1 **(a)** Find the set of values of k for which the system of equations

$$x + 2y + 3z = 1,$$
$$kx + 4y + 6z = 0,$$
$$7x + 8y + 9z = 3,$$

has a unique solution. [3]

(b) Interpret the situation geometrically in the case where the system of equations does not have a unique solution. [2]

2 A curve has equation

$$(x+1)y+y^2=2.$$

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{2}{3}$ at the point $(0,-2)$. [3]

(b) Find the value of $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ at the point $(0,-2)$. [4]

3 **(a)** A curve has equation $y = e^x + \frac{1}{4}e^{-x}$, for $0 \leqslant x \leqslant 1$. Find, in terms of π and e, the area of the surface generated when the curve is rotated through 2π radians about the x-axis. [6]

(b) Using standard results from the list of formulae (MF19), or otherwise, find the Maclaurin's series for $e^x + \frac{1}{4}e^{-x}$ up to and including the term in x^2. [2]

4 Find the solution of the differential equation

$$(4t^2 - 1)\frac{dx}{dt} + 4x = 4t^2 - 1$$

for which $x = 3$ when $t = 1$. Give your answer in the form $x = f(t)$. [9]

5 **(a)** Write down the fourth roots of unity. [1]

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(b) Use de Moivre's theorem to show that

$$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1.$$ [4]

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(c) Hence obtain the real roots of the equation

$$16(8x^4 - 8x^2 + 1)^4 - 9 = 0$$

in the form $\cos(q\pi)$, where q is a rational number. [5]

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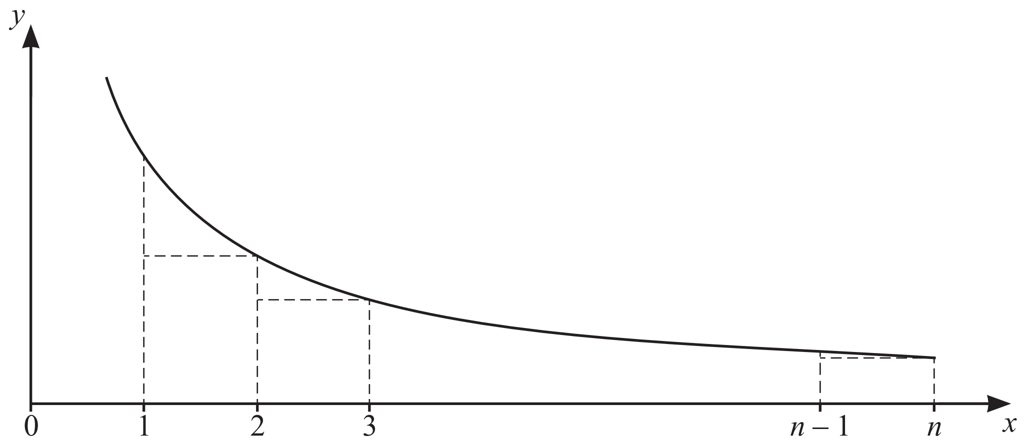

The diagram shows the curve $y = \frac{1}{\sqrt{x^2 + 2x}}$ for $x > 0$, together with a set of $(n-1)$ rectangles of unit width.

By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^{n} \frac{1}{\sqrt{r^2 + 2r}} < \ln\left(n + 1 + \sqrt{n^2 + 2n}\right) + \tfrac{1}{3}\sqrt{3} - \ln\left(2 + \sqrt{3}\right).$$ [10]

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7 **(a)** It is given that λ is an eigenvalue of the non-singular square matrix $\mathbf{A}$, with corresponding eigenvector $\mathbf{e}$.

Show that λ^{-1} is an eigenvalue of $\mathbf{A}^{-1}$ for which $\mathbf{e}$ is a corresponding eigenvector. [2]

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The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & 3 \\ 15 & -4 & 3 \\ 3 & 0 & 2 \end{pmatrix}.$$

(b) Given that -1 is an eigenvalue of $\mathbf{A}$, find a corresponding eigenvector. [2]

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(c) It is also given that $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ are eigenvectors of $\mathbf{A}$. Find the corresponding eigenvalues. [2]

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(d) Hence find a matrix **P** and a diagonal matrix **D** such that $\mathbf{A}^{-1} = \mathbf{PDP}^{-1}$. [2]

(e) Use the characteristic equation of **A** to show that $\mathbf{A}^{-1} = p\mathbf{A}^2 + q\mathbf{I}$, where p and q are rational numbers to be determined. [4]

8 It is given that $y = \cosh u$, where $u > 0$, and

$$\sqrt{\cosh^2 u - 1}\left(\frac{d^2u}{dx^2} + \frac{du}{dx}\right) + \cosh u\left(\frac{du}{dx}\right)^2 - 2\cosh u = 4e^{-x}.$$

(a) Show that

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 4e^{-x}.$$ [4]

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(b) Find u in terms of x, given that, when $x = 0$, $u = \ln 3$ and $\frac{du}{dx} = 3$. [10]

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