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- 1 *EITHER*: State or imply non-modular equation $(x-2)^2 = \left(\frac{1}{3}x\right)^2$,
- or pair of equations $x-2 = \pm\frac{1}{3}x$ M1
- Obtain answer $x = 3$ A1
- Obtain answer $x = \frac{3}{2}$, or equivalent A1
- OR*: Obtain answer $x = 3$ by solving an equation or by inspection B1
- State or imply the equation $x-2 = -\frac{1}{3}$, or equivalent M1
- Obtain answer $x = \frac{3}{2}$, or equivalent A1 [3]
- 2 (i) Use the iterative formula correctly at least once M1
- Obtain final answer 3.6840 A1
- Show sufficient iterations to at least 6 d.p. to justify 3.6840, or show there is a sign change in the interval (3.68395, 3.68405) A1 [3]
- (ii) State a suitable equation, e.g. $x = \frac{x(x^3+100)}{2(x^3+25)}$ B1
- State that the value of α is $3\sqrt{50}$, or exact equivalent B1 [2]
- 3 *EITHER*: State or imply $\ln y = \ln A - kx^2$ B1
- Substitute values of $\ln y$ and x^2 , and solve for k or $\ln A$ M1
- Obtain $k = 0.42$ or $A = 2.80$ A1
- Solve for $\ln A$ or k M1
- Obtain $A = 2.80$ or $k = 0.42$ A1
- OR1*: State or imply $\ln y = \ln A - kx^2$ B1
- Using values of $\ln y$ and x^2 , equate gradient of line to $-k$ and solve for k M1
- Obtain $k = 0.42$ A1
- Solve for $\ln A$ M1
- Obtain $A = 2.80$ A1
- OR2*: Obtain two correct equations in k and A and substituting y - and x^2 - values in $y = Ae^{-kx^2}$ B1
- Solve for k M1
- Obtain $k = 0.42$ A1
- Solve for A M1
- Obtain $A = 2.80$ A1 [5]
- [SR: If unsound substitutions are made, e.g. using $x = 0.64$ and $y = 0.76$, give B1M0A0M1A0 in the *EITHER* and *OR1* schemes, and B0M1A0M1A0 in the *OR2* scheme.]

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- 4 (i) Substitute $x = -\frac{1}{3}$, or divide by $3x + 1$, and obtain a correct equation,
 e.g. $-\frac{1}{27}a - \frac{20}{9} - \frac{1}{3} + 3 = 0$ B1
 Solve for a an equation obtained by a valid method M1
 Obtain $a = 12$ A1 [3]
- (ii) Commence division by $3x + 1$ reaching a partial quotient $\frac{1}{3}ax^2 + kx$ M1
 Obtain quadratic factor $4x^2 - 8x + 3$ A1
 Obtain factorisation $(3x + 1)(2x - 1)(2x - 3)$ A1 [3]
- [The M1 is earned if inspection reaches an unknown factor $\frac{1}{3}ax^2 + Bx + C$ and an equation in B and/or C , or an unknown factor $Ax^2 + Bx + 3$ and an equation in A and/or B , or if two coefficients with the correct moduli are stated without working.]
 [If linear factors are found by the factor theorem, give B1B1 for $(2x - 1)$ and $(2x - 3)$, and B1 for the complete factorisation.]
 [Synthetic division giving $12x^2 - 24x + 9$ as quadratic factor earns M1A1, but the final factorisation needs $(x + \frac{1}{3})$, or equivalent, in order to earn the second A1.]
 [SR: If $x = \frac{1}{3}$ is used in substitution or synthetic division, give the M1 in part (i) but give M0 in part (ii).]
- 5 EITHER: State $2ay \frac{dy}{dx}$ as derivative of ay^2 B1
 State $y^2 + 2xy \frac{dy}{dx}$ as derivative of xy^2 B1
 Equate derivative of LHS to zero and set $\frac{dy}{dx}$ equal to zero M1
 Obtain $3x^2 + y^2 - 6ax = 0$, or horizontal equivalent A1
 Eliminate y and obtain an equation in x M1
 Solve for x and obtain answer $x = \sqrt{3}a$ A1
- OR1: Rearrange equation in the form $y^2 = \frac{3ax^2 - x^3}{x + a}$ and attempt differentiation of one side B1
 Use correct quotient or product rule to differentiate RHS M1
 Obtain correct derivative of RHS in any form A1
 Set $\frac{dy}{dx}$ equal to zero and obtain an equation in x M1
 Obtain a correct horizontal equation free of surds A1
 Solve for x and obtain answer $x = \sqrt{3}a$ A1
- OR2: Rearrange equation in the form $y = \left(\frac{3ax^2 - x^3}{x + a} \right)^{\frac{1}{2}}$ and differentiation of RHS B1
 Use correct quotient or product rule and chain rule M1
 Obtain correct derivative in any form A1

