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- 1 *EITHER*: State or imply non-modular inequality $(4x + 3)^2 > x^2$, or corresponding equation or pair of equations $4x + 3 = \pm x$ M1
 Obtain a critical value, e.g. -1 A1
 Obtain a second critical value, e.g. $-\frac{3}{5}$ A1
 State final answer $x < -1, x > -\frac{3}{5}$ A1
- OR*: Obtain critical value $x = -1$, by solving a linear equation or inequality, or from a graphical method or by inspection B1
 Obtain the critical value $-\frac{3}{5}$ similarly B2
 State final answer $x < -1, x > -\frac{3}{5}$ B1 [4]
 [Do not condone \leq or \geq .]
- 2 Use law for the logarithm of a product, quotient or power M1
 Use $\ln e = 1$ or $\exp(1) = e$ M1
 Obtain correct equation free of logarithms in any form, e.g. $\frac{y+1}{y} = ex^3$ A1
 Rearrange as $y = (ex^3 - 1)^{-1}$, or equivalent A1 [4]
- 3 Use correct $\tan 2A$ formula and $\cot x = 1/\tan x$ to form an equation in $\tan x$ M1
 Obtain a correct horizontal equation in any form A1
 Solve an equation in $\tan^2 x$ for x M1
 Obtain answer, e.g. 40.2° A1
 Obtain second answer, e.g. 139.8° , and no other in the given interval A1^h [5]
 [Ignore answers outside the given interval.]
 [Treat answers in radians as a misread and deduct A1 from the marks for the angles.]
 [SR: For the answer $x = 90^\circ$ give B1 and A1 for one of the other angles.]
- 4 (i) State $R = 2$ B1
 Use trig formula to find α M1
 Obtain $\alpha = \frac{1}{6}\pi$ with no errors seen A1 [3]
- (ii) Substitute denominator of integrand and state integral $k \tan(x - \alpha)$ M1*
 State correct indefinite integral $\frac{1}{4} \tan\left(x - \frac{1}{6}\pi\right)$ A1^h
 Substitute limits M1 (dep*)
 Obtain the given answer correctly A1 [4]

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- 5 (i) Substitute $x = -\frac{1}{2}$, or divide by $(2x + 1)$, and obtain a correct equation, e.g. $a - 2b + 8 = 0$ B1
 Substitute $x = \frac{1}{2}$ and equate to 1, or divide by $(2x - 1)$ and equate constant remainder to 1 M1
 Obtain a correct equation, e.g. $a + 2b + 12 = 0$ A1
 Solve for a or for b M1
 Obtain $a = -10$ and $b = -1$ A1 [5]
- (ii) Divide by $2x^2 - 1$ and reach a quotient of the form $4x + k$ M1
 Obtain quotient $4x - 5$ A1
 Obtain remainder $3x - 2$ A1 [3]
- 6 (i) State the correct derivatives $2e^{2x-3}$ and $2/x$ B1
 Equate derivatives and use a law of logarithms on an equation equivalent to $ke^{2x-3} = m/x$ M1
 Obtain the given result correctly (or work *vice versa*) A1 [3]
- (ii) Consider the sign of $a - \frac{1}{2}(3 - \ln a)$ when $a = 1$ and $a = 2$, or equivalent M1
 Complete the argument with correct calculated values A1 [2]
- (iii) Use the iterative formula correctly at least once M1
 Obtain final answer 1.35 A1
 Show sufficient iterations to 4 d.p. to justify 1.35 to 2 d.p., or show there is a sign change in the interval (1.345, 1.355) A1 [3]
- 7 (i) Show that $a^2 + b^2 = (a + ib)(a - ib)$ B1
 Show that $(a + ib - ki)^* = a - ib + ki$ B1 [2]
- (ii) Square both sides and express the given equation in terms of z and z^* M1
 Obtain a correct equation in any form, e.g. $(z - 10i)(z^* + 10i) = 4(z - 4i)(z^* + 4i)$ A1
 Obtain the given equation A1
 Either express $|z - 2i| = 4$ in terms of z and z^* or reduce the given equation to the form
 $|z - u| = r$ M1
 Obtain the given answer correctly A1 [5]
- (iii) State that the locus is a circle with centre $2i$ and radius 5 B1 [1]

