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- 1 EITHER: State or imply non-modular inequality $(x - 3)^2 > (2(x + 1))^2$, or corresponding quadratic equation, or pair of linear equations $(x - 3) = \pm 2(x + 1)$ B1
 Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations M1
 Obtain critical values -5 and $\frac{1}{3}$ A1
 State answer $-5 < x < \frac{1}{3}$ A1
- OR: Obtain the critical value $x = -5$ from a graphical method, or by inspection, or by solving a linear equation or inequality B1
 Obtain the critical value $x = \frac{1}{3}$ similarly B2
 State answer $-5 < x < \frac{1}{3}$ B1 [4]
 [Do not condone \leq for $<$; accept 0.33 for $\frac{1}{3}$.]
- 2 (i) State or imply $3 \ln y = \ln A + 2x$ at any stage B1
 State gradient is $\frac{2}{3}$, or equivalent B1 [2]
- (ii) Substitute $x = 0$, $\ln y = 0.5$ and solve for A M1
 Obtain $A = 4.48$ A1 [2]
- 3 Attempt to use $\tan(A \pm B)$ formula and obtain an equation in $\tan x$ M1
 Obtain 3-term quadratic $2 \tan^2 x + 3 \tan x - 1 = 0$, or equivalent A1
 Solve a 3-term quadratic and find a numerical value of x M1
 Obtain answer 15.7° A1
 Obtain answer 119.3° and no others in the given interval A1 [5]
 [Ignore answers outside the given interval. Treat answers in radians, 0.274 and 2.08, as a misread.]
- 4 Separate variables correctly B1
 Obtain term $k \ln(4 - x^2)$, or terms $k_1 \ln(2 - x) + k_2 \ln(2 + x)$ B1
 Obtain term $-2 \ln(4 - x^2)$, or $-2 \ln(2 - x) - 2 \ln(2 + x)$, or equivalent B1
 Obtain term t , or equivalent B1
 Evaluate a constant or use limits $x = 1$, $t = 0$ in a solution containing terms $a \ln(4 - x^2)$ and bt
 or terms $c \ln(2 - x)$, $d \ln(2 + x)$ and bt M1
 Obtain correct solution in any form, e.g. $-2 \ln(4 - x^2) = t - 2 \ln 3$ A1
 Rearrange and obtain $x^2 = 4 - 3 \exp(-\frac{1}{2}t)$, or equivalent (allow use of $2 \ln 3 = 2.20$) A1 [7]
- 5 (i) State derivative $-e^{-x} - (-2)e^{-2x}$, or equivalent B1 + B1
 Equate derivative to zero and solve for x M1
 Obtain $p = \ln 2$, or exact equivalent A1 [4]
- (ii) State indefinite integral $-e^{-x} - (-\frac{1}{2})e^{-2x}$, or equivalent B1 + B1
 Substitute limits $x = 0$ and $x = p$ correctly M1
 Obtain given answer following full and correct working A1 [4]

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- 6 (i) Use correct quotient or product rule M1
 Obtain correct derivative in any form, e.g. $\frac{1}{x(x+1)} - \frac{\ln x}{(x+1)^2}$ A1
 Equate derivative to zero and obtain the given equation correctly A1
 Consider the sign of $x - \frac{(x+1)}{\ln x}$ at $x = 3$ and $x = 4$, or equivalent M1
 Complete the argument with correct calculated values A1 [5]
- (ii) Use the iterative formula correctly at least once, using or reaching a value in the interval (3, 4) M1
 Obtain final answer 3.59 A1
 Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (3.585, 3.595) A1 [3]
- 7 (i) Use correct $\cos(A + B)$ formula to express $\cos 3\theta$ in terms of trig functions of 2θ and θ M1
 Use correct trig formulae and Pythagoras to express $\cos 3\theta$ in terms of $\cos \theta$ M1
 Obtain a correct expression in terms of $\cos \theta$ in any form A1
 Obtain the given identity correctly A1 [4]
 [SR: Give M1 for using correct formulae to express RHS in terms of $\cos \theta$ and $\cos 2\theta$, then M1A1 for expressing in terms of either only $\cos 3\theta$ and $\cos \theta$, or only $\cos 2\theta$, $\sin 2\theta$, $\cos \theta$, and $\sin \theta$, and A1 for obtaining the given identity correctly.]
