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	i age -	GCE AS/A LEVEL – May/June 2010 9709	Papei 32	
	EITHER	: Attempt to solve for 2^x	M1	
		Obtain $2^x = 6/4$, or equivalent	A1	
		Use correct method for solving an equation of the form $2^x = a$, where $a > 0$	M1	
		Obtain answer $x = 0.585$	A1	
	OR:	State an appropriate iterative formula, e.g. $x_{n+1} = \ln((2^{x_n} + 6) / 5) / \ln 2$	B1	
		Use the iterative formula correctly at least once	M1	
		Obtain answer $x = 0.585$	A1	E.
		Show that the equation has no other root but 0.585	A1	[4
		[For the solution 0.585 with no relevant working, award B1 and a further B1 if 0. be the only root.]	585 is show	wn 1
	Integrate	by parts and reach $\pm x^2 \cos x \pm \int 2x \cos x dx$	M1	
	Obtain -	$-x^2 \cos x + \int 2x \cos x dx$, or equivalent	A1	
	Complet	e the integration, obtaining $-x^2 \cos x + 2x \sin x + 2 \cos x$, or equivalent	A1	
		e limits correctly, having integrated twice	M1	
	Obtain t	ne given answer correctly	A1	[
	(i)	State or imply sin $a = 4/5$	B1	
	(1)	Use $sin(A - B)$ formula and substitute for $cos a$ and $sin a$	M1	
		Obtain answer $\frac{1}{10}(4\sqrt{3}-3)$, or exact equivalent		г
		Obtain answer $\frac{1}{10}(4\sqrt{3}-5)$, of exact equivalent	A1	[
	(ii)	Use $\tan 2A$ formula and substitute for $\tan a$, or use $\sin 2A$ and $\cos 2A$ formulae,	MI	
		substitute sin <i>a</i> and cos <i>a</i> , and divide 24	M1	
		Obtain $\tan 2a = -\frac{24}{7}$, or equivalent	A1	
		Use $tan(A + B)$ formula with $A = 2a$, $B = a$ and substitute for $tan 2a$ and $tan a$	M1	
		Obtain $\tan 3a = -\frac{44}{117}$	A1	[·
		117	711	L
	(i)	Use correct quotient or product rule	M1	
		Obtain correct derivative in any form	A1	
		Equate derivative to zero and solve for x	M1	-
		Obtain the given answer correctly	A1	[
	(ii)	Use the iterative formula correctly at least once	M1	
	(11)	Obtain final answer 4.49	Al	
		Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show		
		there is a sign change in the interval (4.485, 4.495)	A1	[.
	(i)	Substitute $x = -\frac{1}{2}$, equate to zero and obtain a correct equation, e.g.		
		$-\frac{1}{4} + \frac{5}{4} - \frac{1}{2}a + b = 0$	B1	
			M1	
		Substitute $x = -2$ and equate to 9 Obtain a correct equation, e.g. $-16+20-2a+b=9$	A1	
		Solve for <i>a</i> or for <i>b</i>	M1	
		Obtain $a = -4$ and $b = -3$	A1	[:

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<i>(</i> • •)	• · · · ·			271	
(ii)	_	division by $2x + 1$ reaching a partial quotient of $x^2 + kx$		M1	
	-	uadratic factor $x^2 + 2x - 3$		A1	[2]
	Obtain f	actorisation $(2x+1)(x+3)(x-1)$		A1	[3]
	<i>f</i> , or if tv	+ f and an equation of the f and an equation of the f and an equation of the f and $(x + 1)$ and $($			
(i)	EITHER	: State or imply $\frac{1}{y} \frac{dy}{dx}$ as derivative of ln y		B1	
		State correct derivative of LHS, e.g. $\ln y + \frac{x}{y} \frac{dy}{dx}$		B1	
		Differentiate RHS and obtain an expression for $\frac{dy}{dx}$		M1	
		Obtain given answer		A1	
	<i>OR</i> 1:	State $\ln y = \frac{2x+1}{x}$, or equivalent, and differentiate both	n sides	M1	
		State correct derivative of LHS, e.g. $\frac{1}{y} \frac{dy}{dx}$		B1	
		State correct derivative of RHS, e.g. $-1/x^2$		B1	
		Rearrange and obtain given answer		A1	
	<i>OR</i> 2:	State $y = \exp(2+1/x)$, or equivalent, and attempt diffe	rentiation by ch	ain	
		rule		M1	
		State correct derivative of RHS, e.g. $-\exp(2+1/x)/x^2$		B1 + B1	