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- 8 (i) Separate variables correctly and integrate at least one side M1
 Obtain term $\ln t$, or equivalent B1
 Obtain term of the form $a \ln(k - x^3)$ M1
 Obtain term $-\frac{2}{3} \ln(k - x^3)$, or equivalent A1
EITHER: Evaluate a constant or use limits $t = 1, x = 1$ in a solution containing $a \ln t$ and $b \ln(k - x^3)$ M1*
 Obtain correct answer in any form e.g. $\ln t = -\frac{2}{3} \ln(k - x^3) + \frac{2}{3} \ln(k - 1)$ A1
 Use limits $t = 4, x = 2$, and solve for k M1(dep*)
 Obtain $k = 9$ A1
OR: Using limits $t = 1, x = 1$ and $t = 4, x = 2$ in a solution containing $a \ln t$ and $b \ln(k - x^3)$ obtain an equation in k M1*
 Obtain a correct equation in any form, e.g. $\ln 4 = -\frac{2}{3} \ln(k - 8) + \frac{2}{3} \ln(k - 1)$ A1
 Solve for k M1(dep*)
 Obtain $k = 9$ A1
 Substitute $k = 9$ and obtain $x = (9 - 8t^{\frac{3}{2}})^{\frac{1}{3}}$ A1 [9]
- (ii) State that x approaches $9^{\frac{1}{3}}$, or equivalent B1* [1]
- 9 (i) Use product rule M1
 Obtain correct derivative in any form, e.g. $4 \sin 2x \cos 2x \cos x - \sin^2 2x \sin x$ A1
 Equate derivative to zero and use a double angle formula M1*
 Reduce equation to one in a single trig function M1(dep*)
 Obtain a correct equation in any form,
 e.g. $10 \cos^3 x = 6 \cos x, 4 = 6 \tan^2 x$ or $4 = 10 \sin^2 x$ A1
 Solve and obtain $x = 0.685$ A1 [6]
- (ii) Using $du = \pm \cos x dx$, or equivalent, express integral in terms of u and du M1
 Obtain $\int 4u^2(1 - u^2) du$, or equivalent A1
 Use limits $u = 0$ and $u = 1$ in an integral of the form $au^3 + bu^5$ M1
 Obtain answer $\frac{8}{15}$ (or 0.533) A1 [4]
- 10 (i) Equate scalar product of direction vector of l and p to zero M1
 Solve for a and obtain $a = -6$ A1 [2]
- (ii) Express general point of l correctly in parametric form, e.g. $3\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$
 or $(1 - \mu)(3\mathbf{i} + 2\mathbf{j} + \mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k})$ B1
 Equate at least two pairs of corresponding components of l and the second line and solve for λ or for μ M1
 Obtain either $\lambda = \frac{2}{3}$ or $\mu = \frac{1}{3}$; or $\lambda = \frac{2}{a-1}$ or $\mu = \frac{1}{a-1}$; or reach $\lambda(a-4) = 0$
 or $(1 + \mu)(a-4) = 0$ A1
 Obtain $a = 4$ having ensured (if necessary) that all three component equations are satisfied A1 [4]

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- (iii) Using the correct process for the moduli, divide scalar product of direction vector if l and normal to p by the product of their moduli and equate to the sine of the given angle, or form an equivalent horizontal equation M1*
- Use $\frac{2}{\sqrt{5}}$ as sine of the angle A1
- State equation in any form, e.g. $\frac{a+6}{\sqrt{(a^2+4+1)}\sqrt{(1+4+4)}} = \frac{2}{\sqrt{5}}$ A1
- Solve for a M1 (dep*)
- Obtain answers for $a = 0$ and $a = \frac{60}{31}$, or equivalent A1 [5]
- [Allow use of the cosine of the angle to score M1M1.]