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- 1 Use law for the logarithm of a product, power or quotient M1*
 Obtain a correct linear equation, e.g. $(2x - 1)\ln 5 = \ln 2 + x \ln 3$ A1
 Solve a linear equation for x M1(dep*)
 Obtain answer $x = 1.09$ A1 [4]
- [SR: Reduce equation to the form $a^x = b$ M1*, obtain $\left(\frac{25}{3}\right)^x = 10$ A1, use correct method to calculate value of x M1(dep*), obtain answer 1.09 A1.]
- 2 Use correct quotient or product rule M1
 Obtain correct derivative in any form, e.g. $-\frac{3 \ln x}{x^4} + \frac{1}{x^4}$ A1
 Equate derivative to zero and solve for x an equation of the form $\ln x = a$, where $a > 0$ M1
 Obtain answer $\exp\left(\frac{1}{3}\right)$, or 1.40, from correct work A1 [4]
- 3 Attempt integration by parts and reach $k(1-x)e^{\frac{1}{2}x} \pm k \int e^{\frac{1}{2}x} dx$, or equivalent M1
 Obtain $-2(1-x)e^{\frac{1}{2}x} - 2 \int e^{\frac{1}{2}x} dx$, or equivalent A1
 Integrate and obtain $-2(1-x)e^{\frac{1}{2}x} + 4e^{\frac{1}{2}x}$, or equivalent A1
 Use limits $x = 0$ and $x = 1$, having integrated twice M1
 Obtain the given answer correctly A1 [5]
- 4 (i) Use $\tan(A \pm B)$ formula correctly at least once and obtain an equation in $\tan \theta$ M1
 Obtain a correct horizontal equation in any form A1
 Use $\tan 60^\circ = \sqrt{3}$ throughout M1
 Obtain the given equation correctly A1 [4]
- (ii) Set $k = 3\sqrt{3}$ and obtain $\tan^2 \theta = \frac{1}{11}$ B1
 Obtain answer 16.8° B1√
 Obtain answer 163.2° B1√ [3]
 [Ignore answers outside the given interval. Treat answers in radians (0.293 and 2.85) as a misread.]

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- 5 (i) Substitute $x = \frac{1}{2}$ and equate to zero, or divide, and obtain a correct equation, e.g.
- $$\frac{1}{8}a + \frac{1}{4}b + \frac{5}{2} - 2 = 0$$
- B1
- Substitute $x = 2$ and equate result to 12, or divide and equate constant remainder to 12 M1
- Obtain a correct equation, e.g. $8a + 4b + 10 - 2 = 12$ A1
- Solve for a or for b M1
- Obtain $a = 2$ and $b = -3$ A1 [5]
- (ii) Attempt division by $2x - 1$ reaching a partial quotient $\frac{1}{2}ax^2 + kx$ M1
- Obtain quadratic factor $x^2 - x + 2$ A1 [2]
- [The M1 is earned if inspection has an unknown factor $Ax^2 + Bx + 2$ and an equation in A and/or B , or an unknown factor of $\frac{1}{2}ax^2 + Bx + C$ and an equation in B and/or C .]
- 6 (i) Make recognisable sketch of a relevant graph over the given range B1
- Sketch the other relevant graph and justify the given statement B1 [2]
- (ii) Consider the sign of $\cot x - (1 + x^2)$ at $x = 0.5$ and $x = 0.8$, or equivalent M1
- Complete the argument with correct calculated values A1 [2]
- (iii) Use the iterative formula correctly at least once with $0.5 \leq x_n \leq 0.8$ M1
- Obtain final answer 0.62 A1
- Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (0.615, 0.625) A1 [3]
- 7 (i) Use the quadratic formula, completing the square, or the substitution $z = x + iy$ to find a root and use $i^2 = -1$ M1
- Obtain final answers $-\sqrt{3} \pm i$, or equivalent A1 [2]
- (ii) State that the modulus of both roots is 2 B1√
- State that the argument of $-\sqrt{3} + i$ is 150° or $\frac{5}{6}\pi$ (2.62) radians B1√
- State that the argument of $-\sqrt{3} - i$ is -150° (or 210°) or $-\frac{5}{6}\pi$ (-2.62) radians or $\frac{7}{6}\pi$ (3.67) radians B1√ [3]
- (iii) Carry out an attempt to find the sixth power of a root M1
- Verify that one of the roots satisfies $z^6 = -64$ A1
- Verify that the other root satisfies the equation A1 [3]