		Obtain given answer [The B marks are for the exponential term and its multi	olier.]	A1	[4]
(ii)	State or	imply $x = -\frac{1}{2}$ when $y = 1$		B1	
(II)		the and obtain gradient of -4		B1√	
		y form equation of tangent		M1	
		inal answer $y + 4x + 1 = 0$, or equivalent		A1	[4]
(i)	Senarate	variables correctly and attempt integration of both sides		B1	
(1)	-	tan x		B1	
		$\operatorname{erm} -\frac{1}{2} e^{-2t}$		B1	
		a constant or use limits $x = 0$, $t = 0$ in a solution containin	g terms <i>a</i> tan <i>x</i> a		
		orrect solution in any form, e.g. $\tan x = \frac{1}{2} - \frac{1}{2}e^{-2t}$		A1	
		ge as $x = \tan^{-1}(\frac{1}{2} - \frac{1}{2}e^{-2t})$, or equivalent		Al	[6]
(ii)		t x approaches $\tan^{-1}(\frac{1}{2})$		B1	[1]
	State the	t $1 - e^{-2t}$ increases and so does the inverse tangent, or stat	a that $a^{-2t} \cos^2$	n ia	
(iii)	ADVIDE 100				

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<i>OR</i> : Substitute equivalent Obtain co Identify at	Mark Scheme: Teachers' versionSyllabusGCE AS/A LEVEL – May/June 20109709State a correct expression for $ z $ or $ z ^2$, e.g. $(1 + \cos 2\theta)^2 + (\sin 2\theta)^2$ Use double angle formulae throughout or PythagorasObtain given answer 2cos θ correctlyState a correct expression for tangent of argument, e.g. $(\sin 2\theta / (1 + \cos 2\theta))^2$ Use double angle formulae to express it in terms of $\cos \theta$ and $\sin \theta$ Obtain tan θ and state that the argument is θ Use double angle formulae to express z in terms of $\cos \theta$ and $\sin \theta$ Obtain a correct expression, e.g. $1 + \cos^2 \theta - \sin^2 \theta + 2i \sin \theta \cos \theta$ Convert the expression to polar formObtain $2 \cos \theta (\cos \theta + i \sin \theta)$ State that the argument is θ State that the argument is θ for z and multiply numerator and denominator by the conjugate of z, ortrrect real denominator in any form	Paper 32 B1 M1 A1 2θ) B1 M1 A1	[6
<i>OR</i> : Substitute equivalent Obtain co Identify at	Use double angle formulae throughout or Pythagoras Obtain given answer 2cos θ correctly State a correct expression for tangent of argument, e.g. $(\sin 2\theta / (1 + \cos \theta))$ Use double angle formulae to express it in terms of $\cos \theta$ and $\sin \theta$ Obtain tan θ and state that the argument is θ Use double angle formulae to express z in terms of $\cos \theta$ and $\sin \theta$ Obtain a correct expression, e.g. $1 + \cos^2 \theta - \sin^2 \theta + 2i \sin \theta \cos \theta$ Convert the expression to polar form Obtain $2\cos\theta(\cos\theta + i\sin\theta)$ State that the modulus is $2\cos\theta$ State that the argument is θ for z and multiply numerator and denominator by the conjugate of z, or	 M1 A1 2θ) B1 M1 A1 M1 A1 M1 A1 A1<!--</th--><th>[6</th>	[6
<i>OR</i> : Substitute equivalent Obtain co Identify at	Use double angle formulae throughout or Pythagoras Obtain given answer 2cos θ correctly State a correct expression for tangent of argument, e.g. $(\sin 2\theta / (1 + \cos \theta))$ Use double angle formulae to express it in terms of $\cos \theta$ and $\sin \theta$ Obtain tan θ and state that the argument is θ Use double angle formulae to express z in terms of $\cos \theta$ and $\sin \theta$ Obtain a correct expression, e.g. $1 + \cos^2 \theta - \sin^2 \theta + 2i \sin \theta \cos \theta$ Convert the expression to polar form Obtain $2\cos\theta(\cos\theta + i\sin\theta)$ State that the modulus is $2\cos\theta$ State that the argument is θ for z and multiply numerator and denominator by the conjugate of z, or	 M1 A1 2θ) B1 M1 A1 M1 A1 M1 A1 A1<!--</th--><th>[6</th>	[6
