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- 1** *EITHER*: Use law of the logarithm of a power or quotient and remove logarithms M1  
 Obtain a 3-term quadratic equation  $x^2 - x - 3 = 0$ , or equivalent A1  
 Solve 3-term quadratic obtaining 1 or 2 roots M1  
 Obtain answer 2.30 only A1
- OR1*: Use an appropriate iterative formula, e.g.  $x_{n+1} = \exp\left(\frac{1}{2} \ln(3x_n + 4)\right) - 1$  correctly at  
 least once M1  
 Obtain answer 2.30 A1  
 Show sufficient iterations to at least 3 d.p. to justify 2.30 to 2 d.p., or show there is a  
 sign change in the interval (2.295, 2.305) A1  
 Show there is no other root A1
- OR2*: Use calculated values to obtain at least one interval containing the root M1  
 Obtain answer 2.30 A1  
 Show sufficient calculations to justify 2.30 to 3 s.f., e.g. show it lies in (2.295, 2.305) A1  
 Show there is no other root A1 [4]
- 2** (i) Using the formulae  $\frac{1}{2}r^2\theta$  and  $\frac{1}{2}bh$ , form an equation in  $a$  and  $\theta$  M1  
 Obtain given answer A1 [2]
- (ii) Use the iterative formula correctly at least once M1  
 Obtain answer  $\theta = 1.32$  A1  
 Show sufficient iterations to 4 d.p. to justify 1.32 to 2 d.p., or show there is a sign change  
 in the interval (1.315, 1.325) A1 [3]

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- 3 *EITHER*: State a correct unsimplified term in  $x$  or  $x^2$  of  $(1-x)^{\frac{1}{2}}$  or  $(1+x)^{-\frac{1}{2}}$  B1  
 State correct unsimplified expansion of  $(1-x)^{\frac{1}{2}}$  up to the term in  $x^2$  B1  
 State correct unsimplified expansion of  $(1+x)^{-\frac{1}{2}}$  up to the term in  $x^2$  B1  
 Obtain sufficient terms of the product of the expansions of  $(1-x)^{\frac{1}{2}}$  and  $(1+x)^{-\frac{1}{2}}$  M1  
 Obtain final answer  $1-x+\frac{1}{2}x^2$  A1
- OR1*: State that the given expression equals  $(1-x)(1-x^2)^{-\frac{1}{2}}$  and state that the first term of the expansion of  $(1-x^2)^{-\frac{1}{2}}$  is 1 B1  
 State correct unsimplified term in  $x^2$  of  $(1-x^2)^{-\frac{1}{2}}$  B1  
 State correct unsimplified expansion of  $(1-x^2)^{-\frac{1}{2}}$  up to the term in  $x^2$  B1  
 Obtain sufficient terms of the product of  $(1-x)$  and the expansion M1  
 Obtain final answer  $1-x+\frac{1}{2}x^2$  A1
- OR2*: State correct unsimplified expansion of  $(1+x)^{\frac{1}{2}}$  up to the term in  $x^2$  B1  
 Multiply expansion by  $(1-x)$  and obtain  $1-2x+2x^2$  B1  
 Carry out correct method to obtain one non-constant term of the expansion of  $(1-2x+2x^2)^{\frac{1}{2}}$  M1  
 Obtain a correct unsimplified expansion with sufficient terms A1  
 Obtain final answer  $1-x+\frac{1}{2}x^2$  A1 [5]
- [Treat  $(1+x)^{-1}(1-x^2)^{\frac{1}{2}}$  by the *EITHER* scheme.]  
 [Symbolic coefficients, e.g.  $\left(\frac{1}{2}\right)$ , are not sufficient for the B marks.]