- (ii) Use identity and integrate, obtaining terms $\frac{1}{4}(\frac{1}{3} \sin 3\theta)$ and $\frac{1}{4}(3 \sin \theta)$, or equivalent B1 + B1
 Use limits correctly in an integral of the form $k \sin 3\theta + l \sin \theta$ M1
 Obtain answer $\frac{2}{3} - \frac{3}{8} \sqrt{3}$, or any exact equivalent A1 [4]
- 8 (a) EITHER: Substitute $1 + i\sqrt{3}$, attempt complete expansions of the x^3 and x^2 terms M1
 Use $i^2 = -1$ correctly at least once B1
 Complete the verification correctly A1
 State that the other root is $1 - i\sqrt{3}$ B1
 OR1: State that the other root is $1 - i\sqrt{3}$ B1
 State quadratic factor $x^2 - 2x + 4$ B1
 Divide cubic by 3-term quadratic reaching partial quotient $2x + k$ M1
 Complete the division obtaining zero remainder A1
 OR2: State factorisation $(2x + 3)(x^2 - 2x + 4)$, or equivalent B1
 Make reasonable solution attempt at a 3-term quadratic and use $i^2 = -1$ M1
 Obtain the root $1 + i\sqrt{3}$ A1
 State that the other root is $1 - i\sqrt{3}$ B1 [4]
- (b) Show point representing $1 + i\sqrt{3}$ in relatively correct position on an Argand diagram B1
 Show circle with centre at $1 + i\sqrt{3}$ and radius 1 B1√
 Show line for $\arg z = \frac{1}{3}\pi$ making $\frac{1}{3}\pi$ with the real axis B1
 Show line from origin passing through centre of circle, or the diameter which would contain the origin if produced B1
 Shade the relevant region B1√ [5]

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- 9 (i) State or imply partial fractions of the form $\frac{A}{1-2x} + \frac{B}{2+x} + \frac{C}{(2+x)^2}$ B1
- Use any relevant method to determine a constant M1
- Obtain one of the values $A = 1, B = 1, C = -2$ A1
- Obtain a second value A1
- Obtain the third value A1 [5]
- [The form $\frac{A}{1-2x} + \frac{Dx+E}{(2+x)^2}$, where $A = 1, D = 1, E = 0$, is acceptable
- scoring B1M1A1A1A1 as above.]
- (ii) Use correct method to obtain the first two terms of the expansion of $(1-2x)^{-1}, (2+x)^{-1}, (2+x)^{-2}, (1+\frac{1}{2}x)^{-1}$, or $(1+\frac{1}{2}x)^{-2}$ M1
- Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction $A1\sqrt{+} + A1\sqrt{+} + A1\sqrt{+}$
- Obtain answer $1 + \frac{9}{4}x + \frac{15}{4}x^2$, or equivalent A1 [5]
- [Symbolic binomial coefficients, e.g. $\binom{-1}{1}$, are not sufficient for the M1. The f.t. is on A, B, C .]
- [For the A, D, E form of partial fractions, give M1A1 $\sqrt{+}$ A1 $\sqrt{+}$ for the expansions then, if $D \neq 0$, M1 for multiplying out fully and A1 for the final answer.]
- [In the case of an attempt to expand $(4+5x-x^2)(1-2x)^{-1}(2+x)^{-2}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]
- [SR: If B or C omitted from the form of fractions, give B0M1A0A0A0 in (i); M1A1 $\sqrt{+}$ A1 $\sqrt{+}$ in (ii).]
- [SR: If D or E omitted from the form of fractions, give B0M1A0A0A0 in (i); M1A1 $\sqrt{+}$ A1 $\sqrt{+}$ in (ii).]
- 10 (i) Express general point of the line in component form, e.g. $(2 + \lambda, -1 + 2\lambda, -4 + 2\lambda)$ B1
- Substitute in plane equation and solve for λ M1
- Obtain position vector $4\mathbf{i} + 3\mathbf{j}$, or equivalent A1 [3]
- (ii) State or imply a correct vector normal to the plane, e.g. $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ B1
- Using the correct process, evaluate the scalar product of a direction vector for l and a normal for p M1
- Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine or inverse sine of the result M1
- Obtain answer 26.5° (or 0.462 radians) A1 [4]
- (iii) EITHER: State $a + 2b + 2c = 0$ or $3a - b + 2c = 0$ B1
- Obtain two relevant equations and solve for one ratio, e.g. $a : b$ M1
- Obtain $a : b : c = 6 : 4 : -7$, or equivalent A1
- Substitute coordinates of a relevant point in $6x + 4y - 7z = d$ and evaluate d M1
- Obtain answer $6x + 4y - 7z = 36$, or equivalent A1
- OR1: Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) \times (3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ M1
- Obtain two correct components of the product A1
- Obtain correct product, e.g. $6\mathbf{i} + 4\mathbf{j} - 7\mathbf{k}$ A1
- Substitute coordinates of a relevant point in $6x + 4y - 7z = d$ and evaluate d M1
- Obtain answer $6x + 4y - 7z = 36$, or equivalent A1
- OR2: Attempt to form 2-parameter equation with relevant vectors M1
- State a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} - \mathbf{j} - 4\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) + \mu(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ A1
- State three equations in x, y, z, λ, μ A1
- Eliminate λ and μ M1
- Obtain answer $6x + 4y - 7z = 36$, or equivalent A1 [5]