| Question | Answer | Marks |
| :---: | :--- | ---: |
| 1 | Remove logarithm and obtain $1+2^{x}=\mathrm{e}^{2}$ | B1 |
|  | Use correct method to solve an equation of the form $2^{x}=a$, where $a>0$ | M1 |
|  | Obtain answer $x=2.676$ | A1 |
|  |  | Total: |


| Question | Answer | Marks |
| :---: | :--- | ---: |
| 2 | EITHER: | (B1 |
|  | State or imply non-modular inequality $(x-4)^{2}<(2(3 x+1))^{2}$, or corresponding <br> quadratic equation, or pair of linear equations $x-4= \pm 2(3 x+1)$ | M1 |
|  | Make reasonable solution attempt at a 3-term quadratic, or solve two linear <br> equations for $x$ | A1 |
|  | Obtain critical values $x=-\frac{6}{5}$ and $x=\frac{2}{7}$ | A1) |
|  | State final answer $x<-\frac{6}{5}, x>\frac{2}{7}$ | (B1 |
|  | OR: | B2 |
|  | Obtain critical value $x=-\frac{6}{5}$ from a graphical method, or by inspection, or by <br> solving a linear equation or inequality | B1) |
|  | Obtain critical value $x=\frac{2}{7}$ similarly | Total: |


| Question | Answer | Marks |  |  |  |
| :---: | :--- | ---: | :---: | :---: | :---: |
| 3 (i) | Sketch a relevant graph, e.g. $y=\mathrm{e}^{-\frac{1}{2} x}$ | B1 |  |  |  |
|  | Sketch a second relevant graph, e.g. $y=4-x^{2}$, and justify the given statement | B1 |  |  |  |
|  | Total: |  |  |  | $\mathbf{2}$ |
|  | Calculate the value of a relevant expression or values of a pair of expressions at <br> $x=-1$ and $x=-1.5$ | M1 |  |  |  |
|  | complete the argument correctly with correct calculated values | A1 |  |  |  |
|  |  | Total: |  |  |  |


| Question | Answer | Marks |
| :---: | :--- | ---: |
| 3 (iii) | Use the iterative formula correctly at least once | M1 |
|  | Obtain final answer -1.41 | A1 |
|  | Show sufficient iterations to 4 d.p. to justify -1.41 to 2 d.p., or show there is a sign <br> change in the interval $(-1.415,-1.405)$ | A1 |
|  |  | Total: |


| Question | Answer | Marks |
| :---: | :---: | :---: |
| 4(i) | State $R=17$ | B1 |
|  | Use trig formula to find $\alpha$ | M1 |
|  | Obtain $\alpha=61.93^{\circ}$ with no errors seen | A1 |
|  | Total: | 3 |
| 4(ii) | Evaluate $\cos ^{-1}(4 / 17)$ to at least 1d.p. ( $76.39^{\circ}$ to 2 d.p.) | B1 ${ }^{\wedge}$ |
|  | Use a correct method to find a value of $x$ in the interval $0^{\circ}<x<180^{\circ}$ | M1 |
|  | Obtain answer, e.g. $x=7.2^{\circ}$ | A1 |
|  | Obtain second answer, e.g. $x=110.8^{\circ}$ and no others | A1 |
|  | [Ignore answers outside the given interval.] |  |
|  | [Treat answers in radians as a misread.] |  |
|  | Total: | 4 |


| Question | Answer | Marks |
| :---: | :--- | ---: |
| 5 | Use product rule | M1 |
|  | Obtain correct derivative in any form | A1 |
|  | Equate derivative to zero, use Pythagoras and obtain a quadratic equation in $\tan x$ | M1 |
|  | Obtain $\tan ^{2} x-a \tan x+1=0$, or equivalent | A1 |
|  | Use the condition for a quadratic to have only one root | M1 |
|  | Obtain answer $a=2$ | A1 |
|  | Obtain answer $x=\frac{1}{4} \pi$ | A1 |
|  |  | $\mathbf{7}$ |