Substitute equivalent Obtain co Identify an	Obtain given answer 2cos θ correctly State a correct expression for tangent of argument, e.g. $(\sin 2\theta / (1 + \cos \theta))$ Use double angle formulae to express it in terms of $\cos \theta$ and $\sin \theta$ Obtain tan θ and state that the argument is θ Use double angle formulae to express <i>z</i> in terms of $\cos \theta$ and $\sin \theta$ Obtain a correct expression, e.g. $1 + \cos^2 \theta - \sin^2 \theta + 2i \sin \theta \cos \theta$ Convert the expression to polar form Obtain $2 \cos \theta (\cos \theta + i \sin \theta)$ State that the modulus is $2 \cos \theta$ State that the argument is θ for <i>z</i> and multiply numerator and denominator by the conjugate of <i>z</i> , or t	 A1 B1 M1 A1 M1 A1 M1 A1 A1	[6
Substitute equivalent Obtain co Identify an	State a correct expression for tangent of argument, e.g. $(\sin 2\theta / (1 + \cos \theta))$ Use double angle formulae to express it in terms of $\cos \theta$ and $\sin \theta$ Obtain tan θ and state that the argument is θ Use double angle formulae to express z in terms of $\cos \theta$ and $\sin \theta$ Obtain a correct expression, e.g. $1 + \cos^2 \theta - \sin^2 \theta + 2i \sin \theta \cos \theta$ Convert the expression to polar form Obtain $2 \cos \theta (\cos \theta + i \sin \theta)$ State that the modulus is $2 \cos \theta$ State that the argument is θ for z and multiply numerator and denominator by the conjugate of z, or t	2θ) B1 M1 A1 M1 A1 M1 A1 A1 A1	[6
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Substitute equivalent Obtain co Identify an	Obtain tan θ and state that the argument is θ Use double angle formulae to express z in terms of $\cos \theta$ and $\sin \theta$ Obtain a correct expression, e.g. $1 + \cos^2 \theta - \sin^2 \theta + 2i \sin \theta \cos \theta$ Convert the expression to polar form Obtain $2 \cos \theta (\cos \theta + i \sin \theta)$ State that the modulus is $2 \cos \theta$ State that the argument is θ for z and multiply numerator and denominator by the conjugate of z, or t	A1 M1 A1 A1 A1 A1	[6
Substitute equivalent Obtain co Identify an	Use double angle formulae to express z in terms of $\cos \theta$ and $\sin \theta$ Obtain a correct expression, e.g. $1 + \cos^2 \theta - \sin^2 \theta + 2i \sin \theta \cos \theta$ Convert the expression to polar form Obtain $2 \cos \theta (\cos \theta + i \sin \theta)$ State that the modulus is $2 \cos \theta$ State that the argument is θ for z and multiply numerator and denominator by the conjugate of z, or t	M1 A1 A1 A1 A1	[6
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equivalent Obtain co Identify an	for z and multiply numerator and denominator by the conjugate of z, or t		[6
equivalent Obtain co Identify an	t	M1	
equivalent Obtain co Identify an	t	M1	
Obtain co Identify a			
Identify an		Al	
-	nd obtain real part equal to $\frac{1}{2}$	A1	[3
			[*
<i>a</i>			
	nply a correct normal vector to either plane, e.g. $3\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ or $a\mathbf{i} + \mathbf{j} + \mathbf{l}$	K B1	
	alar product of normals to zero and obtain an equation in a , e.g.	M1	
			L.
