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1 Carry out division or equivalent at least as far as two terms of quotient M1
Obtain quotient $2x-4$ A1
Obtain remainder 8 A1 [3]

2 Obtain $1-x$ as first two terms of $(1+2x)^{-\frac{1}{2}}$ B1
Obtain $+\frac{3}{2}x^2$ or unsimplified equivalent as third term of $(1+2x)^{-\frac{1}{2}}$ B1
Multiply $1+3x$ by attempt at $(1+2x)^{-\frac{1}{2}}$, obtaining sufficient terms M1
Obtain final answer $1+2x-\frac{3}{2}x^2$ A1 [4]

3 State or imply correct form $\frac{A}{x}+\frac{Bx+C}{x^2+1}$ B1
Use any relevant method to find at least one constant M1
Obtain $A=2$ A1
Obtain $B=5$ A1
Obtain $C=-3$ A1 [5]

4 **(i)** Either State or imply non-modular equation $(4x-1)^2=(x-3)^2$ or pair of linear equations $4x-1=\pm(x-3)$ B1
Solve a three-term quadratic equation or two linear equations M1
Obtain $-\frac{2}{3}$ and $\frac{4}{5}$ A1

Or Obtain value $-\frac{2}{3}$ from inspection or solving linear equation B1
Obtain value $\frac{4}{5}$ similarly B2 [3]

(ii) State or imply at least $4^y=\frac{4}{5}$, following a positive answer from part (i) B1√
Apply logarithms and use $\log a^b=b\log a$ property M1
Obtain –0.161 and no other answer A1 [3]

5 **(i)** Use correct quotient rule or equivalent M1
Obtain $\frac{(1+e^{2x})2x-(1+x^2)2e^{2x}}{(1+e^{2x})^2}$ or equivalent A1
Substitute $x=0$ and obtain $-\frac{1}{2}$ or equivalent A1 [3]

(ii) Differentiate y^3 and obtain $3y^2\frac{dy}{dx}$ B1
Differentiate $5xy$ and obtain $5y+5x\frac{dy}{dx}$ B1
Obtain $6x^2+5y+5x\frac{dy}{dx}+3y^2\frac{dy}{dx}=0$ B1

Substitute $x = 0, y = 2$ to obtain $-\frac{5}{6}$ or equivalent following correct work — B1 [4]

6 **(i)** State or imply A is (1, 4, –2) — B1
State or imply $\overrightarrow{QP} = 12\mathbf{i} + 6\mathbf{j} - 6\mathbf{k}$ or equivalent — B1
Use QP as normal and A as mid-point to find equation of plane — M1
Obtain $12x + 6y - 6z = 48$ or equivalent — A1 [4]

(ii) Either: State equation of PB is $\mathbf{r} = 7\mathbf{i} + 7\mathbf{j} - 5\mathbf{k} + \lambda\mathbf{i}$ — B1
Set up and solve a relevant equation for λ. — M1
Obtain $\lambda = -9$ and hence B is (–2, 7, –5) — A1
Use correct method to find distance between A and B. — M1
Obtain 5.20 — A1

Or: Obtain 12 for result of scalar product of QP and $\mathbf{i}$ or equivalent — B1
Use correct method involving moduli, scalar product and cosine to find angle APB — M1
Obtain $35.26°$ or equivalent — A1
Use relevant trigonometry to find AB — M1
Obtain 5.20 — A1 [5]

7 **(a)** State or imply $3a + 3b\mathrm{i} + 2\mathrm{i}(a - b\mathrm{i}) = 17 + 8\mathrm{i}$ — B1
Consider real and imaginary parts to obtain two linear equations in a and b — M1*
Solve two simultaneous linear equations for a or b — M1 (dep*)
Obtain $7 - 2\mathrm{i}$ — A1 [4]

(b) Either: Show or imply a triangle with side 2 — B1
State at least two of the angles $\frac{1}{4}\pi, \frac{2}{3}\pi$ and $\frac{1}{12}\pi$ — B1
State or imply argument is $\frac{1}{4}\pi$ — B1
Use sine rule or equivalent to find r — M1
Obtain $6.69\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}}$ — A1

Or: State $y = x$. — B1
State $y = \frac{1}{\sqrt{3}}x + 2$ or $\frac{\sqrt{3}}{2} = \frac{x}{\sqrt{x^2 + (y-2)^2}}$ or $\frac{1}{2} = \frac{y-2}{\sqrt{x^2 + (y-2)^2}}$ — B1
State or imply argument is $\frac{\pi}{4}$ — B1
Solve for x or y. — M1
Obtain $6.69e^{\frac{1}{4}\pi i}$ — A1 [5]

8 **(a)** Carry out integration by parts and reach $ax^2 \ln x + b\int \frac{1}{2}x^2 \mathrm{d}x$ — M1*
Obtain $2x^2 \ln x - \int \frac{1}{x} . 2x^2 \, \mathrm{d}x$ — A1
Obtain $2x^2 \ln x - x^2$ — A1
Use limits, having integrated twice — M1 (dep*)
Confirm given result $56 \ln 2 - 12$ — A1 [5]

(b) State or imply $\frac{du}{dx} = 4\cos 4x$ B1

Carry out complete substitution except limits M1

Obtain $\int(\frac{1}{4} - \frac{1}{4}u^2)\,du$ or equivalent A1

Integrate to obtain form $k_1u + k_2u^3$ with non-zero constants k_1, k_2 M1

Use appropriate limits to obtain $\frac{11}{96}$ A1 [5]

9 **(i)** State or imply $R = 5$ B1

Use relevant trigonometry to find α M1

Obtain $\alpha = 0.6435$ A1 [3]

(ii) **(a)** Carry out appropriate method to find one value in given range M1

Obtain 1.80 A1

Carry out appropriate method to find second value in given range M1

Obtain 5.77 and no other value A1 [4]

(b) Express integrand as $k\sec^2(\theta - \text{their } \alpha)$ for any constant k M1

Integrate to obtain result $k\tan(\theta - \text{their } \alpha)$ A1

Obtain correct answer $2\tan(\theta - 0.6435)$ A1 [3]

10 **(i)** State $\frac{dV}{dt} = 80 - kV$ B1

Correctly separate variables and attempt integration of one side M1

Obtain $a\ln(80 - kV) = t$ or equivalent M1*

Obtain $-\frac{1}{k}\ln(80 - kV) = t$ or equivalent A1

Use $t = 0$ and $V = 0$ to find constant of integration or as limits M1 (dep*)

Obtain $-\frac{1}{k}\ln(80 - kV) = t - \frac{1}{k}\ln 80$ or equivalent A1

Obtain given answer $V = \frac{1}{k}(80 - 80e^{-kt})$ correctly A1 [7]

(ii) Use iterative formula correctly at least once M1

Obtain final answer 0.14 A1

Show sufficient iterations to 4 s.f. to justify answer to 2 s.f. or show a sign change in the interval (0.135, 0.145) A1 [3]

(iii) State a value between 530 and $540\,\text{cm}^3$ inclusive B1

State or imply that volume approaches $569\,\text{cm}^3$ (allowing any value between 567 and 571 inclusive) B1 [2]