Cambridge Assessment International Education

AS & A Level 9709

Questions by Topic

Compiled by : Dr Yu on August 24, 2020

Paper 1

AP/GP

Q1 : 9709_s10_qp_11_Q3

3 The ninth term of an arithmetic progression is 22 and the sum of the first 4 terms is 49.

(i) Find the first term of the progression and the common difference. [4]

The nth term of the progression is 46.

(ii) Find the value of n. [2]

Q2 : 9709_s10_qp_12_Q7

7 **(a)** Find the sum of all the multiples of 5 between 100 and 300 inclusive. [3]

(b) A geometric progression has a common ratio of $-\frac{2}{3}$ and the sum of the first 3 terms is 35. Find

(i) the first term of the progression, [3]

(ii) the sum to infinity. [2]

Q3 : 9709_s10_qp_13_Q1

1 The first term of a geometric progression is 12 and the second term is -6. Find

(i) the tenth term of the progression, [3]

(ii) the sum to infinity. [2]

Q4 : 9709_s11_qp_11_Q8

8 A television quiz show takes place every day. On day 1 the prize money is \$1000. If this is not won the prize money is increased for day 2. The prize money is increased in a similar way every day until it is won. The television company considered the following two different models for increasing the prize money.

Model 1: Increase the prize money by \$1000 each day.

Model 2: Increase the prize money by 10% each day.

On each day that the prize money is not won the television company makes a donation to charity. The amount donated is 5% of the value of the prize on that day. After 40 days the prize money has still not been won. Calculate the total amount donated to charity

(i) if Model 1 is used, [4]

(ii) if Model 2 is used. [3]

Q5 : 9709_s11_qp_12_Q10

10 (a) A circle is divided into 6 sectors in such a way that the angles of the sectors are in arithmetic progression. The angle of the largest sector is 4 times the angle of the smallest sector. Given that the radius of the circle is 5 cm, find the perimeter of the smallest sector. [6]

(b) The first, second and third terms of a geometric progression are $2k + 3$, $k + 6$ and k, respectively. Given that all the terms of the geometric progression are positive, calculate

(i) the value of the constant k, [3]

(ii) the sum to infinity of the progression. [2]

Q6 : 9709_s11_qp_13_Q6

6 (a) A geometric progression has a third term of 20 and a sum to infinity which is three times the first term. Find the first term. [4]

(b) An arithmetic progression is such that the eighth term is three times the third term. Show that the sum of the first eight terms is four times the sum of the first four terms. [4]

Q7 : 9709_s12_qp_11_Q7

7 (a) The first two terms of an arithmetic progression are 1 and $\cos^2 x$ respectively. Show that the sum of the first ten terms can be expressed in the form $a - b\sin^2 x$, where a and b are constants to be found. [3]

(b) The first two terms of a geometric progression are 1 and $\frac{1}{3}\tan^2\theta$ respectively, where $0 < \theta < \frac{1}{2}\pi$.

(i) Find the set of values of θ for which the progression is convergent. [2]

(ii) Find the exact value of the sum to infinity when $\theta = \frac{1}{6}\pi$. [2]

Q8 : 9709_s12_qp_12_Q7

7 (a) In an arithmetic progression, the sum of the first n terms, denoted by S_n, is given by

$$S_n = n^2 + 8n.$$

Find the first term and the common difference. [3]

(b) In a geometric progression, the second term is 9 less than the first term. The sum of the second and third terms is 30. Given that all the terms of the progression are positive, find the first term. [5]

Q9 : 9709_s12_qp_13_Q6

6 The first term of an arithmetic progression is 12 and the sum of the first 9 terms is 135.

(i) Find the common difference of the progression. [2]

The first term, the ninth term and the nth term of this arithmetic progression are the first term, the second term and the third term respectively of a geometric progression.

(ii) Find the common ratio of the geometric progression and the value of n. [5]

Q10 : 9709_s13_qp_11_Q4

4 The third term of a geometric progression is -108 and the sixth term is 32. Find

(i) the common ratio, [3]

(ii) the first term, [1]

(iii) the sum to infinity. [2]

Q11 : 9709_s13_qp_12_Q10

10 (a) The first and last terms of an arithmetic progression are 12 and 48 respectively. The sum of the first four terms is 57. Find the number of terms in the progression. [4]

(b) The third term of a geometric progression is four times the first term. The sum of the first six terms is k times the first term. Find the possible values of k. [4]

Q12 : 9709_s13_qp_13_Q9

9 (a) In an arithmetic progression, the sum, S_n, of the first n terms is given by $S_n = 2n^2 + 8n$. Find the first term and the common difference of the progression. [3]

(b) The first 2 terms of a geometric progression are 64 and 48 respectively. The first 3 terms of the geometric progression are also the 1st term, the 9th term and the nth term respectively of an arithmetic progression. Find the value of n. [5]

Q13 : 9709_s14_qp_11_Q5

5 An arithmetic progression has first term a and common difference d. It is given that the sum of the first 200 terms is 4 times the sum of the first 100 terms.

(i) Find d in terms of a. [3]

(ii) Find the 100th term in terms of a. [2]

Q14 : 9709_s14_qp_12_Q6

6 The 1st, 2nd and 3rd terms of a geometric progression are the 1st, 9th and 21st terms respectively of an arithmetic progression. The 1st term of each progression is 8 and the common ratio of the geometric progression is r, where $r \neq 1$. Find

(i) the value of r, [4]

(ii) the 4th term of each progression. [3]

Q15 : 9709_s14_qp_13_Q2

2 The first term in a progression is 36 and the second term is 32.

(i) Given that the progression is geometric, find the sum to infinity. [2]

(ii) Given instead that the progression is arithmetic, find the number of terms in the progression if the sum of all the terms is 0. [3]

Q16 : 9709_s15_qp_11_Q7

7 **(a)** The third and fourth terms of a geometric progression are $\frac{1}{3}$ and $\frac{2}{9}$ respectively. Find the sum to infinity of the progression. [4]

(b) A circle is divided into 5 sectors in such a way that the angles of the sectors are in arithmetic progression. Given that the angle of the largest sector is 4 times the angle of the smallest sector, find the angle of the largest sector. [4]

Q17 : 9709_s15_qp_12_Q8

8 **(a)** The first, second and last terms in an arithmetic progression are 56, 53 and -22 respectively. Find the sum of all the terms in the progression. [4]

(b) The first, second and third terms of a geometric progression are $2k + 6$, $2k$ and $k + 2$ respectively, where k is a positive constant.

(i) Find the value of k. [3]

(ii) Find the sum to infinity of the progression. [2]

Q18 : 9709_s15_qp_13_Q9

9 **(a)** The first term of an arithmetic progression is -2222 and the common difference is 17. Find the value of the first positive term. [3]

(b) The first term of a geometric progression is $\sqrt{3}$ and the second term is $2\cos\theta$, where $0 < \theta < \pi$. Find the set of values of θ for which the progression is convergent. [5]

Q19 : 9709_s16_qp_11_Q9

9 **(a)** The first term of a geometric progression in which all the terms are positive is 50. The third term is 32. Find the sum to infinity of the progression. [3]

(b) The first three terms of an arithmetic progression are $2\sin x$, $3\cos x$ and $(\sin x + 2\cos x)$ respectively, where x is an acute angle.

(i) Show that $\tan x = \frac{4}{3}$. [3]

(ii) Find the sum of the first twenty terms of the progression. [3]

Q20 : 9709_s16_qp_12_Q9

9 A water tank holds 2000 litres when full. A small hole in the base is gradually getting bigger so that each day a greater amount of water is lost.

(i) On the first day after filling, 10 litres of water are lost and this increases by 2 litres each day.

(a) How many litres will be lost on the 30th day after filling? [2]

(b) The tank becomes empty during the nth day after filling. Find the value of n. [3]

(ii) Assume instead that 10 litres of water are lost on the first day and that the amount of water lost increases by 10% on each succeeding day. Find what percentage of the original 2000 litres is left in the tank at the end of the 30th day after filling. [4]

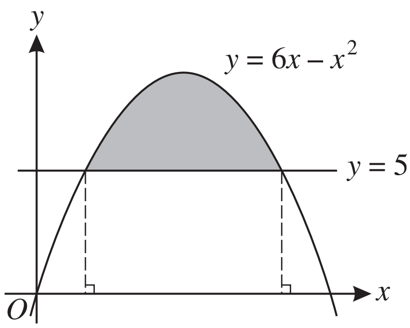
Q21 : 9709_s16_qp_13_Q4

4 The 1st, 3rd and 13th terms of an arithmetic progression are also the 1st, 2nd and 3rd terms respectively of a geometric progression. The first term of each progression is 3. Find the common difference of the arithmetic progression and the common ratio of the geometric progression. [5]

Area under curve

Q22 : 9709_s10_qp_11_Q4

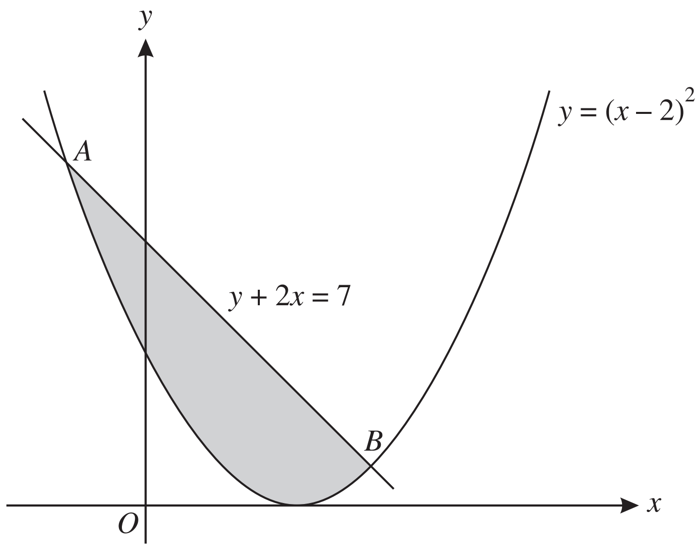
4



The diagram shows the curve $y = 6x - x^2$ and the line $y = 5$. Find the area of the shaded region. [6]

Q23 : 9709_s10_qp_12_Q9

9



The diagram shows the curve $y = (x - 2)^2$ and the line $y + 2x = 7$, which intersect at points A and B. Find the area of the shaded region. [8]

Q24 : 9709_s13_qp_11_Q10

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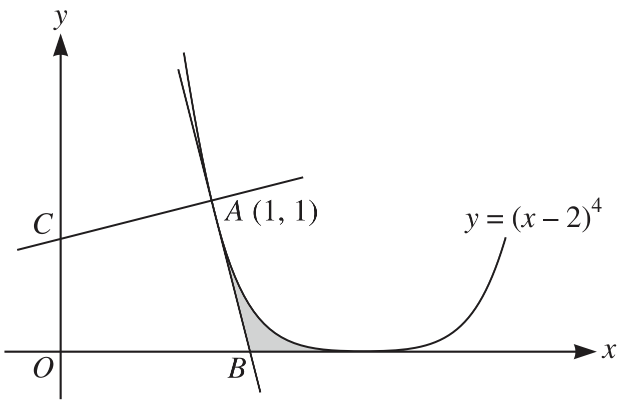


The diagram shows part of the curve $y = (x-2)^4$ and the point $A\,(1,\ 1)$ on the curve. The tangent at A cuts the x-axis at B and the normal at A cuts the y-axis at C.

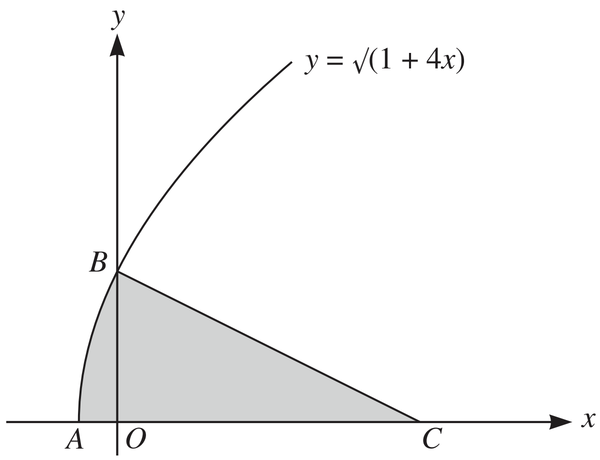
(i) Find the coordinates of B and C. [6]

(ii) Find the distance AC, giving your answer in the form $\frac{\sqrt{a}}{b}$, where a and b are integers. [2]

(iii) Find the area of the shaded region. [4]

Q25 : 9709_s13_qp_12_Q11

11



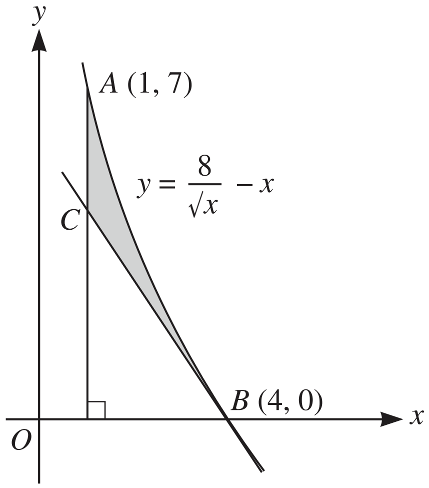
The diagram shows the curve $y = \sqrt{(1+4x)}$, which intersects the x-axis at A and the y-axis at B. The normal to the curve at B meets the x-axis at C. Find

(i) the equation of BC, [5]

(ii) the area of the shaded region. [5]

Q26 : 9709_s13_qp_13_Q11

11



The diagram shows part of the curve $y = \frac{8}{\sqrt{x}} - x$ and points $A\,(1,\ 7)$ and $B\,(4,\ 0)$ which lie on the curve. The tangent to the curve at B intersects the line $x = 1$ at the point C.

(i) Find the coordinates of C. [4]

(ii) Find the area of the shaded region. [5]

Q27 : 9709_s14_qp_11_Q11

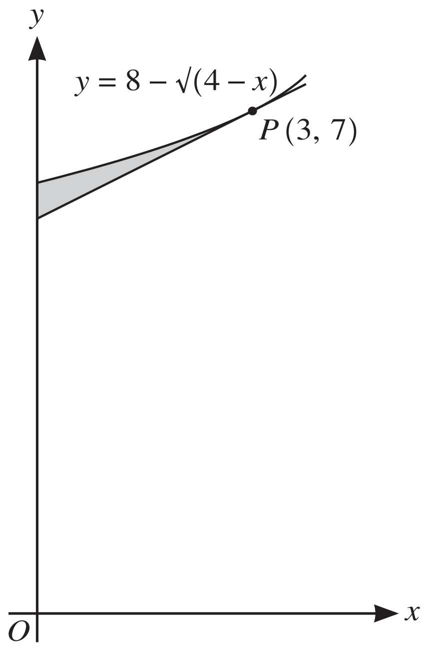
11 A line has equation $y = 2x + c$ and a curve has equation $y = 8 - 2x - x^2$.

(i) For the case where the line is a tangent to the curve, find the value of the constant c. [3]

(ii) For the case where $c = 11$, find the x-coordinates of the points of intersection of the line and the curve. Find also, by integration, the area of the region between the line and the curve. [7]

Q28 : 9709_s14_qp_12_Q9

9



The diagram shows part of the curve $y = 8 - \sqrt{(4 - x)}$ and the tangent to the curve at $P(3, 7)$.

