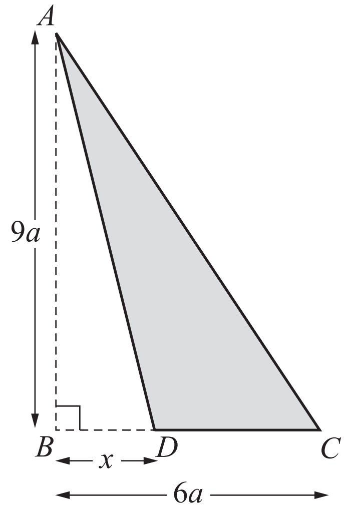
1 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The string is held taut with OP making an angle α with the downward vertical, where $\cos\alpha = \frac{2}{3}$. The particle P is projected perpendicular to OP in an upwards direction with speed $\sqrt{3ag}$. It then starts to move along a circular path in a vertical plane.

Find the cosine of the angle between the string and the upward vertical when the string first becomes slack. [4]



A uniform lamina is in the form of a triangle ABC in which angle B is a right angle, $AB = 9a$ and $BC = 6a$. The point D is on BC such that $BD = x$ (see diagram). The region ABD is removed from the lamina. The resulting shape ADC is placed with the edge DC on a horizontal surface and the plane ADC is vertical.

Find the set of values of x, in terms of a, for which the shape is in equilibrium. [6]

3 One end of a light elastic string, of natural length a and modulus of elasticity $\frac{16}{3}Mg$, is attached to a fixed point O. A particle P of mass $4M$ is attached to the other end of the string and hangs vertically in equilibrium. Another particle of mass $2M$ is attached to P and the combined particle is then released from rest. The speed of the combined particle when it has descended a distance $\frac{1}{4}a$ is v.

Find an expression for v in terms of g and a. [6]

4 A particle P of mass 5 kg moves along a horizontal straight line. At time t s, the velocity of P is $v\,\mathrm{m\,s^{-1}}$ and its displacement from a fixed point O on the line is x m. The forces acting on P are a force of magnitude $\frac{500}{v}$ N in the direction OP and a resistive force of magnitude $\frac{1}{2}v^2$ N. When $t = 0$, $x = 0$ and $v = 5$.

(a) Find an expression for v in terms of x. [6]

(b) State the value that the speed approaches for large values of x. [1]

5 A particle P is projected with speed $u\,\mathrm{m\,s^{-1}}$ at an angle of θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t s are denoted by x m and y m respectively.

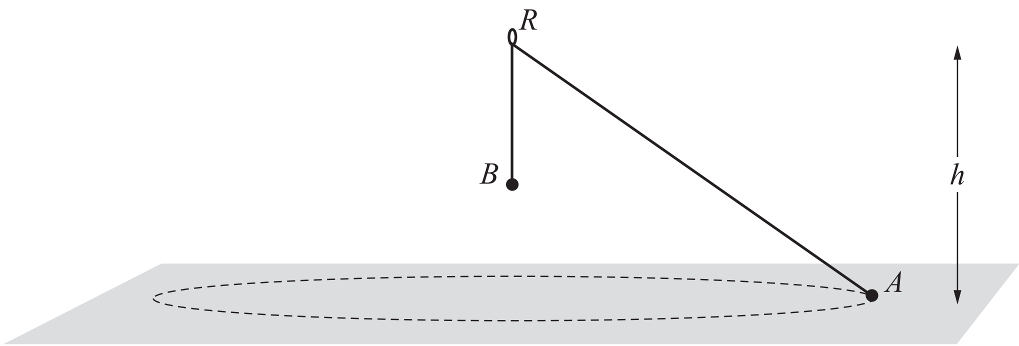
(a) Show that the equation of the trajectory is given by

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta).$$ [4]

In the subsequent motion P passes through the point with coordinates (30, 20).

(b) Given that one possible value of $\tan\theta$ is $\frac{4}{3}$, find the other possible value of $\tan\theta$. [5]

6



A light inextensible string is threaded through a fixed smooth ring R which is at a height h above a smooth horizontal surface. One end of the string is attached to a particle A of mass m. The other end of the string is attached to a particle B of mass $\frac{6}{7}m$. The particle A moves in a horizontal circle on the surface. The particle B hangs in equilibrium below the ring and above the surface (see diagram).

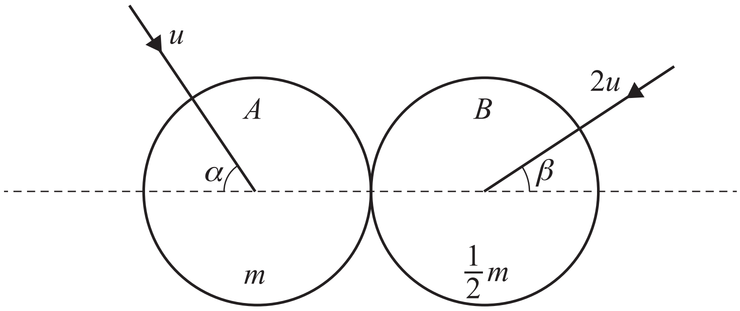
When A has constant angular speed ω, the angle between AR and BR is θ and the normal reaction between A and the surface is N.

When A has constant angular speed $\frac{3}{2}\omega$, the angle between AR and BR is α and the normal reaction between A and the surface is $\frac{1}{2}N$.

(a) Show that $\cos\theta = \frac{4}{9}\cos\alpha$. [5]

(b) Find N in terms of m and g and find the value of $\cos\alpha$. [4]

7



Two uniform smooth spheres A and B of equal radii have masses m and $\frac{1}{2}m$ respectively. The two spheres are moving on a horizontal surface when they collide. Immediately before the collision, sphere A is travelling with speed u and its direction of motion makes an angle α with the line of centres. Sphere B is travelling with speed $2u$ and its direction of motion makes an angle β with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{5}{8}$ and $\alpha + \beta = 90°$.

(a) Find the component of the velocity of B parallel to the line of centres after the collision, giving your answer in terms of u and α. [4]

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The direction of motion of B after the collision is parallel to the direction of motion of A before the collision.

(b) Find the value of $\tan\alpha$. [5]