- 4 Use trig formulae to express equation in terms of  $\cos \theta$  and  $\sin \theta$  M1  
 Use Pythagoras to obtain an equation in  $\sin \theta$  M1  
 Obtain 3-term quadratic  $2\sin^2 \theta - 2\sin \theta - 1 = 0$ , or equivalent A1  
 Solve a 3-term quadratic and obtain a value of  $\theta$  M1  
 Obtain answer, e.g.  $201.5^\circ$  A1  
 Obtain second answer, e.g.  $338.5^\circ$ , and no others in the given interval A1 [6]  
 [Ignore answers outside the given interval. Treat answers in radians (3.52, 5.91) as a misread and deduct A1 from the marks for the angles.]
- 5 Separate variables correctly and attempt integration of both sides B1  
 Obtain term  $-e^{-y}$ , or equivalent B1  
 Obtain term  $\frac{1}{2}e^{2x}$ , or equivalent B1  
 Evaluate a constant, or use limits  $x=0, y=0$  in a solution containing terms  $ae^{-y}$  and  $be^{2x}$  M1  
 Obtain correct solution in any form, e.g.  $-e^{-y} = \frac{1}{2}e^{2x} - \frac{3}{2}$  A1  
 Rearrange and obtain  $y = \ln(2/(3 - e^{2x}))$ , or equivalent A1 [6]

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- 6 (i) State derivative in any correct form, e.g.  $3 \cos x - 12 \cos^2 x \sin x$  B1 + B1  
 Equate derivative to zero and solve for  $\sin 2x$ , or  $\sin x$  or  $\cos x$  M1  
 Obtain answer  $x = \frac{1}{12}\pi$  A1  
 Obtain answer  $x = \frac{5}{12}\pi$  A1  
 Obtain answer  $x = \frac{1}{2}\pi$  and no others in the given interval A1<sup>ft</sup> [6]
- (ii) Carry out a method for determining the nature of the relevant stationary point M1  
 Obtain a maximum at  $\frac{1}{12}\pi$  correctly A1 [2]  
 [Treat answers in degrees as a misread and deduct A1 from the marks for the angles.]
- 7 (i) *EITHER*: Multiply numerator and denominator by  $1 + 3i$ , or equivalent M1  
 Simplify numerator to  $-5 + 5i$ , or denominator to  $10$ , or equivalent A1  
 Obtain final answer  $-\frac{1}{2} + \frac{1}{2}i$ , or equivalent A1  
*OR*: Obtain two equations in  $x$  and  $y$ , and solve for  $x$  or for  $y$  M1  
 Obtain  $x = -\frac{1}{2}$  or  $y = \frac{1}{2}$ , or equivalent A1  
 Obtain final answer  $-\frac{1}{2} + \frac{1}{2}i$ , or equivalent A1 [3]
- (ii) Show  $B$  and  $C$  in relatively correct positions in an Argand diagram B1  
 Show  $u$  in a relatively correct position B1<sup>ft</sup> [2]
- (iii) Substitute exact arguments in the LHS  $\arg(1 + 2i) - \arg(1 - 3i) = \arg u$ , or equivalent M1  
 Obtain and use  $\arg u = \frac{3}{4}\pi$  A1  
 Obtain the given result correctly A1 [3]

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- 8 (i) State or imply  $2u \, du = -dx$ , or equivalent B1  
 Substitute for  $x$  and  $dx$  throughout M1  
 Obtain integrand  $\frac{-10u}{6-u^2+u}$ , or equivalent A1  
 Show correct working to justify the change in limits and obtain the given answer correctly A1 [4]
- (ii) State or imply the form of fractions  $\frac{A}{3-u} + \frac{B}{2+u}$  and use a relevant method to find  $A$   
 or  $B$  M1  
 Obtain  $A = 6$  and  $B = -4$  A1  
 Integrate and obtain  $-6 \ln(3-u) - 4 \ln(2+u)$ , or equivalent  $A1^{\frac{1}{2}} + A1^{\frac{1}{2}}$   
 Substitute limits correctly in an integral of the form  $a \ln(3-u) + b \ln(2+u)$  M1  
 Obtain the given answer correctly having shown sufficient working A1 [6]  
 [The f.t. is on  $A$  and  $B$ .]