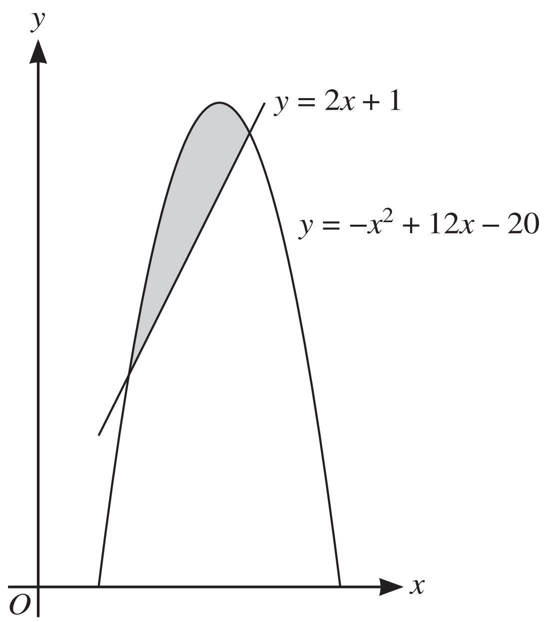
(i) Find expressions for $\frac{dy}{dx}$ and $\int y \, dx$. [5]

(ii) Find the equation of the tangent to the curve at P in the form $y = mx + c$. [2]

(iii) Find, showing all necessary working, the area of the shaded region. [4]

Q29 : 9709_s14_qp_13_Q10

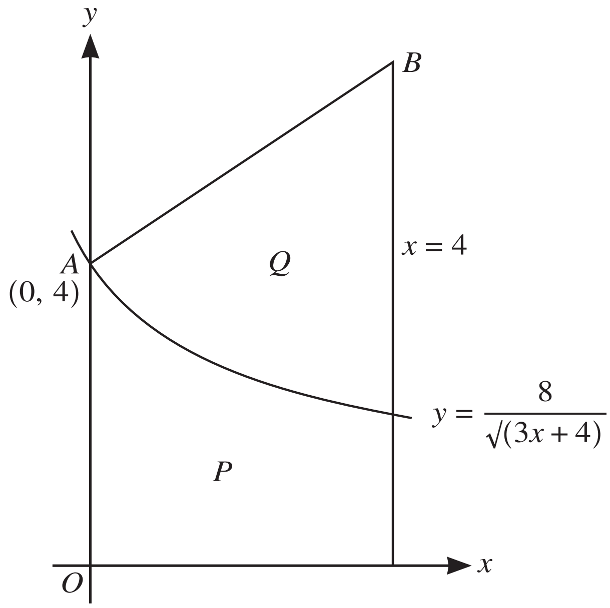
10



The diagram shows the curve $y = -x^2 + 12x - 20$ and the line $y = 2x + 1$. Find, showing all necessary working, the area of the shaded region. [8]

Q30 : 9709_s15_qp_11_Q10

10



The diagram shows part of the curve $y = \dfrac{8}{\sqrt{(3x+4)}}$. The curve intersects the y-axis at $A\ (0, 4)$. The normal to the curve at A intersects the line $x = 4$ at the point B.

(i) Find the coordinates of B. [5]

(ii) Show, with all necessary working, that the areas of the regions marked P and Q are equal. [6]

Binomial

Q31 : 9709_s10_qp_11_Q2

2 **(i)** Find the first 3 terms in the expansion of $\left(2x - \frac{3}{x}\right)^5$ in descending powers of x. [3]

(ii) Hence find the coefficient of x in the expansion of $\left(1 + \frac{2}{x^2}\right)\left(2x - \frac{3}{x}\right)^5$. [2]

Q32 : 9709_s10_qp_12_Q6

6 **(i)** Find the first 3 terms in the expansion of $(1 + ax)^5$ in ascending powers of x. [2]

(ii) Given that there is no term in x in the expansion of $(1 - 2x)(1 + ax)^5$, find the value of the constant a. [2]

(iii) For this value of a, find the coefficient of x^2 in the expansion of $(1 - 2x)(1 + ax)^5$. [3]

Q33 : 9709_s10_qp_13_Q2

2 **(i)** Find the first three terms, in descending powers of x, in the expansion of $\left(x - \frac{2}{x}\right)^6$. [3]

(ii) Find the coefficient of x^4 in the expansion of $(1 + x^2)\left(x - \frac{2}{x}\right)^6$. [2]

Q34 : 9709_s11_qp_11_Q1

1 Find the coefficient of x in the expansion of $\left(x + \frac{2}{x^2}\right)^7$. [3]

Q35 : 9709_s11_qp_12_Q2

2 **(i)** Find the terms in x^2 and x^3 in the expansion of $\left(1 - \frac{3}{2}x\right)^6$. [3]

(ii) Given that there is no term in x^3 in the expansion of $(k + 2x)\left(1 - \frac{3}{2}x\right)^6$, find the value of the constant k. [2]

Q36 : 9709_s11_qp_13_Q1

1 The coefficient of x^3 in the expansion of $(a + x)^5 + (1 - 2x)^6$, where a is positive, is 90. Find the value of a. [5]

Q37 : 9709_s12_qp_11_Q2

2 Find the coefficient of x^6 in the expansion of $\left(2x^3 - \frac{1}{x^2}\right)^7$. [4]

Q38 : `9709_s12_qp_12_Q3`

3 The coefficient of x^3 in the expansion of $(a + x)^5 + (2 - x)^6$ is 90. Find the value of the positive constant a. [5]

Q39 : `9709_s12_qp_13_Q3`

3 The first three terms in the expansion of $(1 - 2x)^2(1 + ax)^6$, in ascending powers of x, are $1 - x + bx^2$. Find the values of the constants a and b. [6]

Q40 : `9709_s13_qp_11_Q2`

2 **(i)** In the expression $(1 - px)^6$, p is a non-zero constant. Find the first three terms when $(1 - px)^6$ is expanded in ascending powers of x. [2]

(ii) It is given that the coefficient of x^2 in the expansion of $(1 - x)(1 - px)^6$ is zero. Find the value of p. [3]

Q41 : `9709_s13_qp_12_Q2`

2 Find the coefficient of x^2 in the expansion of

(i) $\left(2x - \frac{1}{2x}\right)^6$, [2]

(ii) $(1 + x^2)\left(2x - \frac{1}{2x}\right)^6$. [3]

Q42 : `9709_s13_qp_13_Q4`

4 **(i)** Find the first three terms in the expansion of $(2 + ax)^5$ in ascending powers of x. [3]

(ii) Given that the coefficient of x^2 in the expansion of $(1 + 2x)(2 + ax)^5$ is 240, find the possible values of a. [3]

Q43 : `9709_s14_qp_11_Q3`

3 Find the term independent of x in the expansion of $\left(4x^3 + \frac{1}{2x}\right)^8$. [4]

Q44 : `9709_s14_qp_12_Q2`

2 Find the coefficient of x^2 in the expansion of $(1 + x^2)\left(\frac{x}{2} - \frac{4}{x}\right)^6$. [5]

Q45 : 9709_s14_qp_13_Q1

1 Find the coefficient of x in the expansion of $\left(x^2 - \frac{2}{x}\right)^5$. [3]

Q46 : 9709_s15_qp_11_Q3

3 **(i)** Find the first three terms, in ascending powers of x, in the expansion of

(a) $(1 - x)^6$, [2]

(b) $(1 + 2x)^6$. [2]

(ii) Hence find the coefficient of x^2 in the expansion of $[(1 - x)(1 + 2x)]^6$. [3]

Q47 : 9709_s15_qp_12_Q3

3 **(i)** Find the coefficients of x^2 and x^3 in the expansion of $(2 - x)^6$. [3]

(ii) Find the coefficient of x^3 in the expansion of $(3x + 1)(2 - x)^6$. [2]

Q48 : 9709_s15_qp_13_Q3

3 **(i)** Write down the first 4 terms, in ascending powers of x, of the expansion of $(a - x)^5$. [2]

(ii) The coefficient of x^3 in the expansion of $(1 - ax)(a - x)^5$ is -200. Find the possible values of the constant a. [4]

Q49 : 9709_s16_qp_11_Q1

1 Find the term independent of x in the expansion of $\left(x - \frac{3}{2x}\right)^6$. [3]

Q50 : 9709_s16_qp_12_Q4

4 Find the term that is independent of x in the expansion of

(i) $\left(x - \frac{2}{x}\right)^6$, [2]

(ii) $\left(2 + \frac{3}{x^2}\right)\left(x - \frac{2}{x}\right)^6$. [4]

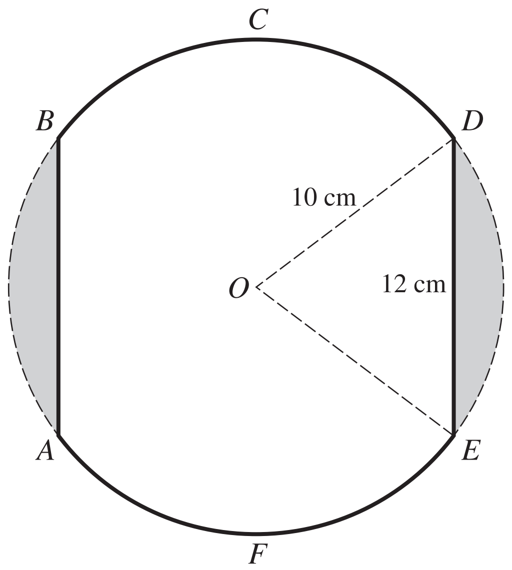
Q51 : 9709_s16_qp_13_Q1

1 Find the coefficient of x in the expansion of $\left(\frac{1}{x} + 3x^2\right)^5$. [3]

Circular measure

Q52 : 9709_s10_qp_13_Q7

7



The diagram shows a metal plate $ABCDEF$ which has been made by removing the two shaded regions from a circle of radius 10 cm and centre O. The parallel edges AB and ED are both of length 12 cm.

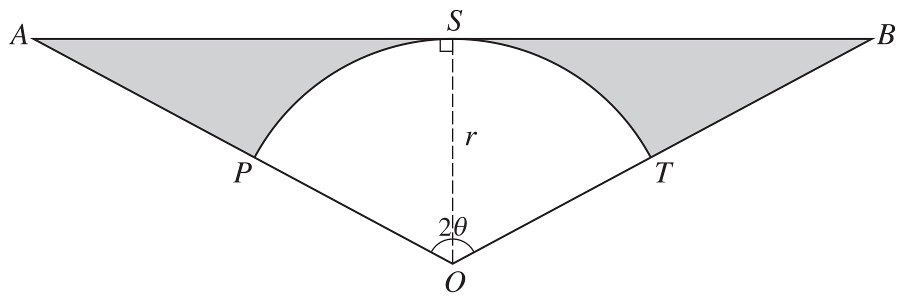
(i) Show that angle DOE is 1.287 radians, correct to 4 significant figures. [2]

(ii) Find the perimeter of the metal plate. [3]

(iii) Find the area of the metal plate. [3]

Q53 : 9709_s11_qp_11_Q9

9



In the diagram, OAB is an isosceles triangle with $OA = OB$ and angle $AOB = 2\theta$ radians. Arc PST has centre O and radius r, and the line ASB is a tangent to the arc PST at S.

(i) Find the total area of the shaded regions in terms of r and θ. [4]

(ii) In the case where $\theta = \frac{1}{3}\pi$ and $r = 6$, find the total perimeter of the shaded regions, leaving your answer in terms of $\sqrt{3}$ and π. [5]

Q54 : 9709_s11_qp_13_Q7

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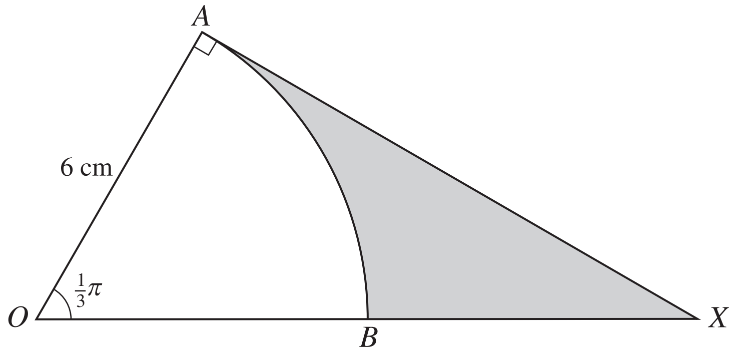


In the diagram, AB is an arc of a circle, centre O and radius 6 cm, and angle $AOB = \frac{1}{3}\pi$ radians. The line AX is a tangent to the circle at A, and OBX is a straight line.

(i) Show that the exact length of AX is $6\sqrt{3}$ cm. [1]

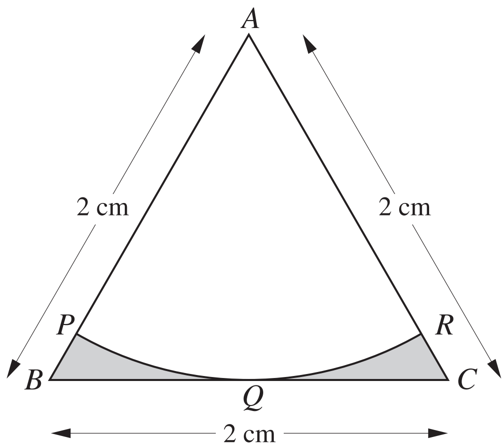
Find, in terms of π and $\sqrt{3}$,

(ii) the area of the shaded region, [3]

(iii) the perimeter of the shaded region. [4]

Q55 : 9709_s12_qp_11_Q3

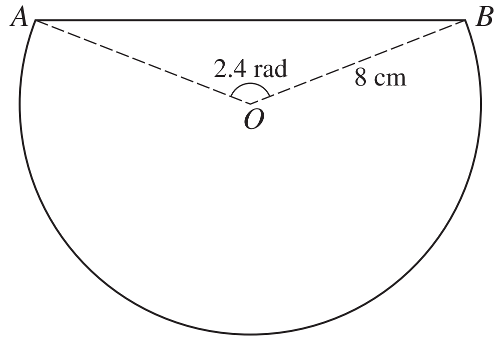
3



In the diagram, ABC is an equilateral triangle of side 2 cm. The mid-point of BC is Q. An arc of a circle with centre A touches BC at Q, and meets AB at P and AC at R. Find the total area of the shaded regions, giving your answer in terms of π and $\sqrt{3}$. [5]

Q56 : 9709_s12_qp_12_Q6

6



The diagram shows a metal plate made by removing a segment from a circle with centre O and radius 8 cm. The line AB is a chord of the circle and angle $AOB = 2.4$ radians. Find

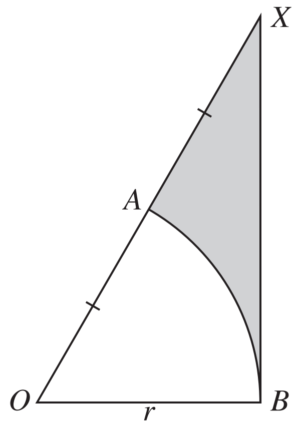
(i) the length of AB, [2]

(ii) the perimeter of the plate, [3]

(iii) the area of the plate. [3]

Q57 : 9709_s12_qp_13_Q8

8



In the diagram, AB is an arc of a circle with centre O and radius r. The line XB is a tangent to the circle at B and A is the mid-point of OX.

(i) Show that angle $AOB = \frac{1}{3}\pi$ radians. [2]

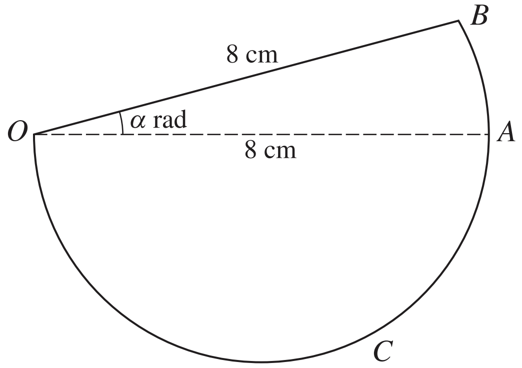
Express each of the following in terms of r, π and $\sqrt{3}$:

(ii) the perimeter of the shaded region, [3]

(iii) the area of the shaded region. [2]

Q58 : 9709_s13_qp_11_Q3

3



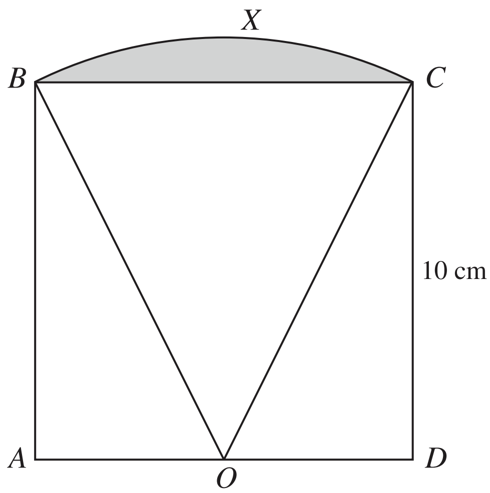
In the diagram, OAB is a sector of a circle with centre O and radius 8 cm. Angle BOA is α radians. OAC is a semicircle with diameter OA. The area of the semicircle OAC is twice the area of the sector OAB.

(i) Find α in terms of π. [3]

(ii) Find the perimeter of the complete figure in terms of π. [2]

Q59 : 9709_s13_qp_12_Q4

4



The diagram shows a square $ABCD$ of side 10 cm. The mid-point of AD is O and BXC is an arc of a circle with centre O.

