Question	Answer	Marks	Guidance
1	$5C3\left[(-)(px)^3\right]$ soi	B1	Can be part of expansion. Condone omission of – sign
	$(-1)10p^3 = -2160$ then $\div$ and cube root	M1	Condone omission of – sign.
	<i>p</i> = 6	A1	
		3	

Question	Answer	Marks	Guidance
2	$y = \frac{1}{3}kx^3 - x^2 (+c)$	M1A1	Attempt integration for M mark
	Sub (0, 2)	DM1	Dep on $c$ present. Expect $c = 2$
	$\operatorname{Sub}(3,-1) \to -1 = 9k - 9 + their c$	DM1	
	k = 2/3	A1	
		5	

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Question	Answer	Marks	Guidance
3	Angle $CBA = \sin^{-1}\left(\frac{7}{8}\right) = 1.0654$ or $CBD = \cos^{-1}\left(\frac{-17}{32}\right) = 2.13$	B1	Accept 61.0°, 66° or 122°
	Sector $BCYD = \frac{1}{2} \times 8^2 \times 2 \times their 1.0654 (rad)$ soi or sector CBY = $\frac{1}{2} \times 8^2 \times their 1.0654 (rad)$	M1	Expect 68.1(9). Angle must be in radians (or <i>their</i> $61/360 \times 2 \times 8^2$ ) Or sector DBY
	$\Delta BCD = 7 \times \sqrt{8^2 - 7^2} \text{ or } \frac{1}{2} \times 8^2 \times \sin(2 \times their1.0654) \text{ soi}$	M1	Expect 27.1(1). Award M1 for ABC or ABD
	Semi-circle $CXD = \frac{1}{2}\pi \times 7^2 = 76.9(7)$	M1	M1M1 for segment area formula used correctly
	Total area = <i>their</i> 68.19 - <i>their</i> 27.11 + <i>their</i> 76.97 = 118.0–118.1	M1A1	Cannot gain M1 without attempt to find angle CBA or CBD
		6	

Question	Answer	Marks	Guidance
4(i)	$dy / dx = -2(2x - 1)^{-2} + 2$	B2,1,0	Unsimplified form ok (-1 for each error in '-2', ' $(2x-1)^{-2}$ ' and '2')
	$d^2 y / dx^2 = 8(2x-1)^{-3}$	<b>B</b> 1	Unsimplified form ok
		3	

Question	Answer	Marks	Guidance
4(ii)	Set $dy / dx$ to zero and attempt to solve – at least one correct step	M1	
	x = 0, 1	A1	Expect $(2x-1)^2 = 1$
	When $x = 0$ , $d^2 y / dx^2 = -8$ (or < 0). Hence MAX	B1	
	When $x = 1$ , $d^2 y / dx^2 = 8$ (or > 0). Hence MIN	B1	Both final marks dependent on correct x and correct $d^2y/dx^2$ and no errors May use change of sign of dy/dx but not at $x = 1/2$
		4	

Question	Answer	Marks	Guidance
5(i)	$\mathbf{u}.\mathbf{v} = 8q + 2q - 2 + 6q^2 - 42$	B1	May be unsimplified
	$6q^2 + 10q - 44 = 0 $ oe	M1	Simplify, set to zero and attempt to solve
	q = 2, -11/3	A1	Both required. Accept –3.67
		3	

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Question	Answer	Marks	Guidance
5(ii)	$\mathbf{u} = \begin{pmatrix} 0\\2\\6 \end{pmatrix} \mathbf{v} = \begin{pmatrix} 8\\-1\\-7 \end{pmatrix} \mathbf{u} \cdot \mathbf{v} = -2 - 42$	M1	Correct method for scalar product
	$ \mathbf{u}  \times  \mathbf{v}  = \sqrt{2^2 + 6^2} \times \sqrt{8^2 + 1^2 + 7^2}$	M1	Prod of mods. At least one methodically correct.
	$\cos\theta = \frac{-44}{\sqrt{40} \times \sqrt{114}} = \frac{-44}{4\sqrt{285}} = \frac{-4}{\sqrt{11}}$	M1	All linked correctly and inverse cos used correctly
	$\theta = 130.7^{\circ} \text{ or } 2.28(05) \text{ rads}$	A1	No other angles between 0° and 180°
		4	

Question	Answer	Marks	Guidance
6(i)	$S_n = \frac{p(2^n - 1)}{2 - 1} $ soi	M1	
	$p(2^{n}-1) > 1000 p \rightarrow 2^{n} > 1001$ AG	A1	
		2	

Question	Answer	Marks	Guidance
6(ii)	p + (n-1)p = 336	B1	Expect $np = 336$
	$\frac{n}{2} \left[ 2p + (n-1)p \right] = 7224$	B1	Expect $\frac{n}{2}(p+np) = 7224$
	Eliminate $n$ or $p$ to an equation in one variable	M1	Expect e.g. $168(1 + n) = 7224$ or $1 + 336/p = 43$ etc
	n = 42, p = 8	A1A1	
		5	

Question	Answer	Marks	Guidance
7(a)	$3(1-\cos^2 2\theta) + 8\cos 2\theta = 0 \rightarrow 3\cos^2 2\theta - 8\cos 2\theta - 3 (=0)$	M1	Use $s^2 = 1 - c^2$ and simplify to 3-term quadratic in $2\theta$
	$\cos 2\theta = -\frac{1}{3}$ soi	A1	Ignore other solution
	$2\theta = 109.(47)^{\circ} \text{ or } 250.(53)^{\circ}$	A1	One solution is sufficient, may be implied by either of the next solns
	<i>θ</i> = 54.7° or 125.3°	A1A1ft	Ft for 180° – other solution Use of double angles leads to $3c^4 - 7c^2 + 2 = 0 \Rightarrow c = \pm 1/\sqrt{3}$ for M1A1A1 then A1A1 for each angle Similar marking if $3\sin^2 2\theta = -8\cos 2\theta$ is squared leading to $9\sin^4 2\theta + 64\sin^2 2\theta - 64 = 0$
		5	

