



2 (a) Show that the system of equations

$$\begin{aligned}x - y + 2z &= 4, \\x - y - 3z &= a, \\x - y + 7z &= 13,\end{aligned}$$

where  $a$  is a constant, does not have a unique solution. [2]

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(b) Given that  $a = -5$ , show that the system of equations in part (a) is consistent. Interpret this situation geometrically. [3]

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(c) Given instead that  $a \neq -5$ , show that the system of equations in part (a) is inconsistent. Interpret this situation geometrically. [2]

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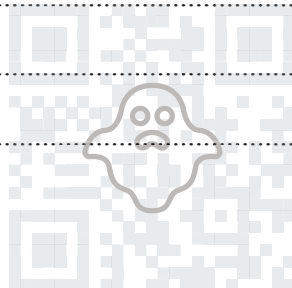
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4 (a) Starting from the definitions of cosh and sinh in terms of exponentials, prove that

$$\cosh^2 x - \sinh^2 x = 1. \quad [3]$$

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(b) Show that  $\frac{d}{dx}(\tan^{-1}(\sinh x)) = \operatorname{sech} x.$  [3]

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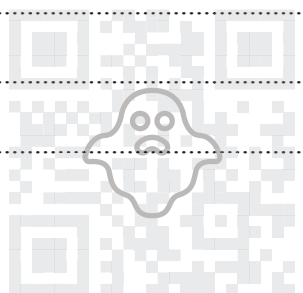
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- (c) Sketch the graph of  $y = \operatorname{sech} x$ , stating the equation of the asymptote. [2]

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- (d) By considering a suitable set of  $n$  rectangles of unit width, use your sketch to show that

$$\sum_{r=1}^n \operatorname{sech} r < \tan^{-1}(\sinh n). \quad [3]$$

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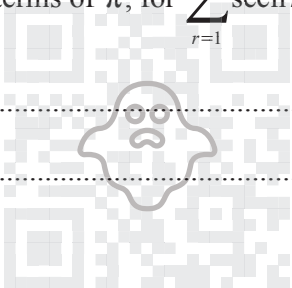
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- (e) Hence state an upper bound, in terms of  $\pi$ , for  $\sum_{r=1}^{\infty} \operatorname{sech} r$ . [1]

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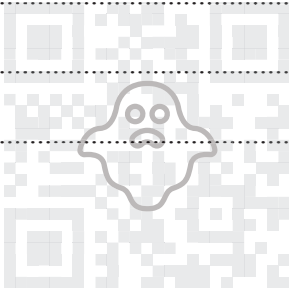
(b) Use the characteristic equation of **A** to show that

$$\mathbf{A}^4 = a\mathbf{A}^2 + b\mathbf{I},$$

where *a* and *b* are integers to be determined.

[4]

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(d) Hence find  $\sum_{k=0}^{6m-1} 2^k \sin\left(\frac{1}{3}k\pi\right)$  in terms of  $m$ . [2]

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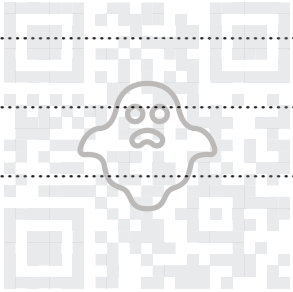
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- 8 (a) Use the substitution  $u = 1 - (\theta - 1)^2$  to find

$$\int \frac{\theta - 1}{\sqrt{1 - (\theta - 1)^2}} d\theta. \quad [3]$$

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- (b) Find the solution of the differential equation

$$\theta \frac{dy}{d\theta} - y = \theta^2 \sin^{-1}(\theta - 1),$$

where  $0 < \theta < 2$ , given that  $y = 1$  when  $\theta = 1$ . Give your answer in the form  $y = f(\theta)$ . [11]

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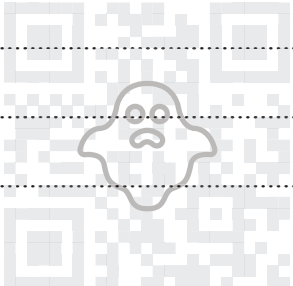
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Handwriting practice lines consisting of 24 horizontal dotted lines.