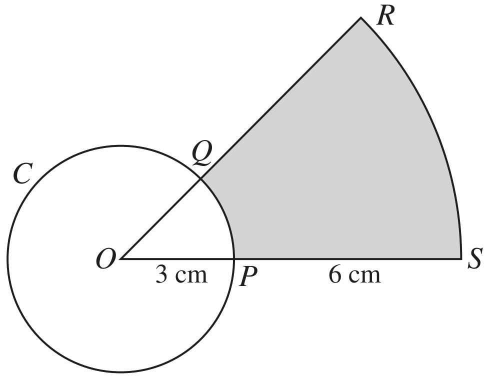
(i) Show that angle BOC is 0.9273 radians, correct to 4 decimal places. [2]

(ii) Find the perimeter of the shaded region. [3]

(iii) Find the area of the shaded region. [2]

Q60 : 9709_s13_qp_13_Q2

2



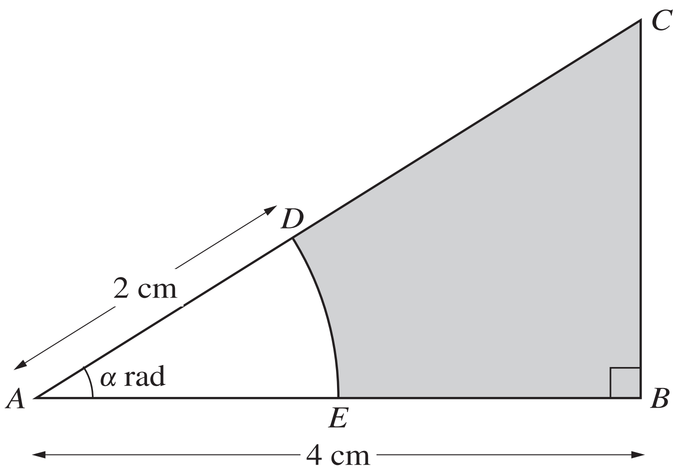
The diagram shows a circle C with centre O and radius 3 cm. The radii OP and OQ are extended to S and R respectively so that ORS is a sector of a circle with centre O. Given that $PS = 6$ cm and that the area of the shaded region is equal to the area of circle C,

(i) show that angle $POQ = \frac{1}{4}\pi$ radians, [3]

(ii) find the perimeter of the shaded region. [2]

Q61 : 9709_s14_qp_11_Q6

6



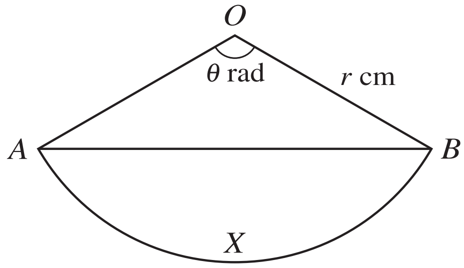
The diagram shows triangle ABC in which AB is perpendicular to BC. The length of AB is 4 cm and angle CAB is α radians. The arc DE with centre A and radius 2 cm meets AC at D and AB at E. Find, in terms of α,

(i) the area of the shaded region, [3]

(ii) the perimeter of the shaded region. [3]

Q62 : 9709_s14_qp_12_Q4

4



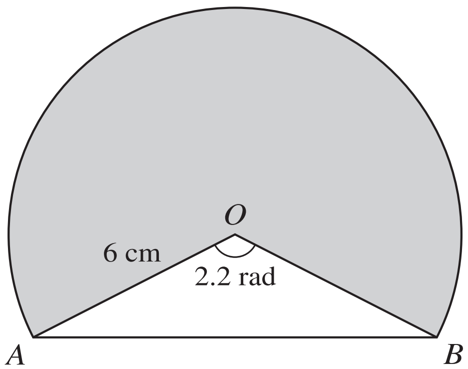
The diagram shows a sector of a circle with radius r cm and centre O. The chord AB divides the sector into a triangle AOB and a segment AXB. Angle AOB is θ radians.

(i) In the case where the areas of the triangle AOB and the segment AXB are equal, find the value of the constant p for which $\theta = p \sin \theta$. [2]

(ii) In the case where $r = 8$ and $\theta = 2.4$, find the perimeter of the segment AXB. [3]

Q63 : 9709_s14_qp_13_Q3

3



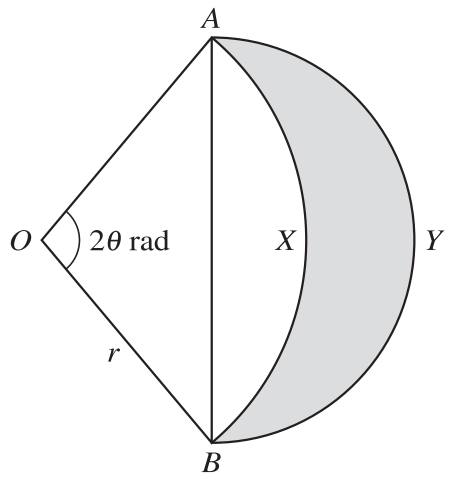
The diagram shows part of a circle with centre O and radius 6 cm. The chord AB is such that angle $AOB = 2.2$ radians. Calculate

(i) the perimeter of the shaded region, [3]

(ii) the ratio of the area of the shaded region to the area of the triangle AOB, giving your answer in the form $k : 1$. [3]

Q64 : 9709_s15_qp_12_Q2

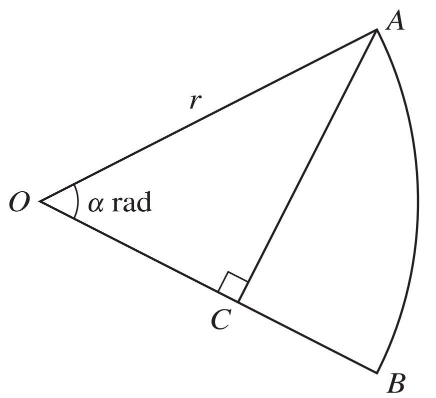
2



In the diagram, AYB is a semicircle with AB as diameter and $OAXB$ is a sector of a circle with centre O and radius r. Angle $AOB = 2\theta$ radians. Find an expression, in terms of r and θ, for the area of the shaded region. [4]

Q65 : 9709_s15_qp_13_Q11

11



In the diagram, OAB is a sector of a circle with centre O and radius r. The point C on OB is such that angle ACO is a right angle. Angle AOB is α radians and is such that AC divides the sector into two regions of equal area.

(i) Show that $\sin\alpha\cos\alpha = \frac{1}{2}\alpha$. [4]

It is given that the solution of the equation in part **(i)** is $\alpha = 0.9477$, correct to 4 decimal places.

(ii) Find the ratio

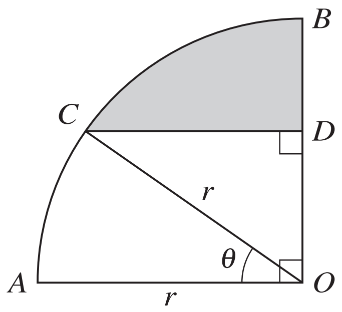
perimeter of region OAC : perimeter of region ACB,

giving your answer in the form $k : 1$, where k is given correct to 1 decimal place. [5]

(iii) Find angle AOB in degrees. [1]

Q66 : 9709_s16_qp_11_Q7

7



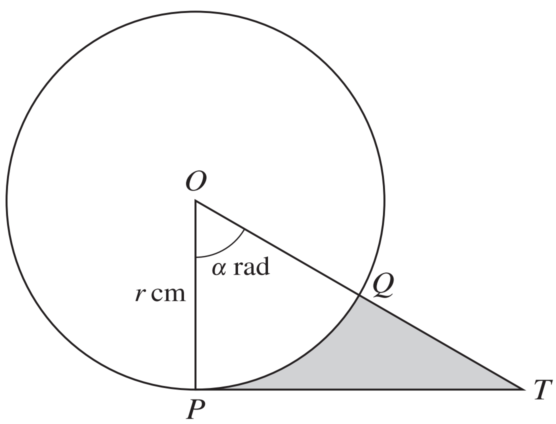
In the diagram, AOB is a quarter circle with centre O and radius r. The point C lies on the arc AB and the point D lies on OB. The line CD is parallel to AO and angle $AOC = \theta$ radians.

(i) Express the perimeter of the shaded region in terms of r, θ and π. [4]

(ii) For the case where $r = 5$ cm and $\theta = 0.6$, find the area of the shaded region. [3]

Q67 : 9709_s16_qp_12_Q6

6



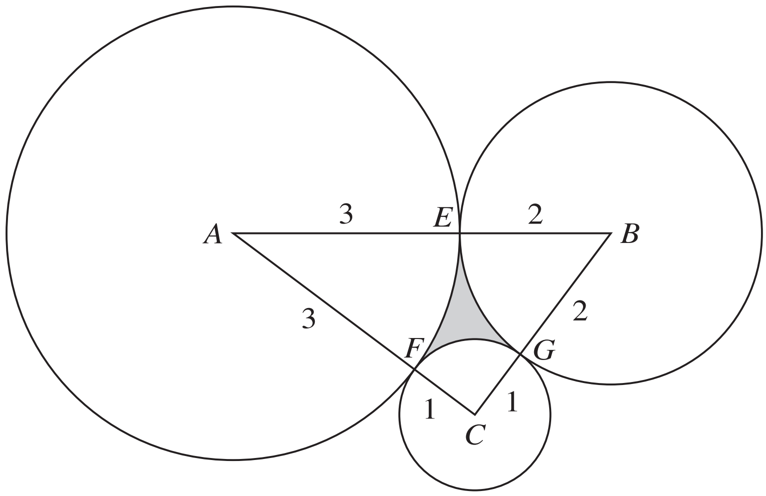
The diagram shows a circle with radius r cm and centre O. The line PT is the tangent to the circle at P and angle $POT = \alpha$ radians. The line OT meets the circle at Q.

(i) Express the perimeter of the shaded region PQT in terms of r and α. [3]

(ii) In the case where $\alpha = \frac{1}{3}\pi$ and $r = 10$, find the area of the shaded region correct to 2 significant figures. [3]

Q68 : 9709_s16_qp_13_Q6

6



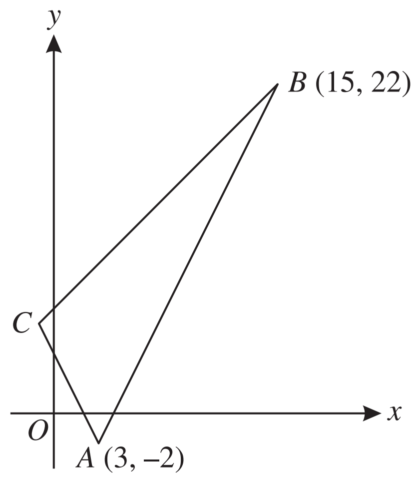
The diagram shows triangle ABC where $AB = 5$ cm, $AC = 4$ cm and $BC = 3$ cm. Three circles with centres at A, B and C have radii 3 cm, 2 cm and 1 cm respectively. The circles touch each other at points E, F and G, lying on AB, AC and BC respectively. Find the area of the shaded region EFG.

[7]

Coordinate geometry

Q69 : 9709_s10_qp_11_Q8

8



The diagram shows a triangle ABC in which A is $(3, -2)$ and B is $(15, 22)$. The gradients of AB, AC and BC are $2m$, $-2m$ and m respectively, where m is a positive constant.

(i) Find the gradient of AB and deduce the value of m. [2]

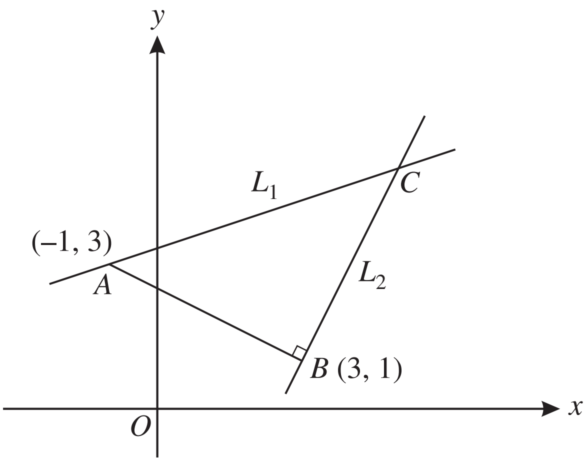
(ii) Find the coordinates of C. [4]

The perpendicular bisector of AB meets BC at D.

(iii) Find the coordinates of D. [4]

Q70 : 9709_s10_qp_12_Q4

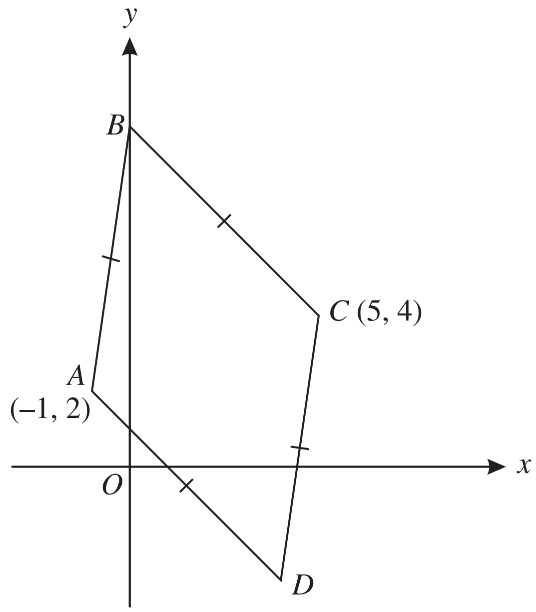
4



In the diagram, A is the point $(-1, 3)$ and B is the point $(3, 1)$. The line L_1 passes through A and is parallel to OB. The line L_2 passes through B and is perpendicular to AB. The lines L_1 and L_2 meet at C. Find the coordinates of C. [6]

Q71 : 9709_s10_qp_13_Q8

8



The diagram shows a rhombus $ABCD$ in which the point A is $(-1, 2)$, the point C is $(5, 4)$ and the point B lies on the y-axis. Find

(i) the equation of the perpendicular bisector of AC, [3]

(ii) the coordinates of B and D, [3]

(iii) the area of the rhombus. [3]

Q72 : 9709_s11_qp_11_Q10

10 **(i)** Express $2x^2 - 4x + 1$ in the form $a(x + b)^2 + c$ and hence state the coordinates of the minimum point, A, on the curve $y = 2x^2 - 4x + 1$. [4]

The line $x - y + 4 = 0$ intersects the curve $y = 2x^2 - 4x + 1$ at points P and Q. It is given that the coordinates of P are $(3, 7)$.

(ii) Find the coordinates of Q. [3]

(iii) Find the equation of the line joining Q to the mid-point of AP. [3]

Q73 : 9709_s11_qp_12_Q7

7 The line L_1 passes through the points $A\,(2, 5)$ and $B\,(10, 9)$. The line L_2 is parallel to L_1 and passes through the origin. The point C lies on L_2 such that AC is perpendicular to L_2. Find

(i) the coordinates of C, [5]

(ii) the distance AC. [2]

Q74 : 9709_s11_qp_13_Q3

3 The line $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are positive constants, meets the x-axis at P and the y-axis at Q. Given that $PQ = \sqrt{(45)}$ and that the gradient of the line PQ is $-\frac{1}{2}$, find the values of a and b. [5]

Q75 : 9709_s12_qp_11_Q9

9 The coordinates of A are $(-3, 2)$ and the coordinates of C are $(5, 6)$. The mid-point of AC is M and the perpendicular bisector of AC cuts the x-axis at B.