| Question | Answer | Marks |
| :---: | :---: | :---: |
| 6 (i) | Verify that the point with position vector $\mathbf{i}+2 \mathbf{j}-3 \mathbf{k}$ lies in the plane | B1 |
|  | EITHER: |  |
|  | Find a second point on $l$ and substitute its coordinates in the equation of $p$ | (M1 |
|  | Verify that the second point, e.g. $(3,1,-2)$, lies in the plane | A1) |
|  | OR: |  |
|  | Expand scalar product of a normal to $p$ and the direction vector of $l$ | (M1 |
|  | Verify scalar product is zero | A1) |
|  | Total: | 3 |


| Question | Answer | Marks |
| :---: | :---: | :---: |
| 6(ii) | EITHER: |  |
|  | Use scalar product to obtain a relevant equation in $a, b$ and $c$, e.g. $2 a-b+c=0$ | (B1 |
|  | Obtain a second relevant equation, e.g. $3 a+b-5 c=0$, and solve for one ratio e.g. $a: b$ | M1 |
|  | Obtain $a: b: c=4: 13: 5$, or equivalent | A1 |
|  | Substitute ( $3,-1,2)$ and the values of $a, b$ and $c$ in the general equation and find $d$ | M1 |
|  | Obtain answer $4 x+13 y+5 z=9$, or equivalent | A1) |
|  | OR1: |  |
|  | Attempt to calculate vector product of relevant vectors, e.g. $(2 \mathbf{i}-\mathbf{j}+\mathbf{k}) \times(3 \mathbf{i}+\mathbf{j}-5 \mathbf{k})$ | (M1 |
|  | Obtain two correct components | A1 |
|  | Obtain correct answer, e.g. $4 \mathbf{i}+13 \mathbf{j}+5 \mathbf{k}$ | A1 |
|  | Substitute $(3,-1,2)$ in $4 x+13 y+5 z=d$, or equivalent, and find $d$ | M1 |
|  | Obtain answer $4 x+13 y+5 z=9$, or equivalent | A1) |
|  | OR2: |  |
|  | Using the relevant point and relevant vectors form a 2-parameter equation for the plane | (M1 |
|  | State a correct equation, e.g. $\mathbf{r}=3 \mathbf{i}-\mathbf{j}+2 \mathbf{k}+\lambda(2 \mathbf{i}-\mathbf{j}+\mathbf{k})+\mu(3 \mathbf{i}+\mathbf{j}-5 \mathbf{k})$ | A1 |
|  | State three correct equations in $x, y, z, \lambda$ and $\mu$ | A1 |
|  | Eliminate $\lambda$ and $\mu$ | M1 |
|  | Obtain answer $4 x+13 y+5 z=9$, or equivalent | A1) |
|  | OR3: |  |
|  | Using the relevant point and relevant vectors form a determinant equation for the plane | (M1 |
|  | State a correct equation, e.g. $\left\|\begin{array}{ccc}x-3 & y+1 & z-2 \\ 2 & -1 & 1 \\ 3 & 1 & -5\end{array}\right\|=0$ | A1 |
|  | Attempt to expand the determinant | M1 |
|  | Obtain or imply two correct cofactors | A1 |


| Question | Answer | Marks |
| :---: | :---: | :---: |
|  | Obtain answer $4 x+13 y+5 z=9$, or equivalent | A1) |
|  |  | Total: |


| Question | Answer |  | Marks |
| :---: | :---: | :---: | :---: |
| 7(i) | State or imply $\frac{\mathrm{d} V}{\mathrm{~d} t}=2 \frac{\mathrm{~d} h}{\mathrm{~d} t}$ |  | B1 |
|  | State or imply $\frac{\mathrm{d} V}{\mathrm{~d} t}=1-0.2 \sqrt{h}$ |  | B1 |
|  | Obtain the given answer correctly |  | B1 |
|  |  | Total: | 3 |
| 7(ii) | State or imply $\mathrm{d} u=-\frac{1}{2 \sqrt{h}} \mathrm{~d} h$, or equivalent |  | B1 |
|  | Substitute for $h$ and $\mathrm{d} h$ throughout |  | M1 |
|  | Obtain $T=\int_{3}^{5} \frac{20(5-u)}{u} \mathrm{~d} u$, or equivalent |  | A1 |
|  | Integrate and obtain terms $100 \ln u-20 u$, or equivalent |  | A1 |
|  | Substitute limits $u=3$ and $u=5$ correctly |  | M1 |
|  | Obtain answer 11.1, with no errors seen |  | A1 |
|  |  | Total: | 6 |


