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- 1 State or imply $4 - 2^x = -10$ and 10 B1
 Use correct method for solving equation of form $2^x = a$ M1
 Obtain 3.81 A1 [3]
- 2 (i) Either Obtain correct (unsimplified) version of x or x^2 term from $(1 - 4x)^{\frac{1}{2}}$ M1
 Obtain $1 + 2x$ A1
 Obtain $+ 6x^2$ A1
Or Differentiate and evaluate $f(0)$ and $f'(0)$ where $f(x) = k(1 - 4x)^{-\frac{3}{2}}$ M1
 Obtain $1 + 2x$ A1
 Obtain $+ 6x^2$ A1 [3]
- (ii) Combine both x^2 terms from product of $1 + 2x$ and answer from part (i) M1
 Obtain 5 A1 [2]
- 3 (i) Substitute $x = 2$ and equate to zero, or divide by $x - 2$ and equate constant remainder to zero, or equivalent M1
 Obtain $a = 4$ A1 [2]
- (ii) (a) Find further (quadratic or linear) factor by division, inspection or factor theorem or equivalent M1
 Obtain $x^2 + 2x - 8$ or $x + 4$ A1
 State $(x - 2)^2(x + 4)$ or equivalent A1 [3]
- (b) State any two of the four (or six) roots B1 \sqrt{h}
 State all roots $(\pm\sqrt{2}, \pm 2i)$, provided two are purely imaginary B1 \sqrt{h} [2]
- 4 (i) Either Expand $(1 + 2i)^2$ to obtain $-3 + 4i$ or unsimplified equivalent B1
 Multiply numerator and denominator by $2 - i$ M1
 Obtain correct numerator $-2 + 11i$ or correct denominator 5 A1
 Obtain $-\frac{2}{5} + \frac{11}{5}i$ or equivalent A1
Or Expand $(1 + 2i)^2$ to obtain $-3 + 4i$ or unsimplified equivalent B1
 Obtain two equations in x and y and solve for x or y M1
 Obtain final answer $x = -\frac{2}{5}$ A1
 Obtain final answer $y = \frac{11}{5}$ A1 [4]
- (ii) Draw a circle M1
 Show centre at relatively correct position, following their u A1 \sqrt{h}
 Draw circle passing through the origin A1 [3]

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- 5 (i) Differentiate to obtain $4\cos\frac{1}{2}x - \frac{1}{2}\sec^2\frac{1}{2}x$ B1
- Equate to zero and find value of $\cos\frac{1}{2}x$ M1
- Obtain $\cos\frac{1}{2}x = \frac{1}{2}$ and confirm $\alpha = \frac{2}{3}\pi$ A1 [3]
- (ii) Integrate to obtain $-16\cos\frac{1}{2}x \dots$ B1
- $\dots + 2\ln\cos\frac{1}{2}x$ or equivalent B1
- Using limits 0 and $\frac{2}{3}\pi$ in $a\cos\frac{1}{2}x + b\ln\cos\frac{1}{2}x$ M1
- Obtain $8 + 2\ln\frac{1}{2}$ or exact equivalent A1 [4]
- 6 (i) Obtain $2y\frac{dy}{dx}$ as derivative of y^2 B1
- Obtain $-4y - 4x\frac{dy}{dx}$ as derivative of $-4xy$ B1
- Substitute $x = 2$ and $y = -3$ and find value of $\frac{dy}{dx}$
- (dependent on at least one B1 being earned and $\frac{d(45)}{dx} = 0$) M1
- Obtain $\frac{12}{7}$ or equivalent A1 [4]
- (ii) Substitute $\frac{dy}{dx} = 1$ in an expression involving $\frac{dy}{dx}$, x and y and obtain $ay = bx$ M1
- Obtain $y = x$ or equivalent A1
- Uses $y = x$ in original equation and demonstrate contradiction A1 [3]
- 7 Separate variables correctly and attempt integration on at least one side M1
- Obtain $\frac{1}{3}y^3$ or equivalent on left-hand side A1
- Use integration by parts on right-hand side (as far as $axe^{3x} + \int be^{3x} dx$) M1
- Obtain or imply $2xe^{3x} + \int 2e^{3x} dx$ or equivalent A1
- Obtain $2xe^{3x} - \frac{2}{3}e^{3x}$ A1
- Substitute $x = 0, y = 2$ in an expression containing terms Ay^3, Bxe^{3x}, Ce^{3x} , where $ABC \neq 0$, and find the value of c M1
- Obtain $\frac{1}{3}y^3 = 2xe^{3x} - \frac{2}{3}e^{3x} + \frac{10}{3}$ or equivalent A1
- Substitute $x = 0.5$ to obtain $y = 2.44$ A1 [8]