Obtain <i>u</i> -	2	AI	[3
Express g	eneral point of the line in component form, e.g. $(\lambda, 1+2\lambda, -1+2\lambda)$	B1	
		M1*	
Obtain λ =	= 1 for point A	A1	
Obtain λ =	= 2 for point B	A1	
		M1(dep*)	
Obtain an	swer $AB = 3$	A1	[6
[The second M mark is dependent on both values of λ being found by correct met			
ETTHER:	•		
	· · · · · · · · · · · · · · · · · · ·		
OR^{\cdot}			
0111			
	Use any relevant method to obtain <i>B</i> , <i>C</i> or <i>D</i>	M1	
	Obtain one correct answer	A1	
	Obtain $B = 2$, $C = 1$ and $D = -3$	A1	[5
	[SR: If $A = 1$ stated without working give B1.]		
T , ,		Dal	
	<i>x z</i>	B3√	
	•		
	· · ·	M1	
Obtain giv	ven answer correctly following full and exact working	A1	[5
	Obtain $a =$ Express g Either sub componer Obtain $\lambda =$ Obtain $\lambda =$ Carry out Obtain an [The seco <i>EITHER</i> : <i>OR</i> : Integrate a (The f.t. is Substitute	Obtain answer $AB = 3$ [The second M mark is dependent on both values of λ being found by correct meth <i>EITHER</i> : Divide by denominator and obtain quadratic remainder Obtain $A = 1$ Use any relevant method to obtain B, C or D Obtain one correct answer Obtain $B = 2, C = 1$ and $D = -3$ <i>OR</i> : Reduce RHS to a single fraction and equate numerators, or equivalent Obtain $A = 1$ Use any relevant method to obtain B, C or D Obtain one correct answer Obtain $B = 2, C = 1$ and $D = -3$	Obtain $a = -2$ A1Express general point of the line in component form, e.g. $(\lambda, 1 + 2\lambda, -1 + 2\lambda)$ B1Either substitute components in the equation of p and solve for λ , or substitute components and the value of a in the equation of q and solve for λ or substitute components and the value of a in the equation of q and solve for λ M1*Obtain $\lambda = 1$ for point A A1Obtain $\lambda = 2$ for point B A1Carry out correct process for finding the length of AB M1(dep*)Obtain answer $AB = 3$ A1[The second M mark is dependent on both values of λ being found by correct methods.]EITHER:Divide by denominator and obtain quadratic remainderM1Obtain $A = 1$ A1Use any relevant method to obtain B, C or D M1Obtain $B = 2, C = 1$ and $D = -3$ A1OR:Reduce RHS to a single fraction and equate numerators, or equivalentM1Obtain $B = 2, C = 1$ and $D = -3$ A1Use any relevant method to obtain B, C or D M1Obtain $A = 1$ A1Use any relevant method to obtain B, C or D M1Obtain $B = 2, C = 1$ and $D = -3$ A1Is relevant method to obtain B, C or D M1Obtain $B = 2, C = 1$ and $D = -3$ A1Is relevant method to obtain B, C or D M1Obtain $B = 2, C = 1$ and $D = -3$ A1Is relevant method to obtain B, C or D M1Obtain $B = 2, C = 1$ and $D = -3$ A1Is relevant method to obtain B, C or D M1Obtain $B = 2, C = 1$ and $D = -3$ A1 </td