(i) Find the equation of MB and the coordinates of B. [5]

(ii) Show that AB is perpendicular to BC. [2]

(iii) Given that $ABCD$ is a square, find the coordinates of D and the length of AD. [2]

Q76 : 9709_s12_qp_12_Q4

4 The point A has coordinates $(-1, -5)$ and the point B has coordinates $(7, 1)$. The perpendicular bisector of AB meets the x-axis at C and the y-axis at D. Calculate the length of CD. [6]

Q77 : 9709_s12_qp_13_Q10

10 The equation of a line is $2y + x = k$, where k is a constant, and the equation of a curve is $xy = 6$.

(i) In the case where $k = 8$, the line intersects the curve at the points A and B. Find the equation of the perpendicular bisector of the line AB. [6]

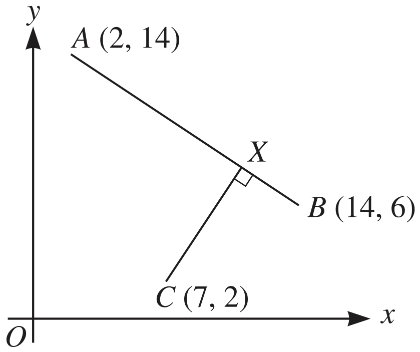
(ii) Find the set of values of k for which the line $2y + x = k$ intersects the curve $xy = 6$ at two distinct points. [3]

Q78 : 9709_s13_qp_12_Q7

7 The point R is the reflection of the point $(-1, 3)$ in the line $3y + 2x = 33$. Find by calculation the coordinates of R. [7]

Q79 : 9709_s13_qp_13_Q7

7



The diagram shows three points $A\,(2,\ 14)$, $B\,(14,\ 6)$ and $C\,(7,\ 2)$. The point X lies on AB, and CX is perpendicular to AB. Find, by calculation,

(i) the coordinates of X, [6]

(ii) the ratio $AX : XB$. [2]

Q80 : 9709_s14_qp_11_Q7

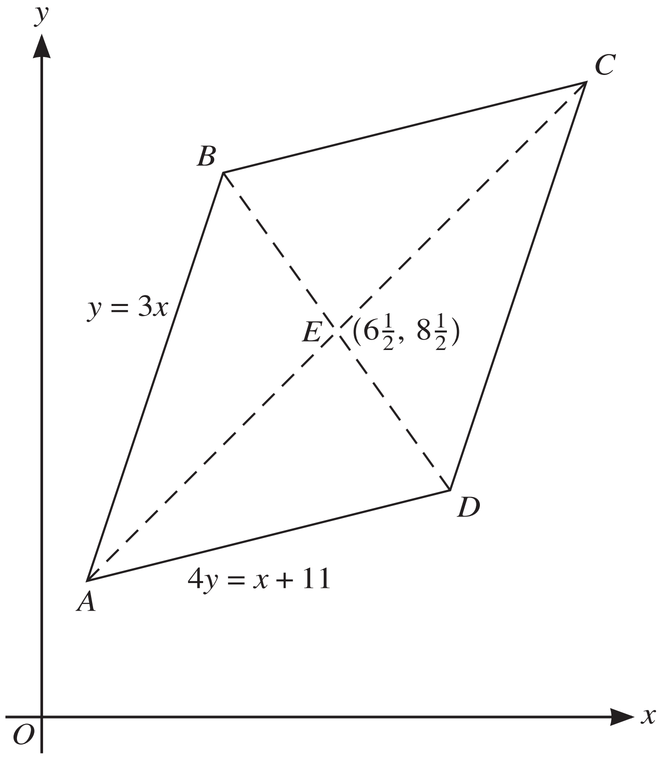
7 The coordinates of points A and B are $(a,\ 2)$ and $(3,\ b)$ respectively, where a and b are constants. The distance AB is $\sqrt{(125)}$ units and the gradient of the line AB is 2. Find the possible values of a and of b. [6]

Q81 : 9709_s14_qp_12_Q1

1 Find the coordinates of the point at which the perpendicular bisector of the line joining $(2,\ 7)$ to $(10,\ 3)$ meets the x-axis. [5]

Q82 : 9709_s14_qp_13_Q11

11



The diagram shows a parallelogram $ABCD$, in which the equation of AB is $y = 3x$ and the equation of AD is $4y = x + 11$. The diagonals AC and BD meet at the point $E\left(6\frac{1}{2}, 8\frac{1}{2}\right)$. Find, by calculation, the coordinates of A, B, C and D. [9]

Q83 : 9709_s15_qp_11_Q6

6 The line with gradient -2 passing through the point $P(3t, 2t)$ intersects the x-axis at A and the y-axis at B.

(i) Find the area of triangle AOB in terms of t. [3]

The line through P perpendicular to AB intersects the x-axis at C.

(ii) Show that the mid-point of PC lies on the line $y = x$. [4]

Q84 : 9709_s15_qp_12_Q7

7 The point C lies on the perpendicular bisector of the line joining the points $A(4, 6)$ and $B(10, 2)$. C also lies on the line parallel to AB through $(3, 11)$.

(i) Find the equation of the perpendicular bisector of AB. [4]

(ii) Calculate the coordinates of C. [3]

Q85 : `9709_s15_qp_13_Q7`

7 The point A has coordinates $(p, 1)$ and the point B has coordinates $(9, 3p + 1)$, where p is a constant.

(i) For the case where the distance AB is 13 units, find the possible values of p. [3]

(ii) For the case in which the line with equation $2x + 3y = 9$ is perpendicular to AB, find the value of p. [4]

Q86 : `9709_s16_qp_11_Q8`

8 A curve has equation $y = 3x - \frac{4}{x}$ and passes through the points $A\,(1, -1)$ and $B\,(4, 11)$. At each of the points C and D on the curve, the tangent is parallel to AB. Find the equation of the perpendicular bisector of CD. [7]

Q87 : `9709_s16_qp_12_Q8`

8 Three points have coordinates $A\,(0, 7)$, $B\,(8, 3)$ and $C\,(3k, k)$. Find the value of the constant k for which

(i) C lies on the line that passes through A and B, [4]

(ii) C lies on the perpendicular bisector of AB. [4]

Q88 : `9709_s16_qp_13_Q11`

11 Triangle ABC has vertices at $A\,(-2, -1)$, $B\,(4, 6)$ and $C\,(6, -3)$.

(i) Show that triangle ABC is isosceles and find the exact area of this triangle. [6]

(ii) The point D is the point on AB such that CD is perpendicular to AB. Calculate the x-coordinate of D. [6]

Equation of curve

Q89 : 9709_s10_qp_11_Q6

6 A curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 6$ and the point (9, 2) lies on the curve.

(i) Find the equation of the curve. [4]

(ii) Find the x-coordinate of the stationary point on the curve and determine the nature of the stationary point. [3]

Q90 : 9709_s11_qp_11_Q7

7 A curve is such that $\frac{dy}{dx} = \frac{3}{(1+2x)^2}$ and the point $(1, \frac{1}{2})$ lies on the curve.

(i) Find the equation of the curve. [4]

(ii) Find the set of values of x for which the gradient of the curve is less than $\frac{1}{3}$. [3]

Q91 : 9709_s11_qp_12_Q1

1 Find $\int \left(x^3 + \frac{1}{x^3}\right) dx$. [3]

Q92 : 9709_s11_qp_13_Q9

9 A curve is such that $\frac{dy}{dx} = \frac{2}{\sqrt{x}} - 1$ and P (9, 5) is a point on the curve.

(i) Find the equation of the curve. [4]

(ii) Find the coordinates of the stationary point on the curve. [3]

(iii) Find an expression for $\frac{d^2y}{dx^2}$ and determine the nature of the stationary point. [2]

(iv) The normal to the curve at P makes an angle of $\tan^{-1}k$ with the positive x-axis. Find the value of k. [2]

Q93 : 9709_s12_qp_13_Q9

9 A curve is such that $\frac{d^2y}{dx^2} = -4x$. The curve has a maximum point at (2, 12).

(i) Find the equation of the curve. [6]

A point P moves along the curve in such a way that the x-coordinate is increasing at 0.05 units per second.

(ii) Find the rate at which the y-coordinate is changing when $x = 3$, stating whether the y-coordinate is increasing or decreasing. [2]

Q94 : 9709_s13_qp_12_Q1

1 A curve is such that $\frac{dy}{dx} = \frac{6}{x^2}$ and (2, 9) is a point on the curve. Find the equation of the curve. [3]

Q95 : 9709_s13_qp_13_Q1

1 A curve is such that $\frac{dy}{dx} = \sqrt{(2x+5)}$ and (2, 5) is a point on the curve. Find the equation of the curve. [4]

Q96 : 9709_s14_qp_11_Q12

12 A curve is such that $\frac{dy}{dx} = x^{\frac{1}{2}} - x^{-\frac{1}{2}}$. The curve passes through the point $\left(4, \frac{2}{3}\right)$.

(i) Find the equation of the curve. [4]

(ii) Find $\frac{d^2y}{dx^2}$. [2]

(iii) Find the coordinates of the stationary point and determine its nature. [5]

Q97 : 9709_s14_qp_13_Q6

6 A curve is such that $\frac{dy}{dx} = \frac{12}{\sqrt{(4x+a)}}$, where a is a constant. The point $P(2, 14)$ lies on the curve and the normal to the curve at P is $3y + x = 5$.

(i) Show that $a = 8$. [3]

(ii) Find the equation of the curve. [4]

Q98 : 9709_s15_qp_12_Q1

1 The function f is such that $f'(x) = 5 - 2x^2$ and (3, 5) is a point on the curve $y = f(x)$. Find $f(x)$. [3]

Q99 : 9709_s15_qp_13_Q2

2 A curve is such that $\frac{dy}{dx} = (2x+1)^{\frac{1}{2}}$ and the point (4, 7) lies on the curve. Find the equation of the curve. [4]

Q100 : 9709_s16_qp_11_Q4

4 A curve is such that $\frac{dy}{dx} = 2 - 8(3x + 4)^{-\frac{1}{2}}$.

(i) A point P moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.3 units per second. Find the rate of change of the y-coordinate as P crosses the y-axis. [2]

The curve intersects the y-axis where $y = \frac{4}{3}$.

(ii) Find the equation of the curve. [4]

Q101 : 9709_s16_qp_12_Q2

2 A curve is such that $\frac{dy}{dx} = \frac{8}{(5 - 2x)^2}$. Given that the curve passes through $(2, 7)$, find the equation of the curve. [4]

Q102 : 9709_s16_qp_13_Q3

3 A curve is such that $\frac{dy}{dx} = 6x^2 + \frac{k}{x^3}$ and passes through the point $P\,(1, 9)$. The gradient of the curve at P is 2.

(i) Find the value of the constant k. [1]

(ii) Find the equation of the curve. [4]

Functions

Q103 : 9709_s10_qp_11_Q9

9 The function f is defined by $f : x \mapsto 2x^2 - 12x + 7$ for $x \in \mathbb{R}$.

(i) Express $f(x)$ in the form $a(x - b)^2 - c$. [3]

(ii) State the range of f. [1]

(iii) Find the set of values of x for which $f(x) < 21$. [3]

The function g is defined by $g : x \mapsto 2x + k$ for $x \in \mathbb{R}$.

(iv) Find the value of the constant k for which the equation $gf(x) = 0$ has two equal roots. [4]

Q104 : 9709_s10_qp_13_Q10

10 The function $f : x \mapsto 2x^2 - 8x + 14$ is defined for $x \in \mathbb{R}$.

(i) Find the values of the constant k for which the line $y + kx = 12$ is a tangent to the curve $y = f(x)$. [4]

(ii) Express $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(iii) Find the range of f. [1]

The function $g : x \mapsto 2x^2 - 8x + 14$ is defined for $x \geqslant A$.

(iv) Find the smallest value of A for which g has an inverse. [1]

(v) For this value of A, find an expression for $g^{-1}(x)$ in terms of x. [3]

Q105 : 9709_s11_qp_11_Q11

11 Functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto 2x + 1,$$
$$g : x \mapsto x^2 - 2.$$

(i) Find and simplify expressions for $fg(x)$ and $gf(x)$. [2]

(ii) Hence find the value of a for which $fg(a) = gf(a)$. [3]

(iii) Find the value of b $(b \neq a)$ for which $g(b) = b$. [2]

(iv) Find and simplify an expression for $f^{-1}g(x)$. [2]

The function h is defined by

$$h : x \mapsto x^2 - 2, \quad \text{for } x \leqslant 0.$$

(v) Find an expression for $h^{-1}(x)$. [2]

Q106 : `9709_s11_qp_12_Q6`

6 The function f is defined by $f : x \mapsto \frac{x+3}{2x-1}$, $x \in \mathbb{R}$, $x \neq \frac{1}{2}$.

(i) Show that $ff(x) = x$. [3]

(ii) Hence, or otherwise, obtain an expression for $f^{-1}(x)$. [2]

Q107 : `9709_s11_qp_13_Q10`

10 Functions f and g are defined by

$$f : x \mapsto 3x - 4, \quad x \in \mathbb{R},$$
$$g : x \mapsto 2(x-1)^3 + 8, \quad x > 1.$$

(i) Evaluate fg(2). [2]

(ii) Sketch in a single diagram the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between the graphs. [3]

(iii) Obtain an expression for $g'(x)$ and use your answer to explain why g has an inverse. [3]

(iv) Express each of $f^{-1}(x)$ and $g^{-1}(x)$ in terms of x. [4]

Q108 : `9709_s12_qp_11_Q8`

8 The function $f : x \mapsto x^2 - 4x + k$ is defined for the domain $x \geqslant p$, where k and p are constants.

(i) Express $f(x)$ in the form $(x + a)^2 + b + k$, where a and b are constants. [2]

(ii) State the range of f in terms of k. [1]

(iii) State the smallest value of p for which f is one-one. [1]

(iv) For the value of p found in part **(iii)**, find an expression for $f^{-1}(x)$ and state the domain of f^{-1}, giving your answers in terms of k. [4]

Q109 : 9709_s12_qp_12_Q10

10 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 2x + 5 \quad \text{for } x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto \frac{8}{x-3} \quad \text{for } x \in \mathbb{R},\ x \neq 3.$$

(i) Obtain expressions, in terms of x, for $\mathrm{f}^{-1}(x)$ and $\mathrm{g}^{-1}(x)$, stating the value of x for which $\mathrm{g}^{-1}(x)$ is not defined. [4]

(ii) Sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ on the same diagram, making clear the relationship between the two graphs. [3]

(iii) Given that the equation $\mathrm{fg}(x) = 5 - kx$, where k is a constant, has no solutions, find the set of possible values of k. [5]

Q110 : 9709_s12_qp_13_Q11

11 The function f is such that $\mathrm{f}(x) = 8 - (x-2)^2$, for $x \in \mathbb{R}$.

(i) Find the coordinates and the nature of the stationary point on the curve $y = \mathrm{f}(x)$. [3]

The function g is such that $\mathrm{g}(x) = 8 - (x-2)^2$, for $k \leqslant x \leqslant 4$, where k is a constant.

(ii) State the smallest value of k for which g has an inverse. [1]

For this value of k,

(iii) find an expression for $\mathrm{g}^{-1}(x)$, [3]

(iv) sketch, on the same diagram, the graphs of $y = \mathrm{g}(x)$ and $y = \mathrm{g}^{-1}(x)$. [3]

Q111 : 9709_s13_qp_11_Q8

8 **(i)** Express $2x^2 - 12x + 13$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

(ii) The function f is defined by $\mathrm{f}(x) = 2x^2 - 12x + 13$ for $x \geqslant k$, where k is a constant. It is given that f is a one-one function. State the smallest possible value of k. [1]

The value of k is now given to be 7.

(iii) Find the range of f. [1]

(iv) Find an expression for $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [5]

Q112 : 9709_s13_qp_12_Q9

9 A function f is defined by $f(x) = \dfrac{5}{1-3x}$, for $x \geqslant 1$.

(i) Find an expression for $f'(x)$. [2]

(ii) Determine, with a reason, whether f is an increasing function, a decreasing function or neither. [1]

(iii) Find an expression for $f^{-1}(x)$, and state the domain and range of f^{-1}. [5]

Q113 : 9709_s13_qp_13_Q10

10 The function f is defined by $f : x \mapsto 2x + k, x \in \mathbb{R}$, where k is a constant.

(i) In the case where $k = 3$, solve the equation $ff(x) = 25$. [2]

The function g is defined by $g : x \mapsto x^2 - 6x + 8, x \in \mathbb{R}$.

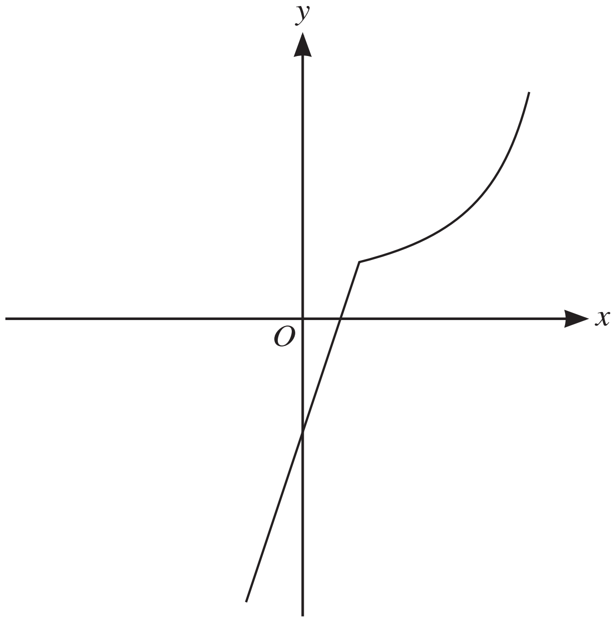
(ii) Find the set of values of k for which the equation $f(x) = g(x)$ has no real solutions. [3]

The function h is defined by $h : x \mapsto x^2 - 6x + 8, x > 3$.

