 0.01N)$ and obtain the given answer $k = 0.02$		B1
			1
	(ii) Separate variables and attempt integration of at least one side		M1
	Integrate and obtain term $0.02t$, or equivalent		A1
	Carry out a relevant method to obtain A or B such that $\frac{1}{N(1 - 0.01N)} \equiv \frac{A}{N} + \frac{B}{1 - 0.01N}$, or		
	equivalent		M1*
	Obtain $A = 1$ and $B = 0.01$, or equivalent		A1
	Integrate and obtain terms $\ln N - \ln(1 - 0.01N)$, or equivalent		A1 ^h
	Evaluate a constant or use limits $t = 0, N = 20$ in a solution with terms $a \ln N$ and $b \ln(1 - 0.01N)$, $ab \neq 0$		M1(dep*)
	Obtain correct answer in any form, e.g. $\ln N - \ln(1 - 0.01N) = 0.02t + \ln 25$		A1
	Rearrange and obtain $t = 50 \ln(4N/(100 - N))$, or equivalent		A1
			8
	(iii) Substitute $N = 40$ and obtain $t = 49.0$		B1
			1

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- 10 (i) *EITHER*: State or imply \vec{AB} and \vec{AC} correctly in component form B1
 Using the correct processes evaluate the scalar product $\vec{AB} \cdot \vec{AC}$, or equivalent M1
 Using the correct process for the moduli divide the scalar product by the product of the moduli M1
 Obtain answer $\frac{20}{21}$ A1
- OR*: Use correct method to find lengths of all sides of triangle ABC M1
 Apply cosine rule correctly to find the cosine of angle BAC M1
 Obtain answer $\frac{20}{21}$ A1 **4**
- (ii) State an exact value for the sine of angle BAC , e.g. $\frac{\sqrt{41}}{21}$ B1[†]
 Use correct area formula to find the area of triangle ABC M1
 Obtain answer $\frac{1}{2}\sqrt{41}$, or exact equivalent A1 **3**
- [SR: Allow use of a vector product, e.g. $\vec{AB} \times \vec{AC} = -6\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ B1[†]. Using correct process for the modulus, divide the modulus by 2 M1. Obtain answer $\frac{1}{2}\sqrt{41}$ A1.]
- (iii) *EITHER*: State or obtain $b = 0$ B1
 Equate scalar product of normal vector and \vec{BC} (or \vec{CB}) to zero M1
 Obtain $a + b - 4c = 0$ (or $a - 4c = 0$) A1
 Substitute a relevant point in $4x + z = d$ and evaluate d M1
 Obtain answer $4x + z = 9$, or equivalent A1
- OR1*: Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{j}) \times (\mathbf{i} + \mathbf{j} - 4\mathbf{k})$ M1
 Obtain two correct components of the product A1
 Obtain correct product, e.g. $-4\mathbf{i} - \mathbf{k}$ A1
 Substitute a relevant point in $4x + z = d$ and evaluate d M1
 Obtain $4x + z = 9$, or equivalent A1
- OR2*: Attempt to form 2-parameter equation for the plane with relevant vectors M1
 State a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k} + \lambda(\mathbf{j}) + \mu(\mathbf{i} + \mathbf{j} - 4\mathbf{k})$ A1
 State 3 equations in x, y, z, λ and μ A1
 Eliminate μ M1
 Obtain answer $4x + z = 9$, or equivalent A1
- OR3*: State or obtain $b = 0$ B1
 Substitute for B and C in the plane equation and obtain $2a + c = d$ and $3a - 3c = d$ (or $2a + 4b + c = d$ and $3a + 5b - 3c = d$) B1
 Solve for one ratio, e.g. $a : d$ M1
 Obtain $a : c : d$, or equivalent M1
 Obtain answer $4x + z = 9$, or equivalent A1
- OR4*: Attempt to form a determinant equation for the plane with relevant vectors M1
 State a correct equation, e.g.
$$\begin{vmatrix} x-2 & y-4 & z-1 \\ 0 & 1 & 0 \\ 1 & 1 & -4 \end{vmatrix} = 0$$
 A1
 Attempt to use a correct method to expand the determinant M1
 Obtain two correct terms of a 3-term expansion, or equivalent A1
 Obtain answer $4x + z = 9$, or equivalent A1 **5**