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	Equate derivative to zero and obtain an equation in x	M1	
	Obtain a correct horizontal equation free of surds	A1	
	Solve for x and obtain answer $x = \sqrt{3a}$	A1	[6]
6	(i) Use correct quotient or chain rule to differentiate $\sec x$	M1	
	Obtain given derivative, $\sec x \tan x$, correctly	A1	
	Use chain rule to differentiate y	M1	
	Obtain the given answer	A1	[4]
	(ii) Using $dx\sqrt{3}\sec^2\theta d\theta$, or equivalent, express integral in terms of θ and $d\theta$	M1	
	Obtain $\int \sec\theta d\theta$	A1	
	Use limits $\frac{1}{6}\pi$ and $\frac{1}{3}\pi$ correctly in an integral form of the form $k \ln(\sec\theta + \tan\theta)$	M1	
	Obtain a correct exact final answer in the given form, e.g. $\ln\left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)$	A1	[4]
7	(i) Use $\cos(A+B)$ formula to express the given expression in terms of $\cos x$ and $\sin x$	M1	
	Collect terms and reach $\frac{\cos x}{\sqrt{2}} - \frac{3}{\sqrt{2}}\sin x$, or equivalent	A1	
	Obtain $R = 2.236$	A1	
	Use trig formula to find α	M1	
	Obtain $\alpha = 71.57^\circ$ with no errors seen	A1	[5]
	(ii) Evaluate $\cos^{-1}(2/2.236)$ to at least 1 d.p. (26.56° to 2 d.p., use of $R = \sqrt{5}$ gives 26.57°)	B1 [✓]	
	Carry out an appropriate method to find a value of x in the interval $0^\circ < x < 360^\circ$	M1	
	Obtain answer, e.g. $x = 315^\circ$ (315.0°)	A1	
	Obtain second answer, e.g. 261.9° and no others in the given interval	A1	[4]
	[Ignore answers outside the given range.]		
	[Treat answers in radians as a misread and deduct A1 from the answers for the angles.]		
	[SR: Conversion of the equation to a correct quadratic in $\sin x$, $\cos x$, or $\tan x$ earns B1, then M1 for solving a 3-term quadratic and obtaining a value of x in the given interval, and A1 + A1 for the two correct answers (candidates must reject spurious roots to earn the final A1).]		
8	(i) Use any relevant method to determine a constant	M1	
	Obtain one of the values $A = 1$, $B = -2$, $C = 4$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	[4]
	[If A and C are found by the cover up rule, give B1 + B1 then M1A1 for finding B . If only one is found by the rule, give B1M1A1A1.]		
	(ii) Separate variables and obtain one term by integrating $\frac{1}{y}$ or a partial fraction	M1	
	Obtain $\ln y = -\frac{1}{2} - 2 \ln(2x+1) + c$, or equivalent	A3 [✓]	

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	Evaluate a constant, or use limits $x = 1, y = 1$, in a solution containing at least three terms of the form $k \ln y, l/x, m \ln x$ and $n \ln(2x + 1)$, or equivalent	M1	
	Obtain solution $\ln y = -\frac{1}{2} - 2 \ln x + 2 \ln(2x + 1) + c$, or equivalent	A1	
	Substitute $x = 2$ and obtain $y = \frac{25}{36}e^{\frac{1}{2}}$, or exact equivalent free of logarithms	A1	[7]
	(The f.t. is on A, B, C . Give A2 [✓] if there is only one error or omission in the integration; A1 [✓] if two.)		
9	(a) Substitute $w = x + iy$ and state a correct equation in x and y	B1	
	Use $i^2 = -1$ and equate real parts	M1	
	Obtain $y = -2$	A1	
	Equate imaginary parts and solve for x	M1	
	Obtain $x = 2\sqrt{2}$, or equivalent, only	A1	[5]
	(b) Show a circle with centre $2i$	B1	
	Show a circle with radius 2	B1	
	Show half line from -2 at $\frac{1}{4}\pi$ to real axis	B1	
	Shade the correct region	B1	
	Carry out a complete method for calculating the greatest value of $ z $	M1	
	Obtain answer 3.70	A1	[6]
10	(i) Carry out a correct method for finding a vector equation for AB	M1	
	Obtain $\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k} + \lambda(3\mathbf{i} + \mathbf{j} - \mathbf{k})$ or		
	$\mathbf{r} = \mu(2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) + (1 - \mu)(5\mathbf{i} - 2\mathbf{j} + \mathbf{k})$, or equivalent	A1	
	Substitute components in equation of p and solve for λ or for μ	M1	
	Obtain $\lambda = \frac{3}{2}$ or $\mu = -\frac{1}{2}$ and final answer $\frac{13}{2}\mathbf{i} - \frac{3}{2}\mathbf{j} + \frac{1}{2}\mathbf{k}$, or equivalent	A1	[4]
	(ii) Either equate scalar product of direction vector of AB and normal to q to zero or substitute for A and B in the equation of q and subtract expressions	M1*	
	Obtain $3 + b - c = 0$, or equivalent	A1	
	Using the correct method for the moduli, divide the scalar product of the normals to p and q by the product of their moduli and equate to $\pm\frac{1}{2}$, or form horizontal equivalent	M1*	
	Obtain correct equation in any form, e.g. $\frac{1+b}{\sqrt{(1+b^2+c^2)}\sqrt{(1+1)}} = \pm\frac{1}{2}$	A1	
	Solve simultaneous equations for b or for c	M1 (dep*)	
	Obtain $b = -4$ and $c = -1$	A1	
	Use a relevant point and obtain final answer $x - 4y - z = 12$, or equivalent	A1 [✓]	[7]
	(The f.t. is on b and c .)		