(iii) Find an expression for $h^{-1}(x)$. [4]

Q114 : 9709_s14_qp_11_Q10

10



The diagram shows the function f defined for $-1 \leqslant x \leqslant 4$, where

$$f(x) = \begin{cases} 3x - 2 & \text{for } -1 \leqslant x \leqslant 1, \\ \dfrac{4}{5-x} & \text{for } 1 < x \leqslant 4. \end{cases}$$

(i) State the range of f. [1]

(ii) Copy the diagram and on your copy sketch the graph of $y = f^{-1}(x)$. [2]

(iii) Obtain expressions to define the function f^{-1}, giving also the set of values for which each expression is valid. [6]

Q115 : 9709_s14_qp_12_Q10

10 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 2x - 3, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto x^2 + 4x, \quad x \in \mathbb{R}.$$

(i) Solve the equation $\mathrm{ff}(x) = 11$. [2]

(ii) Find the range of g. [2]

(iii) Find the set of values of x for which $\mathrm{g}(x) > 12$. [3]

(iv) Find the value of the constant p for which the equation $\mathrm{gf}(x) = p$ has two equal roots. [3]

Function h is defined by $\mathrm{h} : x \mapsto x^2 + 4x$ for $x \geqslant k$, and it is given that h has an inverse.

(v) State the smallest possible value of k. [1]

(vi) Find an expression for $\mathrm{h}^{-1}(x)$. [4]

Q116 : 9709_s14_qp_13_Q5

5 A function f is such that $\mathrm{f}(x) = \dfrac{15}{2x + 3}$ for $0 \leqslant x \leqslant 6$.

(i) Find an expression for $\mathrm{f}'(x)$ and use your result to explain why f has an inverse. [3]

(ii) Find an expression for $\mathrm{f}^{-1}(x)$, and state the domain and range of f^{-1}. [4]

Q117 : 9709_s15_qp_11_Q8

8 The function $\mathrm{f} : x \mapsto 5 + 3\cos\left(\frac{1}{2}x\right)$ is defined for $0 \leqslant x \leqslant 2\pi$.

(i) Solve the equation $\mathrm{f}(x) = 7$, giving your answer correct to 2 decimal places. [3]

(ii) Sketch the graph of $y = \mathrm{f}(x)$. [2]

(iii) Explain why f has an inverse. [1]

(iv) Obtain an expression for $\mathrm{f}^{-1}(x)$. [3]

Q118 : 9709_s15_qp_12_Q11

11 The function f is defined by $f : x \mapsto 2x^2 - 6x + 5$ for $x \in \mathbb{R}$.

(i) Find the set of values of p for which the equation $f(x) = p$ has no real roots. [3]

The function g is defined by $g : x \mapsto 2x^2 - 6x + 5$ for $0 \leqslant x \leqslant 4$.

(ii) Express $g(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

(iii) Find the range of g. [2]

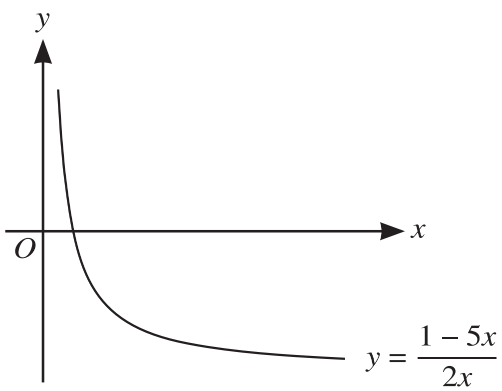
The function h is defined by $h : x \mapsto 2x^2 - 6x + 5$ for $k \leqslant x \leqslant 4$, where k is a constant.

(iv) State the smallest value of k for which h has an inverse. [1]

(v) For this value of k, find an expression for $h^{-1}(x)$. [3]

Q119 : 9709_s15_qp_13_Q6

6



The diagram shows the graph of $y = f^{-1}(x)$, where f^{-1} is defined by $f^{-1}(x) = \dfrac{1-5x}{2x}$ for $0 < x \leqslant 2$.

(i) Find an expression for $f(x)$ and state the domain of f. [5]

(ii) The function g is defined by $g(x) = \dfrac{1}{x}$ for $x \geqslant 1$. Find an expression for $f^{-1}g(x)$, giving your answer in the form $ax + b$, where a and b are constants to be found. [2]

Q120 : 9709_s16_qp_11_Q11

11 The function f is defined by $\mathrm{f} : x \mapsto 4\sin x - 1$ for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

(i) State the range of f. [2]

(ii) Find the coordinates of the points at which the curve $y = \mathrm{f}(x)$ intersects the coordinate axes. [3]

(iii) Sketch the graph of $y = \mathrm{f}(x)$. [2]

(iv) Obtain an expression for $\mathrm{f}^{-1}(x)$, stating both the domain and range of f^{-1}. [4]

Q121 : 9709_s16_qp_12_Q1

1 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 10 - 3x, \quad x \in \mathbb{R},$$

$$\mathrm{g} : x \mapsto \frac{10}{3 - 2x}, \quad x \in \mathbb{R},\ x \neq \frac{3}{2}.$$

Solve the equation $\mathrm{ff}(x) = \mathrm{gf}(2)$. [3]

Q122 : 9709_s16_qp_12_Q11

11 The function f is defined by $\mathrm{f} : x \mapsto 6x - x^2 - 5$ for $x \in \mathbb{R}$.

(i) Find the set of values of x for which $\mathrm{f}(x) \leqslant 3$. [3]

(ii) Given that the line $y = mx + c$ is a tangent to the curve $y = \mathrm{f}(x)$, show that $4c = m^2 - 12m + 16$. [3]

The function g is defined by $\mathrm{g} : x \mapsto 6x - x^2 - 5$ for $x \geqslant k$, where k is a constant.

(iii) Express $6x - x^2 - 5$ in the form $a - (x - b)^2$, where a and b are constants. [2]

(iv) State the smallest value of k for which g has an inverse. [1]

(v) For this value of k, find an expression for $\mathrm{g}^{-1}(x)$. [2]

Q123 : 9709_s16_qp_13_Q10

10 The function f is such that $f(x) = 2x + 3$ for $x \geqslant 0$. The function g is such that $g(x) = ax^2 + b$ for $x \leqslant q$, where a, b and q are constants. The function fg is such that $fg(x) = 6x^2 - 21$ for $x \leqslant q$.

(i) Find the values of a and b. [3]

(ii) Find the greatest possible value of q. [2]

It is now given that $q = -3$.

(iii) Find the range of fg. [1]

(iv) Find an expression for $(fg)^{-1}(x)$ and state the domain of $(fg)^{-1}$. [3]

Quadratic

Q124 : 9709_s10_qp_12_Q3

3 The functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto 4x - 2x^2,$$
$$g : x \mapsto 5x + 3.$$

(i) Find the range of f. [2]

(ii) Find the value of the constant k for which the equation $gf(x) = k$ has equal roots. [3]

Q125 : 9709_s11_qp_12_Q3

3 The equation $x^2 + px + q = 0$, where p and q are constants, has roots -3 and 5.

(i) Find the values of p and q. [2]

(ii) Using these values of p and q, find the value of the constant r for which the equation $x^2 + px + q + r = 0$ has equal roots. [3]

Q126 : 9709_s11_qp_13_Q2

2 Find the set of values of m for which the line $y = mx + 4$ intersects the curve $y = 3x^2 - 4x + 7$ at two distinct points. [5]

Q127 : 9709_s13_qp_11_Q7

7 A curve has equation $y = x^2 - 4x + 4$ and a line has equation $y = mx$, where m is a constant.

(i) For the case where $m = 1$, the curve and the line intersect at the points A and B. Find the coordinates of the mid-point of AB. [4]

(ii) Find the non-zero value of m for which the line is a tangent to the curve, and find the coordinates of the point where the tangent touches the curve. [5]

Q128 : 9709_s13_qp_12_Q3

3 The straight line $y = mx + 14$ is a tangent to the curve $y = \frac{12}{x} + 2$ at the point P. Find the value of the constant m and the coordinates of P. [5]

Q129 : 9709_s14_qp_11_Q2

2 **(i)** Express $4x^2 - 12x$ in the form $(2x + a)^2 + b$. [2]

(ii) Hence, or otherwise, find the set of values of x satisfying $4x^2 - 12x > 7$. [2]

Q130 : 9709_s14_qp_13_Q8

8 **(i)** Express $2x^2 - 10x + 8$ in the form $a(x+b)^2 + c$, where a, b and c are constants, and use your answer to state the minimum value of $2x^2 - 10x + 8$. [4]

(ii) Find the set of values of k for which the equation $2x^2 - 10x + 8 = kx$ has no real roots. [4]

Q131 : 9709_s15_qp_11_Q5

5 A piece of wire of length 24 cm is bent to form the perimeter of a sector of a circle of radius r cm.

(i) Show that the area of the sector, A cm^2, is given by $A = 12r - r^2$. [3]

(ii) Express A in the form $a - (r-b)^2$, where a and b are constants. [2]

(iii) Given that r can vary, state the greatest value of A and find the corresponding angle of the sector. [2]

Q132 : 9709_s15_qp_13_Q1

1 Express $2x^2 - 12x + 7$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

Q133 : 9709_s16_qp_11_Q6

6 **(a)** Find the values of the constant m for which the line $y = mx$ is a tangent to the curve $y = 2x^2 - 4x + 8$. [3]

(b) The function f is defined for $x \in \mathbb{R}$ by $f(x) = x^2 + ax + b$, where a and b are constants. The solutions of the equation $f(x) = 0$ are $x = 1$ and $x = 9$. Find

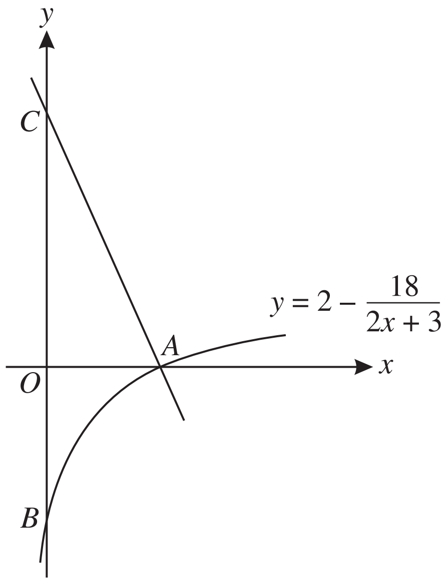
(i) the values of a and b, [2]

(ii) the coordinates of the vertex of the curve $y = f(x)$. [2]

Stationary point/Rate of Change

Q134 : 9709_s10_qp_11_Q7

7



The diagram shows part of the curve $y = 2 - \dfrac{18}{2x+3}$, which crosses the x-axis at A and the y-axis at B. The normal to the curve at A crosses the y-axis at C.

(i) Show that the equation of the line AC is $9x + 4y = 27$. [6]

(ii) Find the length of BC. [2]

Q135 : 9709_s10_qp_12_Q8

8 A solid rectangular block has a square base of side x cm. The height of the block is h cm and the total surface area of the block is 96 cm^2.

(i) Express h in terms of x and show that the volume, $V\text{ cm}^3$, of the block is given by

$$V = 24x - \tfrac{1}{2}x^3.$$ [3]

Given that x can vary,

(ii) find the stationary value of V, [3]

(iii) determine whether this stationary value is a maximum or a minimum. [2]

Q136 : 9709_s11_qp_11_Q2

2 The volume of a spherical balloon is increasing at a constant rate of 50 cm^3 per second. Find the rate of increase of the radius when the radius is 10 cm. [Volume of a sphere $= \frac{4}{3}\pi r^3$.] [4]

Q137 : 9709_s11_qp_11_Q6

6 The variables x, y and z can take only positive values and are such that

$$z = 3x + 2y \quad \text{and} \quad xy = 600.$$

(i) Show that $z = 3x + \frac{1200}{x}$. [1]

(ii) Find the stationary value of z and determine its nature. [6]

Q138 : 9709_s12_qp_11_Q10

10 It is given that a curve has equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = x^3 - 2x^2 + x$.

(i) Find the set of values of x for which the gradient of the curve is less than 5. [4]

(ii) Find the values of $\mathrm{f}(x)$ at the two stationary points on the curve and determine the nature of each stationary point. [5]

Q139 : 9709_s12_qp_11_Q4

4 A watermelon is assumed to be spherical in shape while it is growing. Its mass, M kg, and radius, r cm, are related by the formula $M = kr^3$, where k is a constant. It is also assumed that the radius is increasing at a constant rate of 0.1 centimetres per day. On a particular day the radius is 10 cm and the mass is 3.2 kg. Find the value of k and the rate at which the mass is increasing on this day. [5]

Q140 : 9709_s12_qp_12_Q2

2 The equation of a curve is $y = 4\sqrt{x} + \frac{2}{\sqrt{x}}$.

(i) Obtain an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(ii) A point is moving along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.12 units per second. Find the rate of change of the y-coordinate when $x = 4$. [2]

Q141 : 9709_s13_qp_11_Q9

9 A curve has equation $y = \mathrm{f}(x)$ and is such that $\mathrm{f}'(x) = 3x^{\frac{1}{2}} + 3x^{-\frac{1}{2}} - 10$.

(i) By using the substitution $u = x^{\frac{1}{2}}$, or otherwise, find the values of x for which the curve $y = \mathrm{f}(x)$ has stationary points. [4]

(ii) Find $\mathrm{f}''(x)$ and hence, or otherwise, determine the nature of each stationary point. [3]

(iii) It is given that the curve $y = \mathrm{f}(x)$ passes through the point $(4, -7)$. Find $\mathrm{f}(x)$. [4]

Q142 : 9709_s13_qp_12_Q8

8 The volume of a solid circular cylinder of radius r cm is 250π cm^3.

(i) Show that the total surface area, S cm^2, of the cylinder is given by

$$S = 2\pi r^2 + \frac{500\pi}{r}.$$ [2]

(ii) Given that r can vary, find the stationary value of S. [4]

(iii) Determine the nature of this stationary value. [2]

Q143 : 9709_s13_qp_13_Q6

6 The non-zero variables x, y and u are such that $u = x^2y$. Given that $y + 3x = 9$, find the stationary value of u and determine whether this is a maximum or a minimum value. [7]

Q144 : 9709_s14_qp_12_Q8

8 The equation of a curve is such that $\frac{d^2y}{dx^2} = 2x - 1$. Given that the curve has a minimum point at $(3, -10)$, find the coordinates of the maximum point. [8]

Q145 : 9709_s14_qp_13_Q9

9 The base of a cuboid has sides of length x cm and $3x$ cm. The volume of the cuboid is 288 cm^3.

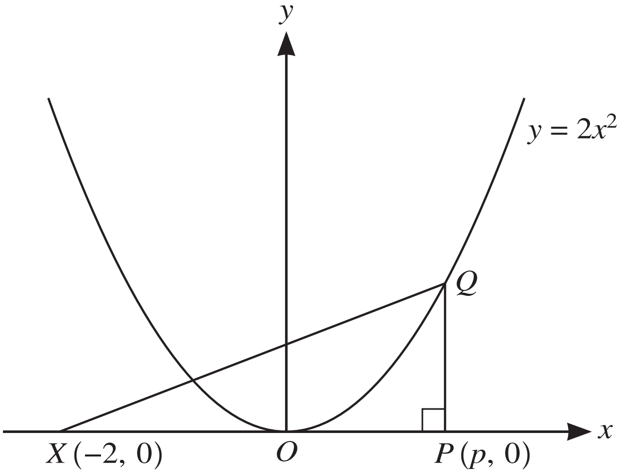
(i) Show that the total surface area of the cuboid, A cm^2, is given by

$$A = 6x^2 + \frac{768}{x}.$$ [3]

(ii) Given that x can vary, find the stationary value of A and determine its nature. [5]

Q146 : 9709_s15_qp_11_Q2

2



The diagram shows the curve $y = 2x^2$ and the points $X(-2, 0)$ and $P(p, 0)$. The point Q lies on the curve and PQ is parallel to the y-axis.

(i) Express the area, A, of triangle XPQ in terms of p. [2]

The point P moves along the x-axis at a constant rate of 0.02 units per second and Q moves along the curve so that PQ remains parallel to the y-axis.

(ii) Find the rate at which A is increasing when $p = 2$. [3]

Q147 : 9709_s15_qp_11_Q9

9 The equation of a curve is $y = x^3 + px^2$, where p is a positive constant.

(i) Show that the origin is a stationary point on the curve and find the coordinates of the other stationary point in terms of p. [4]

(ii) Find the nature of each of the stationary points. [3]

Another curve has equation $y = x^3 + px^2 + px$.