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- 8 (i) Use product and chain rule M1
 Obtain correct derivative in any form, e.g. $15 \sin^2 x \cos^3 x - 10 \sin^4 x \cos x$ A1
 Equate derivative to zero and obtain a relevant equation in one trigonometric function M1
 Obtain $2 \tan^2 x = 3$, $5 \cos^2 x = 2$, or $5 \sin^2 x = 3$ A1
 Obtain answer $x = 0.886$ radians A1 [5]
- (ii) State or imply $du = -\sin x \, dx$, or $\frac{du}{dx} = -\sin x$, or equivalent B1
 Express integral in terms of u and du M1
 Obtain $\pm \int 5(u^2 - u^4) du$, or equivalent A1
 Integrate and use limits $u = 1$ and $u = 0$ (or $x = 0$ and $x = \frac{1}{2}\pi$) M1
 Obtain answer $\frac{2}{3}$, or equivalent, with no errors seen A1 [5]
- 9 (i) State or imply $\frac{dx}{dt} = k(10 - x)(20 - x)$ and show $k = 0.01$ B1 [1]
- (ii) Separate variables correctly and attempt integration of at least one side M1
 Carry out an attempt to find A and B such that $\frac{1}{(10 - x)(20 - x)} \equiv \frac{A}{10 - x} + \frac{B}{20 - x}$, or equivalent M1
 Obtain $A = \frac{1}{10}$ and $B = -\frac{1}{10}$, or equivalent A1
 Integrate and obtain $-\frac{1}{10} \ln(10 - x) + \frac{1}{10} \ln(20 - x)$, or equivalent A1√
 Integrate and obtain term $0.01t$, or equivalent A1
 Evaluate a constant, or use limits $t = 0$, $x = 0$, in a solution containing terms of the form $a \ln(10 - x)$, $b \ln(20 - x)$ and ct M1
 Obtain answer in any form, e.g. $-\frac{1}{10} \ln(10 - x) + \frac{1}{10} \ln(20 - x) = 0.01t + \frac{1}{10} \ln 2$ A1√
 Use laws of logarithms to correctly remove logarithms M1
 Rearrange and obtain $x = 20(\exp(0.1t) - 1) / (2 \exp(0.1t) - 1)$, or equivalent A1 [9]
- (iii) State that x approaches 10 B1 [1]

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- 10 (i) *EITHER*: Express general point of l or m in component form, e.g. $(2 + \lambda, -\lambda, 1 + 2\lambda)$ or $(\mu, 2 + 2\mu, 6 - 2\mu)$ B1
 Equate at least two pairs of components and solve for λ or for μ M1
 Obtain correct answer for λ or μ (possible answers for λ are $-2, \frac{1}{4}, 7$ and for μ are $0, 2\frac{1}{4}, -4\frac{1}{2}$) A1
- OR*: Verify that all three component equations are not satisfied A1
 State a relevant scalar triple product, e.g. $(2\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}) \cdot ((\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \times (\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}))$ B1
 Attempt to use the correct method of evaluation M1
 Obtain at least two correct simplified terms of the three terms of the expansion of the triple product or of the corresponding determinant, e.g. $-4, -8, -15$ A1
 Obtain correct non-zero value, e.g. -27 , and state that the lines do not intersect A1 [4]
- (ii) Carry out the correct process for evaluating scalar product of direction vectors for l and m M1
 Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result M1
 Obtain answer 47.1° or 0.822 radians A1 [3]
- (iii) *EITHER*: Use scalar product to obtain $a - b + 2c = 0$ B1
 Obtain $a + 2b - 2c = 0$, or equivalent, from a scalar product, or by subtracting two point equations obtained from points on m , and solve for one ratio, e.g. $a : b$ M1*
 Obtain $a : b : c = -2 : 4 : 3$, or equivalent A1
 Substitute coordinates of a point on m and values for a, b and c in general equation and evaluate d M1(dep*)
 Obtain answer $-2x + 4y + 3z = 26$, or equivalent A1
- OR1*: Attempt to calculate vector product of direction vectors of l and m M1*
 Obtain two correct components A1
 Obtain $-2\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, or equivalent A1
 Form a plane equation and use coordinates of a relevant point to evaluate d M1(dep*)
 Obtain answer $-2x + 4y + 3z = 26$, or equivalent A1
- OR2*: Form a two-parameter plane equation using relevant vectors M1*
 State a correct equation e.g. $\mathbf{r} = 2\mathbf{j} + 6\mathbf{k} + s(\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + t(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$ A1
 State three correct equations in x, y, z, s and t A1
 Eliminate s and t M1(dep*)
 Obtain answer $-2x + 4y + 3z = 26$, or equivalent A1 [5]