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- 8 (i) Either Obtain $\pm \begin{pmatrix} 2 \\ -1 \\ -15 \end{pmatrix}$ for vector PA (where A is point on line) or equivalent B1
- Use scalar product to find cosine of angle between PA and line M1
- Obtain $\frac{42}{\sqrt{14 \times 230}}$ or equivalent A1
- Use trigonometry to obtain $\sqrt{104}$ or 10.2 or equivalent A1
- Or 1 Obtain $\pm \begin{pmatrix} 2n+2 \\ n-1 \\ 3n-15 \end{pmatrix}$ for PN (where N is foot of perpendicular) B1
- Equate scalar product of PN and line direction to zero
- Or equate derivative of PN^2 to zero
- Or use Pythagoras' theorem in triangle PNA to form equation in n M1
- Solve equation and obtain $n = 3$ A1
- Obtain $\sqrt{104}$ or 10.2 or equivalent A1
- Or 2 Obtain $\pm \begin{pmatrix} 2 \\ -1 \\ -15 \end{pmatrix}$ for vector PA (where A is point on line) B1
- Evaluate vector product of PA and line direction M1
- Obtain $\pm \begin{pmatrix} 12 \\ -36 \\ -4 \end{pmatrix}$ A1
- Divide modulus of this by modulus of line direction and obtain $\sqrt{104}$ or 10.2 or equivalent A1
- Or 3 Obtain $\pm \begin{pmatrix} 2 \\ -1 \\ -15 \end{pmatrix}$ for vector PA (where A is point on line) B1
- Evaluate scalar product of PA and line direction to obtain distance AN M1
- Obtain $3\sqrt{14}$ or equivalent A1
- Use Pythagoras' theorem in triangle PNA and obtain $\sqrt{104}$ or 10.2 or equivalent A1
- Or 4 Obtain $\pm \begin{pmatrix} 2 \\ -1 \\ -15 \end{pmatrix}$ for vector PA (where A is point on line) B1
- Use a second point B on line and use cosine rule in triangle ABP to find angle A or angle B or use vector product to find area of triangle M1
- Obtain correct answer (angle $A = 42.25\dots$) A1
- Use trigonometry to obtain $\sqrt{104}$ or 10.2 or equivalent A1 [4]

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- (ii) Either Use scalar product to obtain a relevant equation in a, b, c , e.g. $2a + b + 3c = 0$ or $2a - b - 15c = 0$ M1
 State two correct equations in a, b and c A1✓^h
 Solve simultaneous equations to obtain one ratio M1
 Obtain $a : b : c = -3 : 9 : -1$ or equivalent A1
 Obtain equation $-3x + 9y - z = 28$ or equivalent A1
- Or 1 Calculate vector product of two of $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 2 \\ -1 \\ -15 \end{pmatrix}$ and $\begin{pmatrix} 8 \\ 2 \\ -6 \end{pmatrix}$ or equiv M1
 Obtain two correct components of the product A1✓^h
 Obtain correct $\begin{pmatrix} -3 \\ 9 \\ -1 \end{pmatrix}$ or equivalent A1
 Substitute in $-3x + 9y - z = d$ to find d or equivalent M1
 Obtain equation $-3x + 9y - z = 28$ or equivalent A1
- Or 2 Form a two-parameter equation of the plane M1
 Obtain $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \\ -15 \end{pmatrix}$ or equivalent A1✓^h
 State three equations in x, y, z, s, t A1
 Eliminate s and t M1
 Obtain equation $3x - 9y + z = -28$ or equivalent A1 [5]
- 9 State or imply form $A + \frac{B}{2x+1} + \frac{C}{x+2}$ B1
 State or obtain $A = 2$ B1
 Use correct method for finding B or C M1
 Obtain $B = 1$ A1
 Obtain $C = -3$ A1
 Obtain $2x + \frac{1}{2} \ln(2x+1) - 3 \ln(x+2)$ [Deduct B1✓^h for each error or omission] B3✓^h
 Substitute limits in expression containing $a \ln(2x+1) + b \ln(x+2)$ M1
 Show full and exact working to confirm that $8 + \frac{1}{2} \ln 9 - 3 \ln 6 + 3 \ln 2$, or an equivalent expression, simplifies to given result $8 - \ln 9$ A1 [10]
- [SR: If A omitted from the form of fractions, give B0B0M1A0A0 in (i); B0✓^hB1✓^hB1✓^hM1A0 in (ii).]
- [SR: For a solution starting with $\frac{M}{2x+1} + \frac{Nx}{x+2}$ or $\frac{Px}{2x+1} + \frac{Q}{x+2}$, give B0B0M1A0A0 in (i); B1✓^hB1✓^hB1✓^h, if recover correct form, M1A0 in (ii).]
- [SR: For a solution starting with $\frac{B}{2x+1} + \frac{Dx+E}{x+2}$, give M1A1 for one of $B = 1, D = 2, E = 1$ and A1 for the other two constants; then give B1B1 for $A = 2, C = -3$.]
- [SR: For a solution starting with $\frac{Fx+G}{2x+1} + \frac{C}{x+2}$, give M1A1 for one of $C = -3, F = 4, G = 3$ and A1 for the other constants or constant; then give B1B1 for $A = 2, B = 1$.]

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- 10 (i)** Use correct identity for $\tan 2x$ and obtains $at^4 + bt^3 + ct^2 + dt = 0$, where b may be zero M1
 Obtain correct horizontal equation, e.g. $4t + 5t^2 - 5t^4 = 0$ A1
 Obtain $kt(t^3 + et + f) = 0$ or equivalent M1
 Confirm given results $t = 0$ and $t = \sqrt[3]{t + 0.8}$ A1 [4]
- (ii)** Consider sign of $t - \sqrt[3]{t + 0.8}$ at 1.2 and 1.3 or equivalent M1
 Justify the given statement with correct calculations (-0.06 and 0.02) A1 [2]
- (iii)** Use the iterative formula correctly at least once with $1.2 < t_n < 1.3$ M1
 Obtain final answer 1.276 A1
 Show sufficient iterations to justify answer or show there is a change of sign in interval
 (1.2755, 1.2765) A1 [3]
- (iv)** Evaluate \tan^{-1} (answer from part **(iii)**) to obtain at least one value M1
 Obtain -2.24 and 0.906 A1
 State $-\pi$, 0 and π B1 [3]
 [SR If A0, B0, allow B1 for any 3 roots]