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- 1 Either: Obtain $1 + \frac{1}{3}kx$, where $k = \pm 6$ or ± 1 M1
 Obtain $1 - 2x$ A1
 Obtain $-4x^2$ A1
 Obtain $-\frac{40}{3}x^3$ or equivalent A1
- Or: Differentiate expression to obtain form $k(1 - 6x)^{-\frac{2}{3}}$ and evaluate $f(0)$ and $f'(0)$ M1
 Obtain $f'(x) = -2(1 - 6x)^{-\frac{2}{3}}$ and hence the correct first two terms $1 - 2x$ A1
 Obtain $f''(x) = -8(1 - 6x)^{-\frac{5}{3}}$ and hence $-4x^2$ A1
 Obtain $f'''(x) = -80(1 - 6x)^{-\frac{8}{3}}$ and hence $-\frac{40}{3}x^3$ or equivalent A1 [4]
- 2 (i) Obtain $\frac{k \cos 2x}{1 + \sin 2x}$ for any non-zero constant k M1
 Obtain $\frac{2 \cos 2x}{1 + \sin 2x}$ A1 [2]
- (ii) Use correct quotient or product rule M1
 Obtain $\frac{x \sec^2 x - \tan x}{x^2}$ or equivalent A1 [2]
- 3 (i) Obtain $\pm \begin{pmatrix} 3 \\ -4 \\ 6 \end{pmatrix}$ as normal to plane B1
 Form equation of p as $3x - 4y + 6z = k$ or $-3x + 4y - 6z = k$ and use relevant point to find k M1
 Obtain $3x - 4y + 6z = 80$ or $-3x + 4y - 6z = -80$ A1 [3]
- (ii) State the direction vector $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ or equivalent B1
 Carry out correct process for finding scalar product of two relevant vectors M1
 Use correct complete process with moduli and scalar product and evaluate \sin^{-1} or \cos^{-1} of result M1
 Obtain 30.8° or 0.538 radians A1 [4]

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- 4 (i) Verify that $-96 + 100 + 8 - 12 = 0$ B1
 Attempt to find quadratic factor by division by $(x + 2)$, reaching a partial quotient
 $12x^2 + kx$, inspection or use of an identity M1
 Obtain $12x^2 + x - 6$ A1
 State $(x + 2)(4x + 3)(3x - 2)$ A1 [4]
 [The M1 can be earned if inspection has unknown factor $Ax^2 + Bx - 6$ and an equation in A and/or B or equation $12x^2 + Bx + C$ and an equation in B and/or C .]
- (ii) State $3^y = \frac{2}{3}$ and no other value B1
 Use correct method for finding y from equation of form $3^y = k$, where $k > 0$ M1
 Obtain -0.369 and no other value A1 [3]
- 5 (i) Use at least one of $e^{2x} = 9$, $e^y = 2$ and $e^{2y} = 4$ B1
 Obtain given result $58 + 2k = c$ AG B1 [2]
- (ii) Differentiate left-hand side term by term, reaching $ae^{2x} + be^y \frac{dy}{dx} + ce^{2y} \frac{dy}{dx}$ M1
 Obtain $12e^{2x} + ke^y \frac{dy}{dx} + 2e^{2y} \frac{dy}{dx}$ A1
 Substitute $(\ln 3, \ln 2)$ in an attempt involving implicit differentiation at least once, where
 RHS = 0 M1
 Obtain $108 - 12k - 48 = 0$ or equivalent A1
 Obtain $k = 5$ and $c = 68$ A1 [5]
- 6 (i) State or imply area of segment is $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$ or $50\theta - 50\sin\theta$ B1
 Attempt to form equation from area of segment = $\frac{1}{5}$ of area of circle, or equivalent M1
 Confirm given result $\theta = \frac{2}{5}\pi + \sin\theta$ A1 [3]
- (ii) Use iterative formula correctly at least once M1
 Obtain value for θ of 2.11 A1
 Show sufficient iterations to justify value of θ or show sign change in interval
 (2.105, 2.115) A1
 Use correct trigonometry to find an expression for the length of AB M1
 e.g. $20 \sin 1.055$ or $\sqrt{200 - 200 \cos 2.11}$
 Hence 17.4 A1 [5]
 [2.1 \rightarrow 2.1198 \rightarrow 2.1097 \rightarrow 2.1149 \rightarrow 2.1122]