(iii) Find the set of values of p for which this curve has no stationary points. [3]

Q148 : 9709_s15_qp_12_Q4

4 Variables u, x and y are such that $u = 2x(y - x)$ and $x + 3y = 12$. Express u in terms of x and hence find the stationary value of u. [5]

Q149 : 9709_s15_qp_13_Q8

8 The function f is defined by $f(x) = \dfrac{1}{x+1} + \dfrac{1}{(x+1)^2}$ for $x > -1$.

(i) Find $f'(x)$. [3]

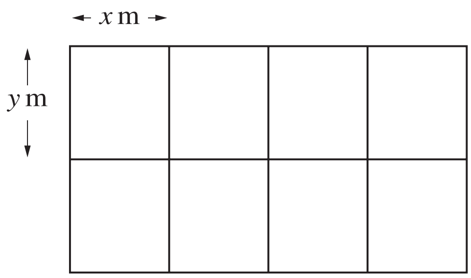
(ii) State, with a reason, whether f is an increasing function, a decreasing function or neither. [1]

The function g is defined by $g(x) = \dfrac{1}{x+1} + \dfrac{1}{(x+1)^2}$ for $x < -1$.

(iii) Find the coordinates of the stationary point on the curve $y = g(x)$. [4]

Q150 : 9709_s16_qp_11_Q5

5



A farmer divides a rectangular piece of land into 8 equal-sized rectangular sheep pens as shown in the diagram. Each sheep pen measures x m by y m and is fully enclosed by metal fencing. The farmer uses 480 m of fencing.

(i) Show that the total area of land used for the sheep pens, $A\ \text{m}^2$, is given by

$$A = 384x - 9.6x^2.$$ [3]

(ii) Given that x and y can vary, find the dimensions of each sheep pen for which the value of A is a maximum. (There is no need to verify that the value of A is a maximum.) [3]

Q151 : 9709_s16_qp_13_Q5

5 A curve has equation $y = 8x + (2x-1)^{-1}$. Find the values of x at which the curve has a stationary point and determine the nature of each stationary point, justifying your answers. [7]

Q152 : 9709_s16_qp_13_Q7

7 The point $P(x, y)$ is moving along the curve $y = x^2 - \frac{10}{3}x^{\frac{3}{2}} + 5x$ in such a way that the rate of change of y is constant. Find the values of x at the points at which the rate of change of x is equal to half the rate of change of y. [7]

Tangent/Normal

Q153 : 9709_s10_qp_12_Q10

10 The equation of a curve is $y = \frac{1}{6}(2x-3)^3 - 4x$.

(i) Find $\frac{dy}{dx}$. [3]

(ii) Find the equation of the tangent to the curve at the point where the curve intersects the y-axis. [3]

(iii) Find the set of values of x for which $\frac{1}{6}(2x-3)^3 - 4x$ is an increasing function of x. [3]

Q154 : 9709_s10_qp_13_Q5

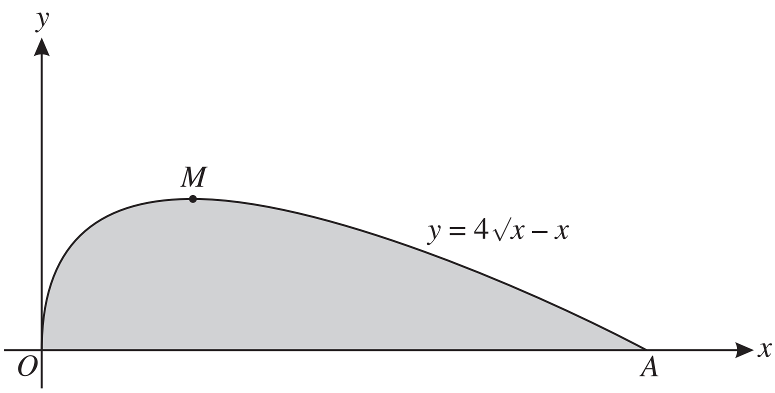
5 The equation of a curve is such that $\frac{dy}{dx} = \frac{6}{\sqrt{(3x-2)}}$. Given that the curve passes through the point $P(2, 11)$, find

(i) the equation of the normal to the curve at P, [3]

(ii) the equation of the curve. [4]

Q155 : 9709_s11_qp_12_Q11

11



The diagram shows part of the curve $y = 4\sqrt{x} - x$. The curve has a maximum point at M and meets the x-axis at O and A.

(i) Find the coordinates of A and M. [5]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the x-axis, giving your answer in terms of π. [6]

Q156 : 9709_s11_qp_12_Q4

4 A curve has equation $y = \frac{4}{3x-4}$ and $P(2, 2)$ is a point on the curve.

(i) Find the equation of the tangent to the curve at P. [4]

(ii) Find the angle that this tangent makes with the x-axis. [2]

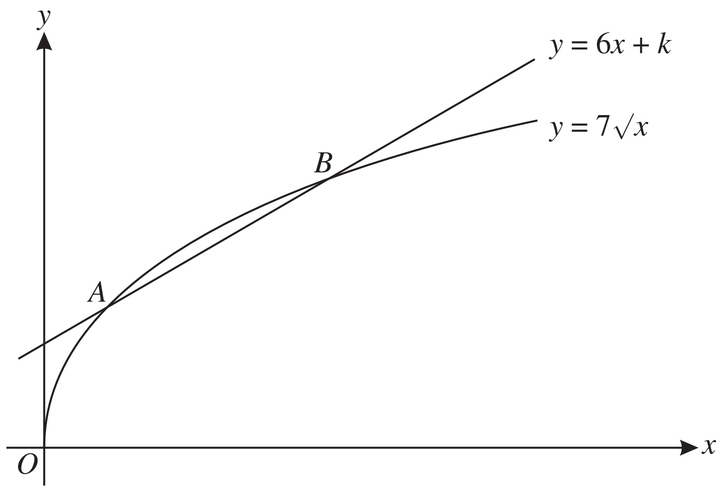
Q157 : 9709_s11_qp_13_Q4

4 **(a)** Differentiate $\dfrac{2x^3 + 5}{x}$ with respect to x. [3]

(b) Find $\int (3x - 2)^5 \, dx$ and hence find the value of $\int_0^1 (3x - 2)^5 \, dx$. [4]

Q158 : 9709_s12_qp_11_Q5

5



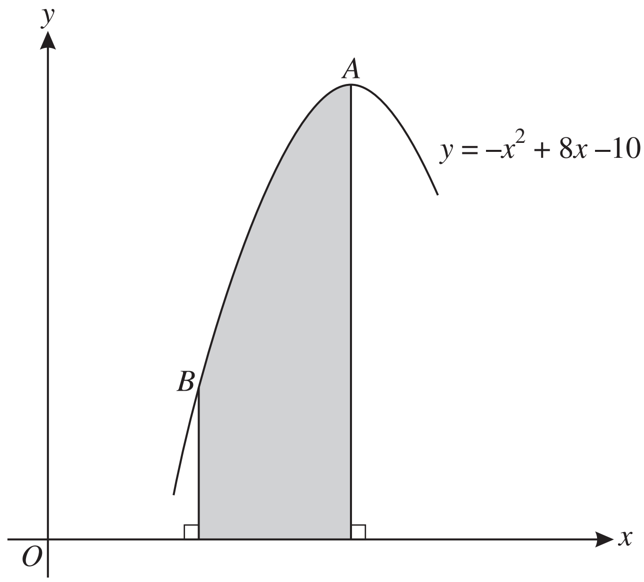
The diagram shows the curve $y = 7\sqrt{x}$ and the line $y = 6x + k$, where k is a constant. The curve and the line intersect at the points A and B.

(i) For the case where $k = 2$, find the x-coordinates of A and B. [4]

(ii) Find the value of k for which $y = 6x + k$ is a tangent to the curve $y = 7\sqrt{x}$. [2]

Q159 : 9709_s12_qp_12_Q9

9



The diagram shows part of the curve $y = -x^2 + 8x - 10$ which passes through the points A and B. The curve has a maximum point at A and the gradient of the line BA is 2.

(i) Find the coordinates of A and B. [7]

(ii) Find $\int y\,dx$ and hence evaluate the area of the shaded region. [4]

Q160 : 9709_s12_qp_13_Q7

7 The curve $y = \frac{10}{2x+1} - 2$ intersects the x-axis at A. The tangent to the curve at A intersects the y-axis at C.

(i) Show that the equation of AC is $5y + 4x = 8$. [5]

(ii) Find the distance AC. [2]

Q161 : 9709_s13_qp_11_Q1

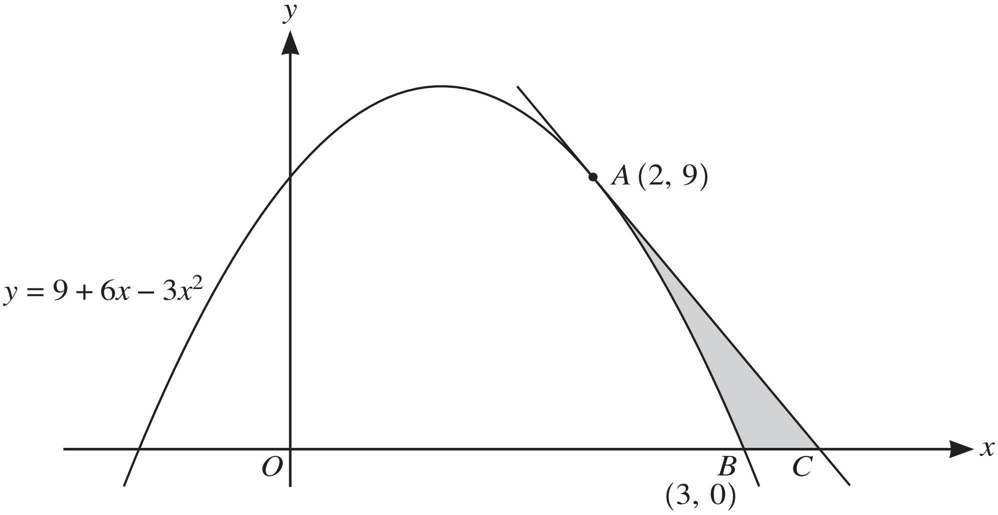
1 It is given that $f(x) = (2x-5)^3 + x$, for $x \in \mathbb{R}$. Show that f is an increasing function. [3]

Q162 : 9709_s14_qp_11_Q4

4 A curve has equation $y = \frac{4}{(3x+1)^2}$. Find the equation of the tangent to the curve at the point where the line $x = -1$ intersects the curve. [5]

Q163 : 9709_s15_qp_13_Q10

10



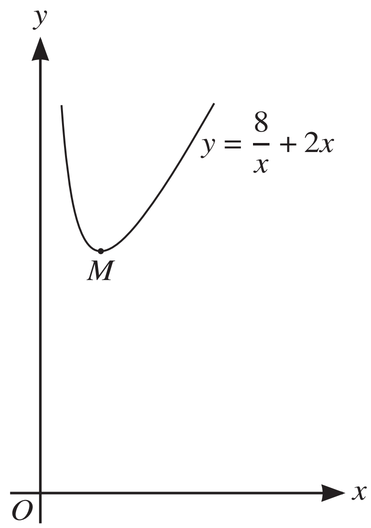
Points $A\,(2,\ 9)$ and $B\,(3,\ 0)$ lie on the curve $y = 9 + 6x - 3x^2$, as shown in the diagram. The tangent at A intersects the x-axis at C. Showing all necessary working,

(i) find the equation of the tangent AC and hence find the x-coordinate of C, [4]

(ii) find the area of the shaded region ABC. [5]

Q164 : 9709_s16_qp_12_Q10

10



The diagram shows the part of the curve $y = \dfrac{8}{x} + 2x$ for $x > 0$, and the minimum point M.

(i) Find expressions for $\dfrac{dy}{dx}$, $\dfrac{d^2y}{dx^2}$ and $\displaystyle\int y^2\,dx$. [5]

(ii) Find the coordinates of M and determine the coordinates and nature of the stationary point on the part of the curve for which $x < 0$. [5]

(iii) Find the volume obtained when the region bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$ is rotated through 360° about the x-axis. [2]

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Trigonometry

Q165 : 9709_s10_qp_11_Q1

1 The acute angle x radians is such that $\tan x = k$, where k is a positive constant. Express, in terms of k,

(i) $\tan(\pi - x)$, [1]

(ii) $\tan(\frac{1}{2}\pi - x)$, [1]

(iii) $\sin x$. [2]

Q166 : 9709_s10_qp_11_Q5

5 The function f is such that $f(x) = 2\sin^2 x - 3\cos^2 x$ for $0 \leqslant x \leqslant \pi$.

(i) Express $f(x)$ in the form $a + b\cos^2 x$, stating the values of a and b. [2]

(ii) State the greatest and least values of $f(x)$. [2]

(iii) Solve the equation $f(x) + 1 = 0$. [3]

Q167 : 9709_s10_qp_12_Q1

1 **(i)** Show that the equation

$$3(2\sin x - \cos x) = 2(\sin x - 3\cos x)$$

can be written in the form $\tan x = -\frac{3}{4}$. [2]

(ii) Solve the equation $3(2\sin x - \cos x) = 2(\sin x - 3\cos x)$, for $0° \leqslant x \leqslant 360°$. [2]

Q168 : 9709_s10_qp_12_Q11

11 The function $f : x \mapsto 4 - 3\sin x$ is defined for the domain $0 \leqslant x \leqslant 2\pi$.

(i) Solve the equation $f(x) = 2$. [3]

(ii) Sketch the graph of $y = f(x)$. [2]

(iii) Find the set of values of k for which the equation $f(x) = k$ has no solution. [2]

The function $g : x \mapsto 4 - 3\sin x$ is defined for the domain $\frac{1}{2}\pi \leqslant x \leqslant A$.

(iv) State the largest value of A for which g has an inverse. [1]

(v) For this value of A, find the value of $g^{-1}(3)$. [2]

Q169 : 9709_s10_qp_13_Q3

3 The function $f : x \mapsto a + b\cos x$ is defined for $0 \leqslant x \leqslant 2\pi$. Given that $f(0) = 10$ and that $f(\frac{2}{3}\pi) = 1$, find

(i) the values of a and b, [2]

(ii) the range of f, [1]

(iii) the exact value of $f(\frac{5}{6}\pi)$. [2]

Q170 : 9709_s10_qp_13_Q4

4 **(i)** Show that the equation $2\sin x \tan x + 3 = 0$ can be expressed as $2\cos^2 x - 3\cos x - 2 = 0$. [2]

(ii) Solve the equation $2\sin x \tan x + 3 = 0$ for $0° \leqslant x \leqslant 360°$. [3]

Q171 : 9709_s11_qp_11_Q5

5 **(i)** Show that the equation $2\tan^2\theta \sin^2\theta = 1$ can be written in the form

$$2\sin^4\theta + \sin^2\theta - 1 = 0.$$ [2]

(ii) Hence solve the equation $2\tan^2\theta \sin^2\theta = 1$ for $0° \leqslant \theta \leqslant 360°$. [4]

Q172 : 9709_s11_qp_12_Q5

5 **(i)** Prove the identity $\dfrac{\cos\theta}{\tan\theta(1-\sin\theta)} \equiv 1 + \dfrac{1}{\sin\theta}$. [3]

(ii) Hence solve the equation $\dfrac{\cos\theta}{\tan\theta(1-\sin\theta)} = 4$, for $0° \leqslant \theta \leqslant 360°$. [3]

Q173 : 9709_s11_qp_12_Q9

9 The function f is such that $f(x) = 3 - 4\cos^k x$, for $0 \leqslant x \leqslant \pi$, where k is a constant.

(i) In the case where $k = 2$,

(a) find the range of f, [2]

(b) find the exact solutions of the equation $f(x) = 1$. [3]

(ii) In the case where $k = 1$,

(a) sketch the graph of $y = f(x)$, [2]

(b) state, with a reason, whether f has an inverse. [1]

Q174 : 9709_s11_qp_13_Q8

8 **(i)** Prove the identity $\left(\dfrac{1}{\sin\theta} - \dfrac{1}{\tan\theta}\right)^2 \equiv \dfrac{1-\cos\theta}{1+\cos\theta}$. [3]

(ii) Hence solve the equation $\left(\dfrac{1}{\sin\theta} - \dfrac{1}{\tan\theta}\right)^2 = \frac{2}{5}$, for $0° \leqslant \theta \leqslant 360°$. [4]

Q175 : 9709_s12_qp_11_Q1

1 Solve the equation $\sin 2x = 2\cos 2x$, for $0° \leqslant x \leqslant 180°$. [4]

Q176 : 9709_s12_qp_12_Q5

5 (i) Prove the identity $\tan x + \dfrac{1}{\tan x} \equiv \dfrac{1}{\sin x \cos x}$. [2]

(ii) Solve the equation $\dfrac{2}{\sin x \cos x} = 1 + 3\tan x$, for $0° \leqslant x \leqslant 180°$. [4]

Q177 : 9709_s12_qp_13_Q1

1 (i) Prove the identity $\tan^2\theta - \sin^2\theta \equiv \tan^2\theta \sin^2\theta$. [3]

(ii) Use this result to explain why $\tan\theta > \sin\theta$ for $0° < \theta < 90°$. [1]

Q178 : 9709_s12_qp_13_Q4

4 (i) Solve the equation $\sin 2x + 3\cos 2x = 0$ for $0° \leqslant x \leqslant 360°$. [5]

(ii) How many solutions has the equation $\sin 2x + 3\cos 2x = 0$ for $0° \leqslant x \leqslant 1080°$? [1]

Q179 : 9709_s13_qp_11_Q5

5 (i) Show that $\dfrac{\sin\theta}{\sin\theta + \cos\theta} + \dfrac{\cos\theta}{\sin\theta - \cos\theta} \equiv \dfrac{1}{\sin^2\theta - \cos^2\theta}$. [3]

(ii) Hence solve the equation $\dfrac{\sin\theta}{\sin\theta + \cos\theta} + \dfrac{\cos\theta}{\sin\theta - \cos\theta} = 3$, for $0° \leqslant \theta \leqslant 360°$. [4]

Q180 : 9709_s13_qp_12_Q5

5 It is given that $a = \sin\theta - 3\cos\theta$ and $b = 3\sin\theta + \cos\theta$, where $0° \leqslant \theta \leqslant 360°$.

