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- 1 *EITHER*: State or imply non-modular inequality $(x + 3a)^2 > (2(x - 2a))^2$, or corresponding quadratic equation, or pair of linear equations $(x + 3a) = \pm 2(x - 2a)$ B1
 Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations M1
 Obtain critical values $x = \frac{1}{3}a$ and $x = 7a$ A1
 State answer $\frac{1}{3}a < x < 7a$ A1
- OR*: Obtain the critical value $x = 7a$ from a graphical method, or by inspection, or by solving a linear equation or inequality B1
 Obtain the critical value $x = \frac{1}{3}a$ similarly B2
 State answer $\frac{1}{3}a < x < 7a$ B1 [4]
 [Do not condone \leq for $<$; accept 0.33 for $\frac{1}{3}$.]
- 2 Use correct $\cos 2A$ formula and obtain an equation in $\sin \theta$ M1
 Obtain $4\sin^2 \theta + \sin \theta - 3 = 0$, or equivalent A1
 Make reasonable attempt to solve a 3-term quadratic in $\sin \theta$ M1
 Obtain answer 48.6° A1
 Obtain answer 131.4° and no others in the given range A1 ✓
 Obtain answer 270° and no others in the given range A1 [6]
 [Treat the giving of answers in radians as a misread. Ignore answers outside the given range.]
- 3 (i) *EITHER*: State or imply $n \ln x + \ln y = \ln C$ B1
 Substitute x - and y -values and solve for n M1
 Obtain $n = 1.50$ A1
 Solve for C M1
 Obtain $C = 6.00$ A1
- OR*: Obtain two correct equations by substituting x - and y -values in $x^n y = C$ B1
 Solve for n M1
 Obtain $n = 1.50$ A1
 Solve for C M1
 Obtain $C = 6.00$ A1 [5]
- (ii) State that the graph of $\ln y$ against $\ln x$ has equation $n \ln x + \ln y = \ln C$ which is *linear* in $\ln y$ and $\ln x$, or has equation of the form $nX + Y = \ln C$, where $X = \ln x$ and $Y = \ln y$, and is thus a straight line B1 [1]
- 4 (i) State correct expansion of $\cos(3x - x)$ or $\cos(3x + x)$ B1
 Substitute expansions in $\frac{1}{2}(\cos 2x - \cos 4x)$, or equivalent M1
 Simplify and obtain the given identity correctly A1 [3]
- (ii) Obtain integral $\frac{1}{4}\sin 2x - \frac{1}{8}\sin 4x$ B1
 Substitute limits correctly in an integral of the form $a \sin 2x + b \sin 4x$ M1
 Obtain given answer following full, correct and exact working A1 [3]

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- 5** Separate variables correctly B1
 Integrate and obtain term $\ln x$ B1
 Integrate and obtain term $\frac{1}{2} \ln(y^2 + 4)$ B1
 Evaluate a constant or use limits $y = 0, x = 1$ in a solution containing $a \ln x$ and $b \ln(y^2 + 4)$ M1
 Obtain correct solution in any form, e.g. $\frac{1}{2} \ln(y^2 + 4) = \ln x + \frac{1}{2} \ln 4$ A1
 Rearrange as $y^2 = 4(x^2 - 1)$, or equivalent A1 [6]
- 6**
- (i)** Using the formulae $\frac{1}{2} r^2 \theta$ and $\frac{1}{2} r^2 \sin \theta$, or equivalent, form an equation M1
 Obtain a correct equation in r and x and/or $x/2$ in any form A1
 Obtain the given equation correctly A1 [3]
- (ii)** Consider the sign of $x - (\frac{3}{4} \pi - \sin x)$ at $x = 1.3$ and $x = 1.5$, or equivalent M1
 Complete the argument with correct calculations A1 [2]
- (iii)** Use the iterative formula correctly at least once M1
 Obtain final answer 1.38 A1
 Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.375, 1.385) A1 [3]
- 7**
- (i)** Obtain modulus $\sqrt{8}$ B1
 Obtain argument $\frac{1}{4} \pi$ or 45° B1 [2]
- (ii)** Show 1, i and u in relatively correct positions on an Argand diagram B1
 Show the perpendicular bisector of the line joining 1 and i B1
 Show a circle with centre u and radius 1 B1
 Shade the correct region B1 [4]
- (iii)** State or imply relevance of the appropriate tangent from O to the circle B1 \checkmark
 Carry out complete strategy for finding $|z|$ for the critical point M1
 Obtain answer $\sqrt{7}$ A1 [3]
- 8**
- (i)** State or imply the form $\frac{A}{x+1} + \frac{B}{x+3}$ and use a relevant method to find A or B M1
 Obtain $A = 1, B = -1$ A1 [2]
- (ii)** Square the result of part **(i)** and substitute the fractions of part **(i)** M1
 Obtain the given answer correctly A1 [2]
- (iii)** Integrate and obtain $-\frac{1}{x+1} - \ln(x+1) + \ln(x+3) - \frac{1}{x+3}$ B3
 Substitute limits correctly in an integral containing at least two terms of the correct form M1
 Obtain given answer following full and exact working A1 [5]

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- 9
- (i) Use quotient or product rule to differentiate $(1-x)/(1+x)$ M1
 Obtain correct derivative in any form A1
 Use chain rule to find $\frac{dy}{dx}$ M1
 Obtain a correct expression in any form A1
 Obtain the gradient of the normal in the given form correctly A1 [5]
- (ii) Use product rule M1
 Obtain correct derivative in any form A1
 Equate derivative to zero and solve for x M1
 Obtain $x = \frac{1}{2}$ A1 [4]
- 10
- (i) Express general point of l or m in component form, e.g. $(1+s, 1-s, 1+2s)$ or $(4+2t, 6+2t, 1+t)$ B1
 Equate at least two corresponding pairs of components and solve for s or t M1
 Obtain $s = -1$ or $t = -2$ A1
 Verify that all three component equations are satisfied A1 [4]
- (ii) Carry out correct process for evaluating the scalar product of the direction vectors of l and m M1
 Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result M1
 Obtain answer 74.2° (or 1.30 radians) A1 [3]
- (iii) EITHER: Use scalar product to obtain $a-b+2c=0$ and $2a+2b+c=0$ B1
 Solve and obtain one ratio, e.g. $a:b$ M1
 Obtain $a:b:c = 5:-3:-4$, or equivalent A1
 Substitute coordinates of a relevant point and values for a, b and c in general equation of plane and evaluate d M1
 Obtain answer $5x-3y-4z=-2$, or equivalent A1
 OR 1: Using two points on l and one on m , or vice versa, state three equations in a, b, c and d B1
 Solve and obtain one ratio, e.g. $a:b$ M1
 Obtain a ratio of three of the unknowns, e.g. $a:b:c = -5:3:4$ A1
 Use coordinates of a relevant point and found ratio to find the fourth unknown, e.g. d M1
 Obtain answer $-5x+3y+4z=2$, or equivalent A1
 OR 2: Form a correct 2-parameter equation for the plane, B1
 e.g. $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + \mu(2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ M1
 State three equations in x, y, z, λ and μ M1
 State three correct equations A1
 Eliminate λ and μ M1
 Obtain answer $5x-3y-4z=-2$, or equivalent A1
 OR 3: Attempt to calculate vector product of direction vectors of l and m M1
 Obtain two correct components of the product A1
 Obtain correct product, e.g. $-5\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ A1
 Form a plane equation and use coordinates of a relevant point to calculate d M1
 Obtain answer $-5x+3y+4z=2$, or equivalent A1 [5]