

Page 4	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9709	31
1	(i) State $\sin 2\alpha = 2\sin\alpha \cos\alpha$ and $\sec\alpha = 1/\cos\alpha$ Obtain $2\sin\alpha$	B1 B1	[2]
	(ii) Use $\cos 2\beta = 2\cos^2\beta - 1$ or equivalent to produce correct equation in $\cos\beta$ Solve three-term quadratic equation for $\cos\beta$ Obtain $\cos\beta = \frac{1}{3}$ only	B1 M1 A1	[3]
2	State $\frac{du}{dx} = 3\sec^2 x$ or equivalent Express integral in terms of u and du (accept unsimplified and without limits) Obtain $\int \frac{1}{3}u^{\frac{1}{2}} du$ Integrate $Cu^{\frac{1}{2}}$ to obtain $\frac{2C}{3}u^{\frac{3}{2}}$ Obtain $\frac{14}{9}$	B1 M1 A1 M1 A1	[5]
3	Obtain $\frac{2}{2t+3}$ for derivative of x Use quotient of product rule, or equivalent, for derivative of y Obtain $\frac{5}{(2t+3)^2}$ or unsimplified equivalent Obtain $t = -1$ Use $\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}$ in algebraic or numerical form Obtain gradient $\frac{5}{2}$	B1 M1 A1 B1 M1 A1	[6]
4	Separate variables correctly and recognisable attempt at integration of at least one side Obtain $\ln y$, or equivalent Obtain $k \ln(2 + e^{3x})$ Use $y(0) = 36$ to find constant in $y = A(2 + e^{3x})^k$ or $\ln y = k \ln(2 + e^{3x}) + c$ or equivalent Obtain equation correctly without logarithms from $\ln y = \ln\left(A(2 + e^{3x})^k\right)$ Obtain $y = 4(2 + e^{3x})^2$	M1 B1 B1 M1* *M1 A1	[6]

Page 5	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9709	31

- 5 (i) Either Multiply numerator and denominator by $\sqrt{3} + i$ and use $i^2 = -1$ M1
 Obtain correct numerator $18 + 18\sqrt{3}i$ or correct denominator 4 B1
 Obtain $\frac{9}{2} + \frac{9}{2}\sqrt{3}i$ or $(18 + 18\sqrt{3}i)/4$ A1
 Obtain modulus or argument M1
 Obtain $9e^{\frac{1}{3}\pi i}$ A1 [5]
- OR Obtain modulus and argument of numerator or denominator, or both moduli or both arguments M1
 Obtain moduli and argument 18 and $\frac{1}{6}\pi$ or 2 and $-\frac{1}{6}\pi$
 or moduli 18 and 2 or arguments $\frac{1}{6}\pi$ and $-\frac{1}{6}\pi$ (allow degrees) B1
 Obtain $18e^{\frac{1}{6}\pi i} \div 2e^{-\frac{1}{6}\pi i}$ or equivalent A1
 Divide moduli and subtract arguments M1
 Obtain $9e^{\frac{1}{3}\pi i}$ A1 [5]
- (ii) State $3e^{\frac{1}{6}\pi i}$, following through their answer to part (i) B1✓
 State $3e^{\frac{1}{6}\pi i \pm \frac{1}{2}\pi i}$, following through their answer to part (i) B1✓
 Obtain $3e^{-\frac{5}{6}\pi i}$ B1 [3]
- 6 (i) Use law for the logarithm for a product or quotient or exponentiation AND for a power M1
 Obtain $(4x - 5)^2(x + 1) = 27$ B1
 Obtain given equation correctly $16x^3 - 24x^2 - 15x - 2 = 0$ A1 [3]
- (ii) Obtain $x = 2$ is root or $(x - 2)$ is a factor, or likewise with $x = -\frac{1}{4}$ B1
 Divide by $(x - 2)$ to reach a quotient of the form $16x^2 + kx$ M1
 Obtain quotient $16x^2 + 8x + 1$ A1
 Obtain $(x - 2)(4x + 1)^2$ or $(x - 2), (4x + 1), (4x + 1)$ A1 [4]
- (iii) State $x = 2$ only A1 [1]
- 7 (i) Obtain $2x - 3y + 6z$ for LHS of equation B1
 Obtain $2x - 3y + 6z = 23$ B1 [2]
- (ii) Either Use correct formula to find perpendicular distance M1
 Obtain unsimplified value $\frac{\pm 23}{\sqrt{2^2 + (-3)^2 + 6^2}}$, following answer to (i) A1✓
 Obtain $\frac{23}{7}$ or equivalent A1 [3]