(i) Show that $a^2 + b^2$ has a constant value for all values of θ. [3]

(ii) Find the values of θ for which $2a = b$. [4]

Q181 : 9709_s13_qp_13_Q3

3 (i) Express the equation $2\cos^2\theta = \tan^2\theta$ as a quadratic equation in $\cos^2\theta$. [2]

(ii) Solve the equation $2\cos^2\theta = \tan^2\theta$ for $0 \leqslant \theta \leqslant \pi$, giving solutions in terms of π. [3]

Q182 : 9709_s13_qp_13_Q5

5 (i) Sketch, on the same diagram, the curves $y = \sin 2x$ and $y = \cos x - 1$ for $0 \leqslant x \leqslant 2\pi$. [4]

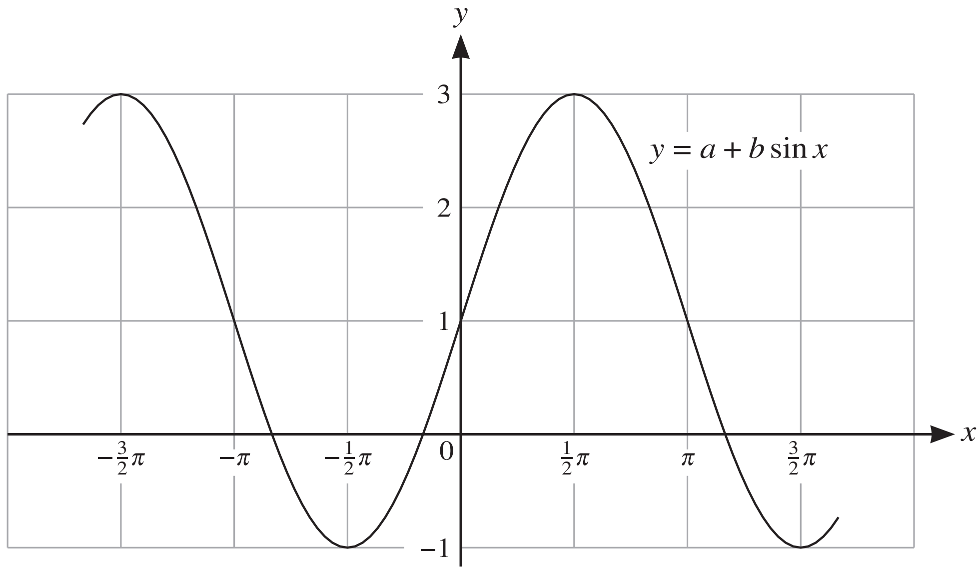
(ii) Hence state the number of solutions, in the interval $0 \leqslant x \leqslant 2\pi$, of the equations

(a) $2\sin 2x + 1 = 0$, [1]

(b) $\sin 2x - \cos x + 1 = 0$. [1]

Q183 : 9709_s14_qp_11_Q1

1



The diagram shows part of the graph of $y = a + b \sin x$. State the values of the constants a and b. [2]

Q184 : 9709_s14_qp_11_Q9

9 **(i)** Prove the identity $\dfrac{\sin\theta}{1-\cos\theta} - \dfrac{1}{\sin\theta} \equiv \dfrac{1}{\tan\theta}$. [4]

(ii) Hence solve the equation $\dfrac{\sin\theta}{1-\cos\theta} - \dfrac{1}{\sin\theta} = 4\tan\theta$ for $0° < \theta < 180°$. [3]

Q185 : 9709_s14_qp_12_Q3

3 The reflex angle θ is such that $\cos\theta = k$, where $0 < k < 1$.

(i) Find an expression, in terms of k, for

(a) $\sin\theta$, [2]

(b) $\tan\theta$. [1]

(ii) Explain why $\sin 2\theta$ is negative for $0 < k < 1$. [2]

Q186 : 9709_s14_qp_12_Q5

5 **(i)** Prove the identity $\dfrac{1}{\cos\theta} - \dfrac{\cos\theta}{1+\sin\theta} \equiv \tan\theta$. [4]

(ii) Solve the equation $\dfrac{1}{\cos\theta} - \dfrac{\cos\theta}{1+\sin\theta} + 2 = 0$ for $0° \leqslant \theta \leqslant 360°$. [3]

Q187 : 9709_s14_qp_13_Q4

4 **(i)** Prove the identity $\dfrac{\tan x + 1}{\sin x \tan x + \cos x} \equiv \sin x + \cos x$. [3]

(ii) Hence solve the equation $\dfrac{\tan x + 1}{\sin x \tan x + \cos x} = 3\sin x - 2\cos x$ for $0 \leqslant x \leqslant 2\pi$. [3]

Q188 : 9709_s15_qp_11_Q1

1 Given that θ is an obtuse angle measured in radians and that $\sin\theta = k$, find, in terms of k, an expression for

(i) $\cos\theta$, [1]

(ii) $\tan\theta$, [2]

(iii) $\sin(\theta + \pi)$. [1]

Q189 : 9709_s15_qp_12_Q5

5 **(i)** Prove the identity $\dfrac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} \equiv \dfrac{\tan\theta - 1}{\tan\theta + 1}$. [1]

(ii) Hence solve the equation $\dfrac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} = \dfrac{\tan\theta}{6}$, for $0° \leqslant \theta \leqslant 180°$. [4]

Q190 : 9709_s15_qp_12_Q6

6 A tourist attraction in a city centre is a big vertical wheel on which passengers can ride. The wheel turns in such a way that the height, h m, of a passenger above the ground is given by the formula $h = 60(1 - \cos kt)$. In this formula, k is a constant, t is the time in minutes that has elapsed since the passenger started the ride at ground level and kt is measured in radians.

(i) Find the greatest height of the passenger above the ground. [1]

One complete revolution of the wheel takes 30 minutes.

(ii) Show that $k = \frac{1}{15}\pi$. [2]

(iii) Find the time for which the passenger is above a height of 90 m. [3]

Q191 : 9709_s15_qp_13_Q4

4 **(i)** Express the equation $3\sin\theta = \cos\theta$ in the form $\tan\theta = k$ and solve the equation for $0° < \theta < 180°$. [2]

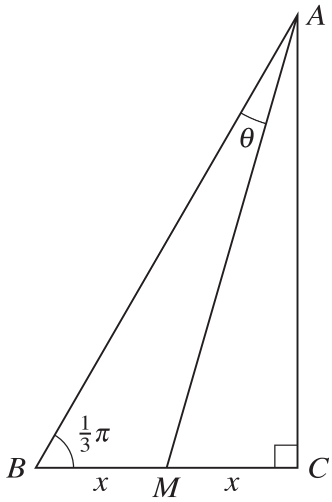
(ii) Solve the equation $3\sin^2 2x = \cos^2 2x$ for $0° < x < 180°$. [4]

Q192 : 9709_s16_qp_11_Q2

2 Solve the equation $3\sin^2\theta = 4\cos\theta - 1$ for $0° \leqslant \theta \leqslant 360°$. [4]

Q193 : 9709_s16_qp_12_Q5

5



In the diagram, triangle ABC is right-angled at C and M is the mid-point of BC. It is given that angle $ABC = \frac{1}{3}\pi$ radians and angle $BAM = \theta$ radians. Denoting the lengths of BM and MC by x,

(i) find AM in terms of x, [3]

(ii) show that $\theta = \frac{1}{6}\pi - \tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$. [2]

Q194 : 9709_s16_qp_12_Q7

7 **(i)** Prove the identity $\dfrac{1+\cos\theta}{1-\cos\theta} - \dfrac{1-\cos\theta}{1+\cos\theta} \equiv \dfrac{4}{\sin\theta\tan\theta}$. [4]

(ii) Hence solve, for $0° < \theta < 360°$, the equation

$$\sin\theta\left(\frac{1+\cos\theta}{1-\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta}\right) = 3.$$ [3]

Q195 : 9709_s16_qp_13_Q8

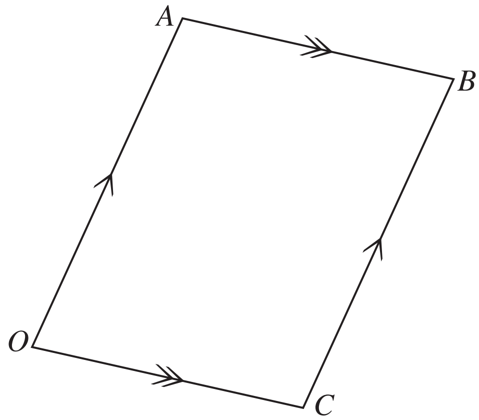
8 **(i)** Show that $3\sin x\tan x - \cos x + 1 = 0$ can be written as a quadratic equation in $\cos x$ and hence solve the equation $3\sin x\tan x - \cos x + 1 = 0$ for $0 \leqslant x \leqslant \pi$. [5]

(ii) Find the solutions to the equation $3\sin 2x\tan 2x - \cos 2x + 1 = 0$ for $0 \leqslant x \leqslant \pi$. [3]

Vectors

Q196 : 9709_s10_qp_11_Q10

10



The diagram shows the parallelogram $OABC$. Given that $\overrightarrow{OA} = \mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{OC} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$, find

(i) the unit vector in the direction of $\overrightarrow{OB}$, [3]

(ii) the acute angle between the diagonals of the parallelogram, [5]

(iii) the perimeter of the parallelogram, correct to 1 decimal place. [3]

Q197 : 9709_s10_qp_12_Q5

5 Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 1 \\ p \end{pmatrix}.$$

(i) Find the value of p for which $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OB}$. [2]

(ii) Find the values of p for which the magnitude of $\overrightarrow{AB}$ is 7. [4]

Q198 : 9709_s10_qp_13_Q6

6 Relative to an origin O, the position vectors of the points A, B and C are given by

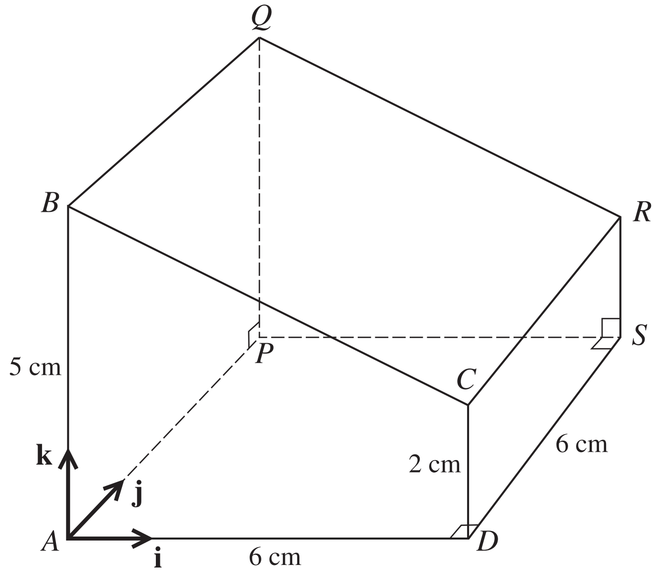
$$\overrightarrow{OA} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k}, \quad \overrightarrow{OB} = 3\mathbf{i} + 2\mathbf{j} + 8\mathbf{k}, \quad \overrightarrow{OC} = -\mathbf{i} - 2\mathbf{j} + 10\mathbf{k}.$$

(i) Use a scalar product to find angle ABC. [6]

(ii) Find the perimeter of triangle ABC, giving your answer correct to 2 decimal places. [2]

Q199 : 9709_s11_qp_11_Q4

4



The diagram shows a prism *ABCDPQRS* with a horizontal square base *APSD* with sides of length 6 cm. The cross-section *ABCD* is a trapezium and is such that the vertical edges *AB* and *DC* are of lengths 5 cm and 2 cm respectively. Unit vectors **i**, **j** and **k** are parallel to *AD*, *AP* and *AB* respectively.

(i) Express each of the vectors $\overrightarrow{CP}$ and $\overrightarrow{CQ}$ in terms of **i**, **j** and **k**. [2]

(ii) Use a scalar product to calculate angle *PCQ*. [4]

Q200 : 9709_s11_qp_12_Q8

8 Relative to the origin *O*, the position vectors of the points *A*, *B* and *C* are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 10 \\ 0 \\ 6 \end{pmatrix}.$$

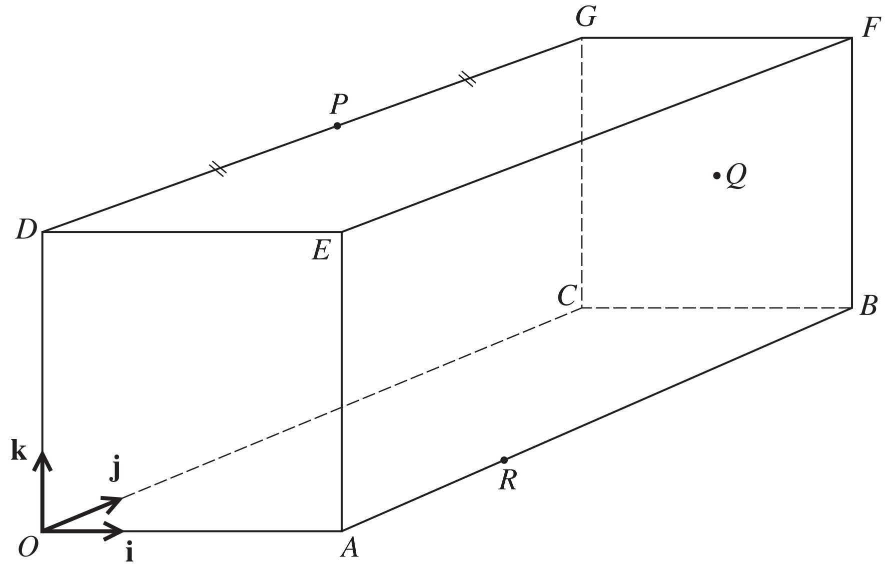
(i) Find angle *ABC*. [6]

The point *D* is such that *ABCD* is a parallelogram.

(ii) Find the position vector of *D*. [2]

Q201 : 9709_s11_qp_13_Q5

5



In the diagram, $OABCDEFG$ is a rectangular block in which $OA = OD = 6$ cm and $AB = 12$ cm. The unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to $\overrightarrow{OA}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively. The point P is the mid-point of DG, Q is the centre of the square face $CBFG$ and R lies on AB such that $AR = 4$ cm.

(i) Express each of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{RQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [3]

(ii) Use a scalar product to find angle RQP. [4]

Q202 : 9709_s12_qp_11_Q6

6 Two vectors $\mathbf{u}$ and $\mathbf{v}$ are such that $\mathbf{u} = \begin{pmatrix} p^2 \\ -2 \\ 6 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 2 \\ p-1 \\ 2p+1 \end{pmatrix}$, where p is a constant.

