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| 1 | | | Use law of the logarithm of a power, quotient or product | **M1** | |
| | | | Remove logarithms and obtain a correct equation in $x$, e.g. $x^2 + 4 = 4x^2$ | **A1** | |
| | | | Obtain final answer $x = 2/\sqrt{3}$, or exact equivalent | **A1** | [3] |
| 2 | | | Use tan($A \pm B$) formula and obtain an equation in tan $\theta$ | **M1** | |
| | | | Using $\tan 45° = 1$, obtain a horizontal equation in tan $\theta$ in any correct form | **A1** | |
| | | | Reduce the equation to $7\tan^2\theta - 2\tan\theta - 1 = 0$, or equivalent | **A1** | |
| | | | Solve a 3-term quadratic for tan $\theta$ | **M1** | |
| | | | Obtain a correct answer, e.g. $\theta = 28.7°$ | **A1** | |
| | | | Obtain a second answer, e.g. $\theta = 165.4°$, and no others | **A1** | [6] |
| | | | [Ignore answers outside the given interval. Treat answers in radians as a misread (0.500, 2.89).] | | |
| 3 | **(i)** | | Consider sign of $x^5 - 3x^3 + x^2 - 4$ at $x = 1$ and $x = 2$, or equivalent | **M1** | |
| | | | Complete the argument correctly with correct calculated values | **A1** | [2] |
| | **(ii)** | | Rearrange the given quintic equation in the given form, or work *vice versa* | **B1** | [1] |
| | **(iii)** | | Use the iterative formula correctly at least once | **M1** | |
| | | | Obtain final answer 1.78 | **A1** | |
| | | | Show sufficient iterations to 4 d.p. to justify 1.78 to 2 d.p., or show there is a sign change in the interval (1.775, 1.785) | **A1** | [3] |
| 4 | **(i)** | | Substitute $x = -\frac{1}{2}$ and equate to zero, or divide by $(2x+1)$ and equate constant remainder to zero | **M1** | |
| | | | Obtain $a = 3$ | **A1** | [2] |
| | **(ii)** | **(a)** | Commence division by $(2x + 1)$ reaching a partial quotient of $2x^2 + kx$ | **M1** | |
| | | | Obtain factorisation $(2x+1)(2x^2 - x + 2)$ | **A1** | [2] |
| | | | [The M1 is earned if inspection reaches an unknown factor $2x^2 + Bx + C$ and an equation in $B$ and/or $C$, or an unknown factor $Ax^2 + Bx + 2$ and an equation in $A$ and/or $B$.] | | |
| | | **(b)** | State or imply critical value $x = -\frac{1}{2}$ | **B1** | |
| | | | Show that $2x^2 - x + 2$ is always positive, or that the gradient of $4x^3 + 3x + 2$ is always positive | **B1*** | |
| | | | Justify final answer $x > -\frac{1}{2}$ | **B1(dep*)** | [3] |
| 5 | **(i)** | | State or imply $\mathrm{d}x = \sqrt{3}\sec^2\theta \, \mathrm{d}\theta$ | **B1** | |
| | | | Substitute for $x$ and $\mathrm{d}x$ throughout | **M1** | |
| | | | Obtain the given answer correctly | **A1** | [3] |

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| | **(ii)** | Replace integrand by $\frac{1}{2}\cos 2\theta + \frac{1}{2}$ | **B1** | |
| | | Obtain integral $\frac{1}{4}\sin 2\theta + \frac{1}{2}\theta$ | **B1**√ | |
| | | Substitute limits correctly in an integral of the form $c\sin 2\theta + b\theta$, where $cb \neq 0$ | **M1** | |
| | | Obtain answer $\frac{1}{12}\sqrt{3}\pi + \frac{3}{8}$, or exact equivalent | **A1** | [4] |
| | | [The f.t. is on integrands of the form $a\cos 2\theta + b$, where $ab \neq 0$.] | | |
| **6** | **(i)** | *EITHER*: State correct derivative of $\sin y$ with respect to $x$ | **B1** | |
| | | Use product rule to differentiate the LHS | **M1** | |
| | | Obtain correct derivative of the LHS | **A1** | |
| | | Obtain a complete and correct derived equation in any form | **A1** | |
| | | Obtain a correct expression for $\frac{dy}{dx}$ in any form | **A1** | |
| | | *OR*: State correct derivative of $\sin y$ with respect to $x$ | **B1** | |
| | | Rearrange the given equation as $\sin y = x/(\ln x + 2)$ and attempt to differentiate both sides | **B1** | |
| | | Use quotient or product rule to differentiate the RHS | **M1** | |
| | | Obtain correct derivative of the RHS | **A1** | |
| | | Obtain a correct expression for $\frac{dy}{dx}$ in any form | **A1** | [5] |
| | **(ii)** | Equate $\frac{dy}{dx}$ to zero and obtain a horizontal equation in $\ln x$ or $\sin y$ | **M1** | |
| | | Solve for $\ln x$ | **M1** | |
| | | Obtain final answer $x = 1/\mathrm{e}$, or exact equivalent | **A1** | [3] |
| **7** | **(i)** | Separate variables and attempt integration of one side | **M1** | |
| | | Obtain term $-\mathrm{e}^{-y}$ | **A1** | |
| | | Integrate $x\mathrm{e}^x$ by parts reaching $x\mathrm{e}^x \pm \int \mathrm{e}^x \, \mathrm{d}x$ | **M1** | |
| | | Obtain integral $x\mathrm{e}^x - \mathrm{e}^x$ | **A1** | |
| | | Evaluate a constant, or use limits $x = 0$, $y = 0$ | **M1** | |
| | | Obtain correct solution in any form | **A1** | |
| | | Obtain final answer $y = -\ln(\mathrm{e}^x(1-x))$, or equivalent | **A1** | [7] |
| | **(ii)** | Justify the given statement | **B1** | [1] |

