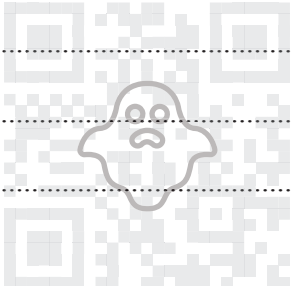


- 1 The number, x , of pine trees was counted in each of 40 randomly chosen regions of equal size in country A . The number, y , of pine trees was counted in each of 60 randomly chosen regions of the same equal size in country B . The results are summarised as follows.

$$\Sigma x = 752 \quad \Sigma x^2 = 14320 \quad \Sigma y = 1548 \quad \Sigma y^2 = 40200$$

Find a 95% confidence interval for the difference between the mean number of pine trees in regions of this size in countries A and B . [7]



- 2 It is claimed that the heights of a particular age group of boys follow a normal distribution with mean 125 cm and standard deviation 12 cm. Observations for a randomly chosen group of 60 boys in this age group are summarised in the following table. The table also gives the expected frequencies, correct to 2 decimal places, based on the normal distribution with mean 125 cm and standard deviation 12 cm.

Height, x cm	$x < 100$	$100 \leq x < 110$	$110 \leq x < 120$	$120 \leq x < 130$	$130 \leq x < 140$	$x \geq 140$
Observed frequency	0	3	15	23	11	8
Expected frequency	1.12	5.22	13.97	19.38	13.97	6.34

- (a) Show how the expected frequency for $130 \leq x < 140$ is obtained. [2]

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- (b) Carry out a goodness of fit test, at the 5% significance level, to determine whether the claim is supported by the data. [6]

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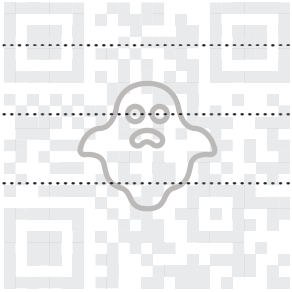
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Handwriting practice area consisting of multiple horizontal dotted lines for writing.



3 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} a + \frac{1}{5}x & 0 \leq x < 1, \\ 2a - \frac{1}{5}x & 1 \leq x \leq 2, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(a) Find the value of a . [3]

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(b) Find $E(X^2)$. [2]

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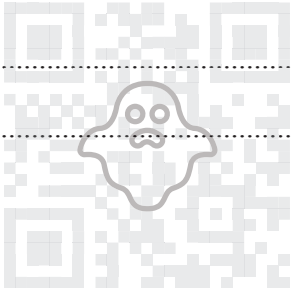
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(c) Find the cumulative distribution function of X .

[3]

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5 The random variable X is such that $P(X = r) = kr^2$ for $r = 1, 2, 3, 4$, where k is a constant.

(a) Find the value of k . [1]

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(b) Find the probability generating function $G_X(t)$ of X . [2]

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The random variable Y has probability generating function $G_Y(t) = \frac{1}{4} + \frac{1}{2}t + \frac{1}{4}t^2$.

The random variable Z is the sum of X and Y .

(c) Assuming that X and Y are independent, find the probability generating function $G_Z(t)$ of Z as a polynomial in t . [3]

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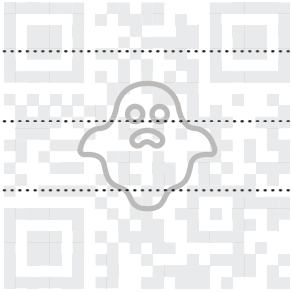
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(d) Given that $E(Z) = \frac{13}{3}$, use $G_Z(t)$ to find $\text{Var}(Z)$. [3]

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