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- 7 (i) State or imply $dx = 2t dt$ or equivalent B1
 Express the integral in terms of x and dx M1
 Obtain given answer $\int_1^5 (2x - 2) \ln x dx$, including change of limits AG A1 [3]
- (ii) Attempt integration by parts obtaining $(ax^2 + bx) \ln x \pm \int (ax^2 + bx) \frac{1}{x} dx$ or equivalent M1
 Obtain $(x^2 - 2x) \ln x - \int (x^2 - 2x) \frac{1}{x} dx$ or equivalent A1
 Obtain $(x^2 - 2x) \ln x - \frac{1}{2}x^2 + 2x$ A1
 Use limits correctly having integrated twice M1
 Obtain $15 \ln 5 - 4$ or exact equivalent A1 [5]
 [Equivalent for M1 is $(2x - 2)(ax \ln x + bx) - \int (ax \ln x + bx) 2dx$]
- 8 (i) Either: Multiply numerator and denominator by $(1 - 2i)$, or equivalent M1
 Obtain $-3i$ A1
 State modulus is 3 A1
 Refer to u being on negative imaginary axis or equivalent and confirm argument as $-\frac{1}{2}\pi$ A1
- Or: Using correct processes, divide moduli of numerator and denominator M1
 Obtain 3 A1
 Subtract argument of denominator from argument of numerator M1
 Obtain $-\tan^{-1} \frac{1}{2} - \tan^{-1} 2$ or $-0.464 - 1.107$ and hence $-\frac{1}{2}\pi$ or -1.57 A1 [4]
- (ii) Show correct half-line from u at angle $\frac{1}{4}\pi$ to real direction B1
 Use correct trigonometry to find required value M1
 Obtain $\frac{3}{2}\sqrt{2}$ or equivalent A1 [3]
- (iii) Show, or imply, locus is a circle with centre $(1 + i)u$ and radius 1 M1
 Use correct method to find distance from origin to furthest point of circle M1
 Obtain $3\sqrt{2} + 1$ or equivalent A1 [3]

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- 9 (i) Express $\cos 4\theta$ as $2 \cos^2 2\theta - 1$ or $\cos^2 2\theta - \sin^2 2\theta$ or $1 - 2 \sin^2 2\theta$ B1
 Express $\cos 4\theta$ in terms of $\cos \theta$ M1
 Obtain $8 \cos^4 \theta - 8 \cos^2 \theta + 1$ A1
 Use $\cos 2\theta = 2 \cos^2 \theta - 1$ to obtain given answer $8 \cos^4 \theta - 3$ AG A1 [4]
- (ii) (a) State or imply $\cos^4 \theta = \frac{1}{2}$ B1
 Obtain 0.572 B1
 Obtain -0.572 B1 [3]
- (b) Integrate and obtain form $k_1 \theta + k_2 \sin 4\theta + k_3 \sin 2\theta$ M1
 Obtain $\frac{3}{8} \theta + \frac{1}{32} \sin 4\theta + \frac{1}{4} \sin 2\theta$ A1
 Obtain $\frac{3}{32} \pi + \frac{1}{4}$ following completely correct work A1 [3]
- 10 (i) Separate variables correctly and integrate of at least one side M1
 Carry out an attempt to find A and B such that $\frac{1}{N(1800 - N)} \equiv \frac{A}{N} + \frac{B}{1800 - N}$, or equivalent M1
 Obtain $\frac{2}{N} + \frac{2}{1800 - N}$ or equivalent A1
 Integrates to produce two terms involving natural logarithms M1
 Obtain $2 \ln N - 2 \ln (1800 - N) = t$ or equivalent A1
 Evaluate a constant, or use $N = 300$ and $t = 0$ in a solution involving $a \ln N$, $b \ln(1800)$ and ct M1
 Obtain $2 \ln N - 2 \ln (1800 - N) = t - 2 \ln 5$ or equivalent A1
 Use laws of logarithms to remove logarithms M1
 Obtain $N = \frac{1800e^{\frac{1}{2}t}}{5 + e^{\frac{1}{2}t}}$ or equivalent A1 [9]
- (ii) State or imply that N approaches 1800 B1 [1]