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| **8** | **(i)** | *EITHER*: | Substitute for **r** in the given equation of $p$ and expand scalar product | **M1** | |
| | | | Obtain equation in $\lambda$ in any correct form | **A1** | |
| | | | Verify this is not satisfied for any value of $\lambda$ | **A1** | |
| | | *OR*1: | Substitute coordinates of a general point of $l$ in the Cartesian equation of plane $p$ | **M1** | |
| | | | Obtain equation in $\lambda$ in any correct form | **A1** | |
| | | | Verify this is not satisfied for any value of $\lambda$ | **A1** | |
| | | *OR*2: | Expand scalar product of the normal to $p$ and the direction vector of $l$ | **M1** | |
| | | | Verify scalar product is zero | **A1** | |
| | | | Verify that one point of $l$ does not lie in the plane | **A1** | |
| | | *OR*3: | Use correct method to find the perpendicular distance of a general point of $l$ from $p$ | **M1** | |
| | | | Obtain a correct unsimplified expression in terms of $\lambda$ | **A1** | |
| | | | Show that the perpendicular distance is $5/\sqrt{6}$, or equivalent, for all $\lambda$ | **A1** | |
| | | *OR*4: | Use correct method to find the perpendicular distance of a particular point of $l$ from $p$ | **M1** | |
| | | | Show that the perpendicular distance is $5/\sqrt{6}$, or equivalent | **A1** | |
| | | | Show that the perpendicular distance of a second point is also $5/\sqrt{6}$, or equivalent | **A1** | [3] |
| | **(ii)** | *EITHER*: | Calling the unknown direction vector $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ state equation $2a+b+3c=0$ | **B1** | |
| | | | State equation $2a-b-c=0$ | **B1** | |
| | | | Solve for one ratio, e.g. $a : b$ | **M1** | |
| | | | Obtain ratio $a : b : c = 1 : 4 : -2$, or equivalent | **A1** | |
| | | *OR*: | Attempt to calculate the vector product of the direction vector of $l$ and the normal vector of the plane $p$, e.g. $(2\mathbf{i}+\mathbf{j}+3\mathbf{k})\times(2\mathbf{i}-\mathbf{j}-\mathbf{k})$ | **M2** | |
| | | | Obtain two correct components of the product | **A1** | |
| | | | Obtain answer $2\mathbf{i}+8\mathbf{j}-4\mathbf{k}$, or equivalent | **A1** | |
| | | | Form line equation with relevant vectors | **M1** | |
| | | | Obtain answer $\mathbf{r}=5\mathbf{i}+3\mathbf{j}+\mathbf{k}+\mu(\mathbf{i}+4\mathbf{j}-2\mathbf{k})$, or equivalent | **A1**√ | [6] |
| **9** | **(i)** | | State or obtain $A = 3$ | **B1** | |
| | | | Use a relevant method to find a constant | **M1** | |
| | | | Obtain one of $B = -4$, $C = 4$ and $D = 0$ | **A1** | |
| | | | Obtain a second value | **A1** | |
| | | | Obtain the third value | **A1** | [5] |
| | **(ii)** | | Integrate and obtain $3x-4\ln x$ | **B1**√ | |
| | | | Integrate and obtain term of the form $k\ln(x^2+2)$ | **M1** | |
| | | | Obtain term $2\ln(x^2+2)$ | **A1**√ | |
| | | | Substitute limits in an integral of the form $ax+b\ln x+c\ln(x^2+2)$, where $abc \neq 0$ | **M1** | |
| | | | Obtain given answer $3-\ln 4$ after full and correct working | **A1** | [5] |
| **10** | **(a)** | | Substitute and obtain a correct equation in $x$ and $y$ | **B1** | |
| | | | Use $\mathrm{i}^2=-1$ and equate real and imaginary parts | **M1** | |
| | | | Obtain two correct equations, e.g. $x + 2y + 1 = 0$ and $y + 2x = 0$ | **A1** | |
| | | | Solve for $x$ or for $y$ | **M1** | |
| | | | Obtain answer $z=\frac{1}{3}-\frac{2}{3}\mathrm{i}$ | **A1** | [5] |

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| **(b)** | **(i)** | Show a circle with centre $-1+3\,\mathrm{i}$ | **B1** | |
| | | Show a circle with radius 1 | **B1** | |
| | | Show the line Im $z = 3$ | **B1** | |
| | | Shade the correct region | **B1** | [4] |
| | **(ii)** | Carry out a complete method to calculate the relevant angle | **M1** | |
| | | Obtain answer 0.588 radians (accept 33.7°) | **A1** | [2] |