Page 6	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9709	31

<u>OR 1</u>	Use scalar product of $(4, -1, 2)$ and a vector normal to the plane	M1	
	Use unit normal to plane to obtain $\pm \frac{(8+3+12)}{\sqrt{49}}$	A1	
	Obtain $\frac{23}{7}$ or equivalent	A1	[3]
<u>OR 2</u>	Find parameter intersection of p and $\mathbf{r} = \mu(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$	M1	
	Obtain $\mu = \frac{23}{49}$ [and $(\frac{46}{49}, -\frac{69}{49}, \frac{138}{49})$ as foot of perpendicular]	A1	
	Obtain distance $\frac{23}{7}$ or equivalent	A1	[3]
(iii) <u>Either</u>	Recognise that plane is $2x - 3y + 6z = k$ and attempt use of formula for perpendicular distance to plane at least once	M1	
	Obtain $\frac{ 23-k }{7} = 14$ or equivalent	A1	
	Obtain $2x - 3y + 6z = 121$ and $2x - 3y + 6z = -75$	A1	[3]
<u>OR</u>	Recognise that plane is $2x - 3y + 6z = k$ and attempt to find at least one point on q using l with $\lambda = \pm 2$	M1	
	Obtain $2x - 3y + 6z = 121$	A1	
	Obtain $2x - 3y + 6z = -75$	A1	[3]
8 (i)	Sketch $y = \operatorname{cosec} x$ for at least $0, x, \pi$	B1	
	Sketch $y = x(\pi - x)$ for at least $0, x, \pi$	B1	
	Justify statement concerning two roots, with evidence of 1 and $\frac{1}{4}\pi^2$ for y -values on graph via scales	B1	[3]
(ii)	Use $\operatorname{cosec} x = \frac{1}{\sin x}$ and commence rearrangement	M1	
	Obtain given equation correctly, showing sufficient detail	A1	[2]
(iii) (a)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 0.66	A1	
	Show sufficient iterations to 4 decimal places to justify answer or show a sign change in the interval $(0.655, 0.665)$	A1	[3]
(b)	Obtain 2.48	B1	[1]

Page 7	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9709	31

- 9 (i) Either State or imply partial fractions are of form $\frac{A}{3-x} + \frac{B}{1+2x} + \frac{C}{(1+2x)^2}$ B1
- Use any relevant method to obtain a constant M1
- Obtain $A = 1$ A1
- Obtain $B = \frac{3}{2}$ A1
- Obtain $C = -\frac{1}{2}$ A1 [5]
- Or State or imply partial fractions are of form $\frac{A}{3-x} + \frac{Dx+E}{(1+2x)^2}$ B1
- Use any relevant method to obtain a constant M1
- Obtain $A = 1$ A1
- Obtain $D = 3$ A1
- Obtain $E = 1$ A1 [5]
- (ii) Obtain the first two terms of one of the expansion of $(3-x)^{-1}, \left(1-\frac{1}{3}x\right)^{-1}$
- $(1+2x)^{-1}$ and $(1+2x)^{-2}$ M1
- Obtain correct unsimplified expansion up to the term in x^2 of each partial fraction, following in each case the value of A, B, C A1✓
A1✓
A1✓
- Obtain answer $\frac{4}{3} - \frac{8}{9}x + \frac{1}{27}x^2$ A1 [5]
- [If A, D, E approach used in part (i), give M1A1✓A1✓ for the expansions, M1 for multiplying out fully and A1 for final answer]
- 10 (i) Use of product or quotient rule M1
- Obtain $-5e^{-\frac{1}{2}x} \sin 4x + 40e^{-\frac{1}{2}x} \cos 4x$ A1
- Equate $\frac{dy}{dx}$ to zero and obtain $\tan 4z = k$ or $R \cos(4x \pm \alpha)$ M1
- Obtain $\tan 4x = 8$ or $\sqrt{65} \cos\left(4x \pm \tan^{-1} \frac{1}{8}\right)$ A1
- Obtain 0.362 or 20.7° A1
- Obtain 1.147 or 65.7° A1 [6]
- (ii) State or imply that x -coordinates of T_n are increasing by $\frac{1}{4}\pi$ or 45° B1
- Attempt solution of inequality (or equation) of form $x_1 + (n-1)k\pi$. 25 M1
- Obtain $n > \frac{4}{\pi}(25 - 0.362) + 1$, following through on their value of x_1 A1✓
- $n = 33$ A1 [4]