(i) Find the values of p for which $\mathbf{u}$ is perpendicular to $\mathbf{v}$. [3]

(ii) For the case where $p = 1$, find the angle between the directions of $\mathbf{u}$ and $\mathbf{v}$. [4]

Q203 : 9709_s12_qp_12_Q8

8 **(i)** Find the angle between the vectors $3\mathbf{i} - 4\mathbf{k}$ and $2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$. [4]

The vector $\overrightarrow{OA}$ has a magnitude of 15 units and is in the same direction as the vector $3\mathbf{i} - 4\mathbf{k}$. The vector $\overrightarrow{OB}$ has a magnitude of 14 units and is in the same direction as the vector $2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$.

(ii) Express $\overrightarrow{OA}$ and $\overrightarrow{OB}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [3]

(iii) Find the unit vector in the direction of $\overrightarrow{AB}$. [3]

Q204 : 9709_s12_qp_13_Q2

2 Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ 3 \\ p \end{pmatrix}.$$

Find

(i) the unit vector in the direction of $\overrightarrow{AB}$, [3]

(ii) the value of the constant p for which angle $BOC = 90°$. [2]

Q205 : 9709_s13_qp_11_Q6

6 Relative to an origin O, the position vectors of three points, A, B and C, are given by

$$\overrightarrow{OA} = \mathbf{i} + 2p\mathbf{j} + q\mathbf{k}, \quad \overrightarrow{OB} = q\mathbf{j} - 2p\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = -(4p^2 + q^2)\mathbf{i} + 2p\mathbf{j} + q\mathbf{k},$$

where p and q are constants.

(i) Show that $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OC}$ for all non-zero values of p and q. [2]

(ii) Find the magnitude of $\overrightarrow{CA}$ in terms of p and q. [2]

(iii) For the case where $p = 3$ and $q = 2$, find the unit vector parallel to $\overrightarrow{BA}$. [3]

Q206 : 9709_s13_qp_12_Q6

6 Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 3\mathbf{i} + p\mathbf{j} + q\mathbf{k},$$

where p and q are constants.

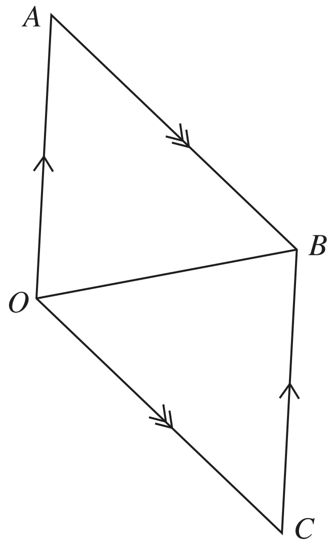
(i) State the values of p and q for which $\overrightarrow{OA}$ is parallel to $\overrightarrow{OB}$. [2]

(ii) In the case where $q = 2p$, find the value of p for which angle BOA is 90°. [2]

(iii) In the case where $p = 1$ and $q = 8$, find the unit vector in the direction of $\overrightarrow{AB}$. [3]

Q207 : 9709_s13_qp_13_Q8

8



The diagram shows a parallelogram *OABC* in which

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 3 \\ -4 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}.$$

(i) Use a scalar product to find angle *BOC*. [6]

(ii) Find a vector which has magnitude 35 and is parallel to the vector $\overrightarrow{OC}$. [2]

Q208 : 9709_s14_qp_11_Q8

8 Relative to an origin *O*, the position vectors of points *A* and *B* are given by

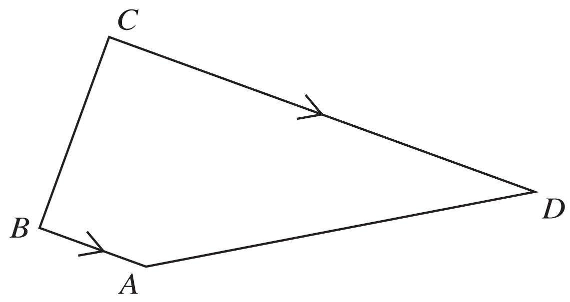
$$\overrightarrow{OA} = \begin{pmatrix} 3p \\ 4 \\ p^2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} -p \\ -1 \\ p^2 \end{pmatrix}.$$

(i) Find the values of p for which angle *AOB* is 90°. [3]

(ii) For the case where $p = 3$, find the unit vector in the direction of $\overrightarrow{BA}$. [3]

Q209 : 9709_s14_qp_12_Q7

7



The diagram shows a trapezium $ABCD$ in which BA is parallel to CD. The position vectors of A, B and C relative to an origin O are given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}.$$

(i) Use a scalar product to show that AB is perpendicular to BC. [3]

(ii) Given that the length of CD is 12 units, find the position vector of D. [4]

Q210 : 9709_s14_qp_13_Q7

7 The position vectors of points A, B and C relative to an origin O are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 6 \\ -1 \\ 7 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix}.$$

(i) Show that angle $BAC = \cos^{-1}\left(\frac{1}{3}\right)$. [5]

(ii) Use the result in part **(i)** to find the exact value of the area of triangle ABC. [3]

Q211 : 9709_s15_qp_11_Q4

4 Relative to the origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 6 \\ -3 \\ 2 \end{pmatrix}.$$

(i) Find the cosine of angle AOB. [3]

The position vector of C is given by $\overrightarrow{OC} = \begin{pmatrix} k \\ -2k \\ 2k-3 \end{pmatrix}$.

(ii) Given that AB and OC have the same length, find the possible values of k. [4]

Q212 : 9709_s15_qp_12_Q9

9 Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = 2\mathbf{i} + 4\mathbf{j} + 4\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 3\mathbf{i} + \mathbf{j} + 4\mathbf{k}.$$

(i) Use a vector method to find angle AOB. [4]

The point C is such that $\overrightarrow{AB} = \overrightarrow{BC}$.

(ii) Find the unit vector in the direction of $\overrightarrow{OC}$. [4]

(iii) Show that triangle OAC is isosceles. [1]

Q213 : 9709_s15_qp_13_Q5

5 Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 5 \\ -1 \\ -2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 6 \\ 1 \\ 2 \end{pmatrix}.$$

(i) Show that angle ABC is 90°. [4]

(ii) Find the area of triangle ABC, giving your answer correct to 1 decimal place. [3]

Q214 : 9709_s16_qp_11_Q10

10 Relative to an origin O, the position vectors of points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 5 \\ -1 \\ k \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix}$$

respectively, where k is a constant.

(i) Find the value of k in the case where angle $AOB = 90°$. [2]

(ii) Find the possible values of k for which the lengths of AB and OC are equal. [4]

The point D is such that $\overrightarrow{OD}$ is in the same direction as $\overrightarrow{OA}$ and has magnitude 9 units. The point E is such that $\overrightarrow{OE}$ is in the same direction as $\overrightarrow{OC}$ and has magnitude 14 units.

(iii) Find the magnitude of $\overrightarrow{DE}$ in the form $\sqrt{n}$ where n is an integer. [4]

Q215 : 9709_s16_qp_12_Q3

3 Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = 2\mathbf{i} - 5\mathbf{j} - 2\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 4\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}.$$

The point C is such that $\overrightarrow{AB} = \overrightarrow{BC}$. Find the unit vector in the direction of $\overrightarrow{OC}$. [4]

Q216 : 9709_s16_qp_13_Q9

9 The position vectors of A, B and C relative to an origin O are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 5 \\ p \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix},$$

where p is a constant.

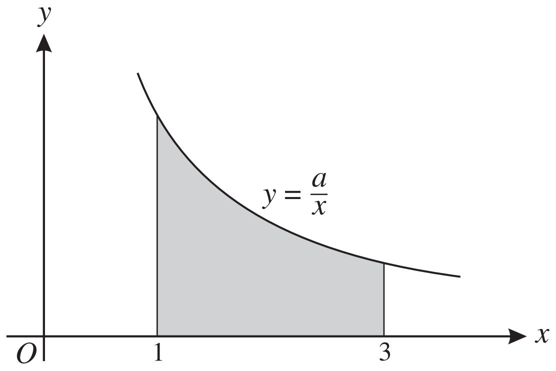
(i) Find the value of p for which the lengths of AB and CB are equal. [4]

(ii) For the case where $p = 1$, use a scalar product to find angle ABC. [4]

Volume of revolution

Q217 : 9709_s10_qp_12_Q2

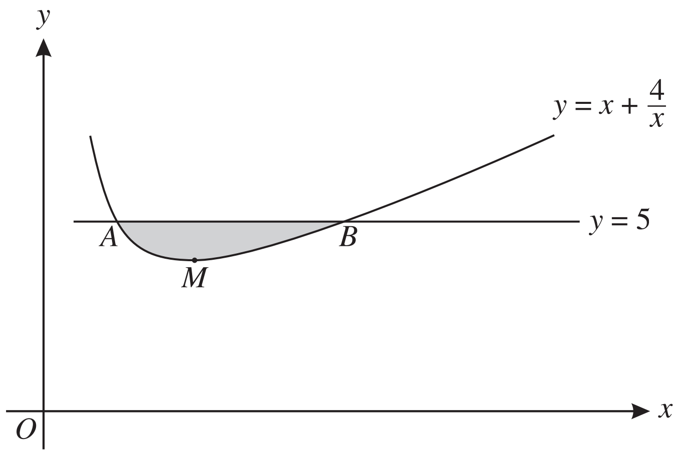
2



The diagram shows part of the curve $y = \frac{a}{x}$, where a is a positive constant. Given that the volume obtained when the shaded region is rotated through 360° about the x-axis is 24π, find the value of a. [4]

Q218 : 9709_s10_qp_13_Q9

9



The diagram shows part of the curve $y = x + \frac{4}{x}$ which has a minimum point at M. The line $y = 5$ intersects the curve at the points A and B.

(i) Find the coordinates of A, B and M. [5]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the x-axis. [6]

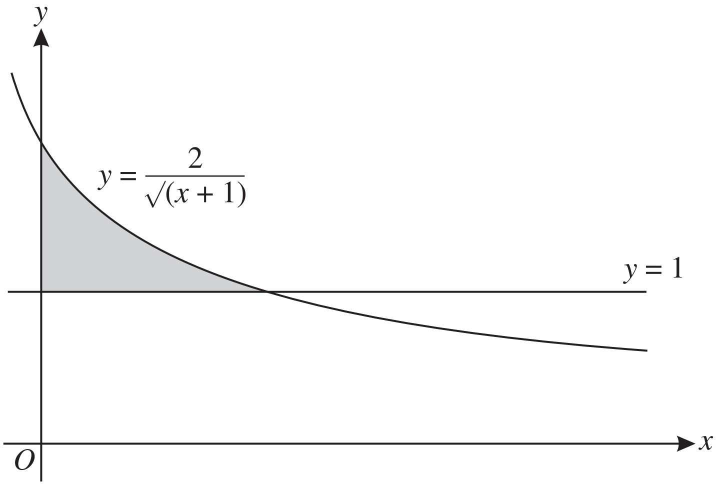
Q219 : 9709_s11_qp_11_Q3

3 **(i)** Sketch the curve $y = (x - 2)^2$. [1]

(ii) The region enclosed by the curve, the x-axis and the y-axis is rotated through 360° about the x-axis. Find the volume obtained, giving your answer in terms of π. [4]

Q220 : 9709_s12_qp_11_Q11

11



The diagram shows the line $y = 1$ and part of the curve $y = \frac{2}{\sqrt{(x+1)}}$.

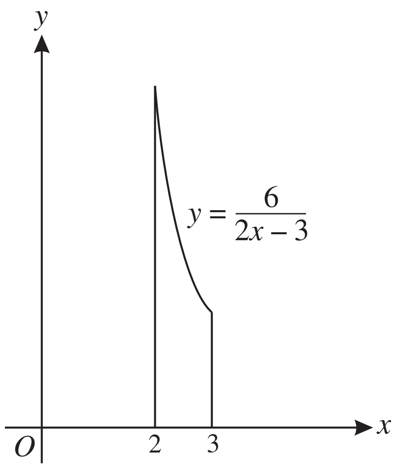
(i) Show that the equation $y = \frac{2}{\sqrt{(x+1)}}$ can be written in the form $x = \frac{4}{y^2} - 1$. [1]

(ii) Find $\int \left(\frac{4}{y^2} - 1\right) \mathrm{d}y$. Hence find the area of the shaded region. [5]

(iii) The shaded region is rotated through 360° about the ***y*-axis**. Find the exact value of the volume of revolution obtained. [5]

Q221 : 9709_s12_qp_12_Q1

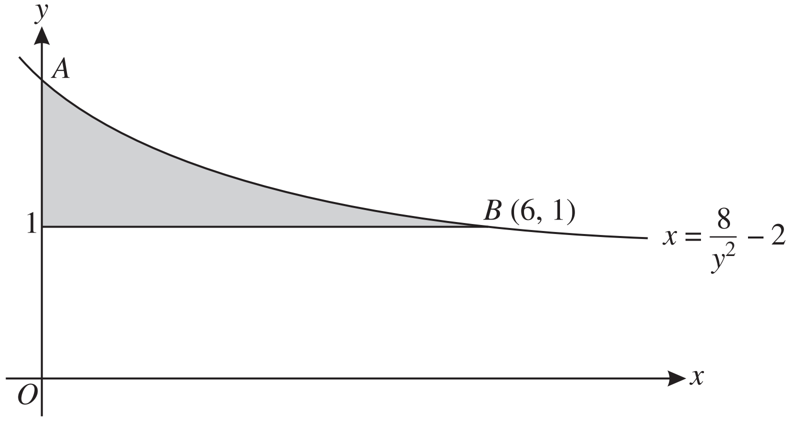
1



The diagram shows the region enclosed by the curve $y = \frac{6}{2x-3}$, the x-axis and the lines $x = 2$ and $x = 3$. Find, in terms of π, the volume obtained when this region is rotated through 360° about the x-axis. [4]

Q222 : 9709_s12_qp_13_Q5

5



The diagram shows part of the curve $x = \frac{8}{y^2} - 2$, crossing the y-axis at the point A. The point $B\,(6, 1)$ lies on the curve. The shaded region is bounded by the curve, the y-axis and the line $y = 1$. Find the exact volume obtained when this shaded region is rotated through 360° about the ***y*-axis**. [6]

Q223 : 9709_s15_qp_12_Q10

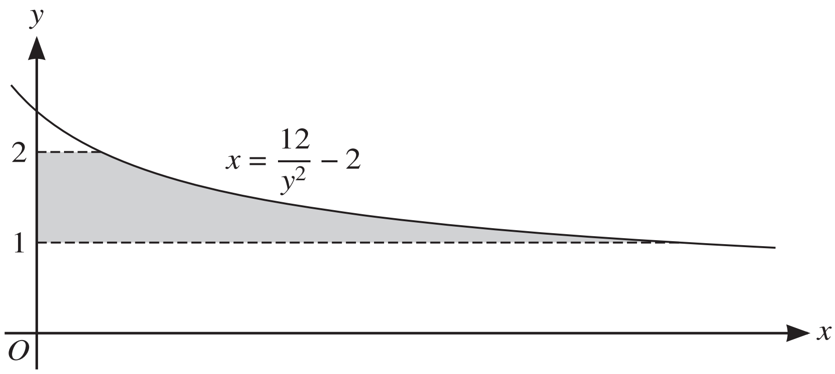
10 The equation of a curve is $y = \frac{4}{2x - 1}$.

(i) Find, showing all necessary working, the volume obtained when the region bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$ is rotated through 360° about the x-axis. [4]

(ii) Given that the line $2y = x + c$ is a normal to the curve, find the possible values of the constant c. [6]

Q224 : 9709_s16_qp_11_Q3

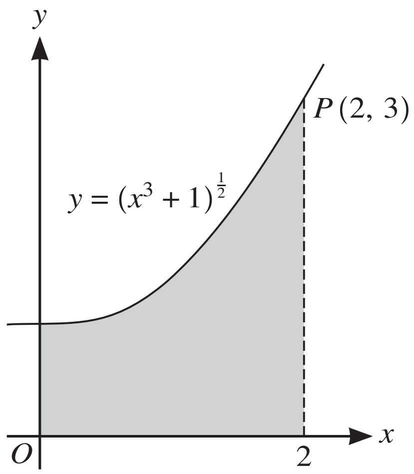
3



The diagram shows part of the curve $x = \frac{12}{y^2} - 2$. The shaded region is bounded by the curve, the y-axis and the lines $y = 1$ and $y = 2$. Showing all necessary working, find the volume, in terms of π, when this shaded region is rotated through 360° about the y-axis. [5]

Q225 : 9709_s16_qp_13_Q2

2



The diagram shows part of the curve $y = (x^3 + 1)^{\frac{1}{2}}$ and the point $P\,(2, 3)$ lying on the curve. Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [4]

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