- 9 (i) Use correct product rule M1  
 Obtain derivative in any correct form, e.g.  $\frac{\ln x}{2\sqrt{x}} + \frac{\sqrt{x}}{x}$  A1  
 Carry out a complete method to form an equation of the tangent at  $x = 1$  M1  
 Obtain answer  $y = x - 1$  A1 [4]
- (ii) State or imply that the indefinite integral for the volume is  $\pi \int x(\ln x)^2 \, dx$  B1  
 Integrate by parts and reach  $ax^2(\ln x)^2 + b \int x^2 \cdot \frac{\ln x}{x} \, dx$  M1\*  
 Obtain  $\frac{1}{2}x^2(\ln x)^2 - \int x \ln x \, dx$ , or unsimplified equivalent A1  
 Attempt second integration by parts reaching  $cx^2 \ln x + d \int x^2 \cdot \frac{1}{x} \, dx$  M1(dep\*)  
 Complete the integration correctly, obtaining  $\frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2 \ln x + \frac{1}{4}x^2$  A1  
 Substitute limits  $x = 1$  and  $x = e$ , having integrated twice M1(dep\*)  
 Obtain answer  $\frac{1}{4}\pi(e^2 - 1)$ , or exact equivalent A1 [7]  
 [If  $\pi$  omitted, or  $2\pi$  or  $\pi/2$  used, give B0 and then follow through.]  
 [Integration using parts  $x \ln x$  and  $\ln x$  is also viable.]

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- 10 (i) *EITHER*: Substitute coordinates of a general point of  $l$  in given equation of plane  $m$  M1  
 Obtain equation in  $\lambda$  in any correct form A1  
 Verify that the equation is not satisfied for any value of  $\lambda$  A1  
*OR1*: Substitute for  $\mathbf{r}$  in the vector equation of plane  $m$  and expand scalar product M1  
 Obtain equation in  $\lambda$  in any correct form A1  
 Verify that the equation is not satisfied for any value of  $\lambda$  A1  
*OR2*: Expand scalar product of a normal to  $m$  and a direction vector of  $l$  M1  
 Verify scalar product is zero A1  
 Verify that one point of  $l$  does not lie in the plane A1  
*OR3*: Use correct method to find perpendicular distance of a general point of  $l$  from  $m$  M1  
 Obtain a correct unsimplified expression in terms of  $\lambda$  A1  
 Show that the perpendicular distance is  $4/3$ , or equivalent, for all  $\lambda$  A1  
*OR4*: Use correct method to find the perpendicular distance of a particular point of  $l$  from  $m$  M1  
 Obtain answer  $4/3$ , or equivalent A1  
 Show that the perpendicular distance of a second point is also  $4/3$ , or equivalent A1 [3]
- (ii) *EITHER*: Express general point of  $l$  in component form, e.g.  $(1 + 2\lambda, 1 + \lambda, -1 + 2\lambda)$  B1  
 Substitute in given equation of  $n$  and solve for  $\lambda$  M1  
 Obtain position vector  $5\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$  from  $\lambda = 2$  A1  
*OR*: State or imply plane  $n$  has vector equation  $\mathbf{r} \cdot (2\mathbf{i} - 2\mathbf{j} + \mathbf{k}) = 7$ , or equivalent B1  
 Substitute for  $\mathbf{r}$ , expand scalar product and solve for  $\lambda$  M1  
 Obtain position vector  $5\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$  from  $\lambda = 2$  A1 [3]
- (iii) Form an equation in  $\lambda$  by equating perpendicular distances of a general point of  $l$  from  $m$  and  $n$  M1\*  
 Obtain a correct modular or non-modular equation in  $\lambda$  in any form A1<sup>h</sup>  
 Solve for  $\lambda$  and obtain a point, e.g.  $7\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$  from  $\lambda = 3$  A1  
 Obtain a second point, e.g.  $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$  from  $\lambda = 1$  A1  
 Use a correct method to find the distance between the two points M1(dep\*)  
 Obtain answer 6 A1 [6]  
 [The f.t. is on the components of  $l$ .]