| Question | Answer | Marks |
| :---: | :---: | :---: |
| 8(i) | Substitute $z=-1+\mathrm{i}$ and attempt expansions of the $z^{2}$ and $z^{4}$ terms | M1 |
|  | Use $\mathrm{i}^{2}=-1$ at least once | M1 |
|  | Complete the verification correctly | A1 |
|  | Total: | 3 |
| 8(ii) | State second root $z=-1-\mathrm{i}$ | B1 |
|  | Carry out a complete method for finding a quadratic factor with zeros $-1+\mathrm{i}$ and $-1-\mathrm{i}$ | M1 |
|  | Obtain $z^{2}+2 z+2$, or equivalent | A1 |
|  | Attempt division of $\mathrm{p}(z)$ by $z^{2}+2 z+2$ and reach a partial quotient $z^{2}+k z$ | M1 |
|  | Obtain quadratic factor $z^{2}-2 z+5$ | A1 |
|  | Solve 3-term quadratic and use $\mathrm{i}^{2}=-1$ | M1 |
|  | Obtain roots $1+2 \mathrm{i}$ and $1-2 \mathrm{i}$ | A1 |
|  | Total: | 7 |


| Question | Answer | Marks |
| :---: | :--- | ---: |
| $9(\mathrm{i})$ | State or imply the form $\frac{A}{2+x}+\frac{B x+C}{4+x^{2}}$ | B1 |
|  | Use a relevant method to determine a constant | M1 |
|  | Obtain one of the values $A=-2, B=1, \mathrm{C}=4$ | A1 |
|  | Obtain a second value | A1 |
|  | Obtain the third value | A1 |
|  |  | $\mathbf{5}$ |


| Question | Answer | Marks |
| :---: | :---: | :---: |
| 9(ii) | Use correct method to obtain the first two terms of the expansion of $\left(1+\frac{1}{2} x\right)^{-1}$, $(2+x)^{-1},\left(1+\frac{1}{4} x^{2}\right)^{-1}$ or $\left(4+x^{2}\right)^{-1}$ | M1 |
|  | Obtain correct unsimplified expansions up to the term in $x^{2}$ of each partial fraction | $\mathbf{A 1} \checkmark^{\wedge}+\mathbf{A 1} \downarrow^{\wedge}$ |
|  | Multiply out up to the term in $x^{2}$ by $B x+C$, where $B C \neq 0$ | M1 |
|  | Obtain final answer $\frac{3}{4} x-\frac{1}{2} x^{2}$ | A1 |
|  | [Symbolic binomial coefficients, e.g. ${ }_{-1} \mathrm{C}_{2}$, are not sufficient for the first M1. The f.t. is on $A, B, C$.] |  |
|  | [In the case of an attempt to expand $x(6-x)(2+x)^{-1}\left(4+x^{2}\right)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.] |  |
|  | Total: | 5 |


| Question | Answer | Marks |
| :---: | :---: | :---: |
| 10(i) | State or imply derivative is $2 \frac{\ln x}{x}$ | B1 |
|  | State or imply gradient of the normal at $x=\mathrm{e}$ is $-\frac{1}{2} \mathrm{e}$, or equivalent | B1 |
|  | Carry out a complete method for finding the $x$-coordinate of $Q$ | M1 |
|  | Obtain answer $x=\mathrm{e}+\frac{2}{\mathrm{e}}$, or exact equivalent | A1 |
|  | Total: | 4 |
| 10(ii) | Justify the given statement by integration or by differentiation | B1 |
|  | Total: | 1 |
| 10(iii) | Integrate by parts and reach $a x(\ln x)^{2}+b \int x \cdot \frac{\ln x}{x} \mathrm{~d} x$ | M1* |
|  | Complete the integration and obtain $x(\ln x)^{2}-2 x \ln x+2 x$, or equivalent | A1 |
|  | Use limits $x=1$ and $x=\mathrm{e}$ correctly, having integrated twice | DM1 |
|  | Obtain exact value e-2 | A1 |
|  | Use $x$ - coordinate of $Q$ found in part (i) and obtain final answer $\mathrm{e}-2+\frac{1}{\mathrm{e}}$ | B1 ${ }^{\text {a }}$ |
|  | Total: | 5 |

