1 **(a)** Use the list of formulae (MF19) to find $\sum_{r=1}^{n} r(r+2)$ in terms of n, simplifying your answer. [2]

(b) Express $\frac{1}{r(r+2)}$ in partial fractions and hence find $\sum_{r=1}^{n} \frac{1}{r(r+2)}$ in terms of n. [4]

(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{r(r+2)}$. [1]

2 The equation $x^4 + 3x^2 + 2x + 6 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

(a) Find a quartic equation whose roots are $\frac{1}{\alpha^2}, \frac{1}{\beta^2}, \frac{1}{\gamma^2}, \frac{1}{\delta^2}$ and state the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} + \frac{1}{\delta^2}$. [4]

(b) Find the value of $\beta^2\gamma^2\delta^2 + \alpha^2\gamma^2\delta^2 + \alpha^2\beta^2\delta^2 + \alpha^2\beta^2\gamma^2$. [3]

(c) Find the value of $\frac{1}{\alpha^4} + \frac{1}{\beta^4} + \frac{1}{\gamma^4} + \frac{1}{\delta^4}$. [2]

3 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}\begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix}$, where k is a constant and $k \neq 0$ or 1.

(a) The matrix **M** represents a sequence of two geometrical transformations.

State the type of each transformation, and make clear the order in which they are applied. [2]

....

....

....

(b) Write $\mathbf{M}^{-1}$ as the product of two matrices, neither of which is **I**. [2]

....

....

....

....

....

(c) Show that the invariant points of the transformation represented by **M** lie on the line $y = \frac{k^2}{1-k}x$. [4]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(d) The triangle ABC in the x-y plane is transformed by $\mathbf{M}$ onto triangle DEF.

Find the value of k for which the area of triangle DEF is equal to the area of triangle ABC. [2]

4 The function f is such that $f''(x) = f(x)$.

Prove by mathematical induction that, for every positive integer n,

$$\frac{d^{2n-1}}{dx^{2n-1}}\big(x\,f(x)\big) = x\,f'(x) + (2n-1)\,f(x).$$ [7]

5 The curve C has polar equation $r = a\sec^2\theta$, where a is a positive constant and $0 \leqslant \theta \leqslant \frac{1}{4}\pi$.

(a) Sketch C, stating the polar coordinates of the point of intersection of C with the initial line and also with the half-line $\theta = \frac{1}{4}\pi$. [3]

....

....

(b) Find the maximum distance of a point of C from the initial line. [2]

....

....

....

....

(c) Find the area of the region enclosed by C, the initial line and the half-line $\theta = \frac{1}{4}\pi$. [4]

....

....

....

....

....

....

....

....

....

....

(d) Find, in the form $y = f(x)$, the Cartesian equation of C. [3]

6 The lines l_1 and l_2 have equations $\mathbf{r} = 2\mathbf{i} + \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ and $\mathbf{r} = 2\mathbf{j} + 6\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$ respectively.

The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2.

(a) Find the length PQ. [5]

The plane Π_1 contains PQ and l_1.

The plane Π_2 contains PQ and l_2.

(b) **(i)** Write down an equation of Π_1, giving your answer in the form $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$. [1]

....

....

(ii) Find an equation of Π_2, giving your answer in the form $ax + by + cz = d$. [4]

....

....

....

....

....

....

....

....

(c) Find the acute angle between Π_1 and Π_2. [5]

....

....

....

....

....

....

....

....

....

....

....

....

....

7 The curve C has equation $y = \frac{x^2 - x}{x + 1}$.

(a) Find the equations of the asymptotes of C. [3]

(b) Find the exact coordinates of the stationary points on C. [4]

(c) Sketch C, stating the coordinates of any intersections with the axes. [3]

....

(d) Sketch the curve with equation $y = \left|\dfrac{x^2 - x}{x+1}\right|$ and find in exact form the set of values of x for which $\left|\dfrac{x^2 - x}{x+1}\right| < 6$. [5]