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Question	Answer	Marks	Guidance
7(b)	$\sqrt{3} = a + \tan 0 \rightarrow a = \sqrt{3}$	B1	b = 8  or  -4  (or  -10, 14  etc)  scores  M1A0
	$0 = \tan(-b\pi/6) + \sqrt{3}$ taken as far as $\tan^{-1}$ , angle units consistent	M1	A0 if $\tan^{-1}(-\sqrt{3})$ is not exact; (b=2 no working scores B2)
	<i>b</i> = 2	A1	
		3	

Question	Answer	Marks	Guidance
8(i)	$\left[\left(x-2\right)^{2}\right]+\left[3\right]$	B1 DB1	2nd B1 dependent on ±2 in 1st bracket
		2	
8(ii)	Largest k is 2 Accept $k \leq 2$	B1	Must be in terms of <i>k</i>
		1	
8(iii)	$y = (x-2)^2 + 3 \implies x-2 = (\pm)\sqrt{y-3}$	M1	
	$\Rightarrow f^{-1}(x) = 2 - \sqrt{x-3} \text{ for } x > 4$	A1B1	
		3	

Question	Answer	Marks	Guidance
8(iv)	$gf(x) = \frac{2}{x^2 - 4x + 7 - 1} = \frac{2}{(x - 2)^2 + 2}$	B1	Either form
	Since $f(x) > 4 \Rightarrow gf(x) < 2/3$ (or since $x < 1$ etc.)	M1A1	2/3 in answer implies M1 www
	range of $gf(x)$ is $0 < gf(x)(< 2/3)$	B1	Accept $0 < y < 2/3$ , $(0, 2/3)$ but $0 < x < 2/3$ is SCM1A1B0
		4	

Question	Answer	Marks	Guidance
9(i)	$V = (\pi) \int (x^3 + x^2) (\mathrm{d}x)$	M1	Attempt $\int y^2 dx$
	$\left[\left(\pi\right)\left[\frac{x^4}{4} + \frac{x^3}{3}\right]_0^3\right]$	A1	
	$\left(\pi\right)\left[\frac{81}{4}+9\left(-0\right)\right]$	DM1	May be implied by a correct answer
	$\frac{117\pi}{4}$ oe	A1	Accept 91.9 If additional areas rotated about x-axis, maximum of M1A0DM1A0
		4	

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Question	Answer	Marks	Guidance
9(ii)	$\frac{dy}{dx} = \frac{1}{2} \left( x^3 + x^2 \right)^{-1/2} \times \left( 3x^2 + 2x \right)$	B2,1,0	Omission of $3x^2 + 2x$ is one error
	(At x = 3,) y = 6	B1	
	At $x = 3$ , $m = \frac{1}{2} \times \frac{1}{6} \times 33 = \frac{11}{4}$ soi	DB1ft	Ft on <i>their</i> dy / dx providing differentiation attempted
	Equation of normal is $y-6 = -\frac{4}{11}(x-3)$	DM1	Equation through (3, <i>their</i> 6) and with gradient $-1/their$ <i>m</i>
	When $x = 0, y = 7\frac{1}{11}$ oe	A1	
		6	

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Question	Answer	Marks	Guidance
10(i)	$4x^{1/2} = x + 3 \rightarrow (x^{1/2})^2 - 4x^{1/2} + 3 (= 0) \text{ OR } 16x = x^2 + 6x + 9$	M1	Eliminate y from the 2 equations and then: Either treat as quad in $x^{1/2}$ OR square both sides and RHS is 3-term
	$x^{1/2} = 1 \text{ or } 3 x^2 - 10x + 9 (= 0)$	A1	If in 1st method $x^{1/2}$ becomes <i>x</i> , allow only M1 unless subsequently squared
	x = 1  or  9	A1	
	y = 4  or  12	A1ft	Ft from <i>their x</i> values If the 2 solutions are found by trial substitution B1 for the first coordinate and B3 for the second coordinate
	$AB^{2} = (9-1)^{2} + (12-4)^{2}$	M1	
	$AB = \sqrt{128} \text{ or } 8\sqrt{2} \text{ oe or } 11.3$	A1	
		6	
10(ii)	$dy/dx = 2 x^{-1/2}$	B1	
	$2x^{-1/2} = 1$	M1	Set <i>their</i> derivative = <i>their</i> gradient of <i>AB</i> and attempt to solve
	(4, 8)	A1	Alternative method without calculus: $M_{AB} = 1$ , tangent is $y = mx + c$ where $m = 1$ and meets $y = 4x^{1/2}$ when $4x^{1/2} = x + c$ . This is a quadratic with $b^2 = 4ac$ , so $16 - 4 \times 1 \times c = 0$ so $c = 4$ B1 Solving $4x^{1/2} = x + 4$ gives $x = 4$ and $y = 8$ M1A1
		3	

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Question	Answer	Marks	Guidance
10(iii)	Equation of normal is $y-8 = -1(x-4)$	M1	Equation through <i>their</i> T and with gradient $-1/their$ gradient of AB. Expect $y = -x + 12$ ,
	Eliminate $y$ (or $x$ ) $\rightarrow -x + 12 = x + 3$ or $y - 3 = 12 - y$	M1	May use <i>their</i> equation of AB
	(4½, 7½)	A1	
		3	