1 The times taken for students at a college to run 200 m have a normal distribution with mean μ s. The times, x s, are recorded for a random sample of 10 students from the college. The results are summarised as follows, where $\bar{x}$ is the sample mean.

$$\bar{x} = 25.6 \qquad \Sigma(x-\bar{x})^2 = 78.5$$

(a) Find a 90% confidence interval for μ. [4]

A test of the null hypothesis $\mu = k$ is carried out on this sample, using a 10% significance level. The test does not support the alternative hypothesis $\mu < k$.

(b) Find the greatest possible value of k. [3]

2 The continuous random variable X has cumulative distribution function F given by

$$F(x) = \begin{cases} 0 & x < -1, \\ \frac{1}{2}(1+x)^2 & -1 \leqslant x \leqslant 0, \\ 1 - \frac{1}{2}(1-x)^2 & 0 < x \leqslant 1, \\ 1 & x > 1. \end{cases}$$

(a) Find the probability density function of X. [2]

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(b) Find $P\left(-\frac{1}{2} \leqslant X \leqslant \frac{1}{2}\right)$. [2]

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(c) Find $E(X^2)$. [2]

(d) Find $Var(X^2)$. [2]

3 A supermarket sells pears in packs of 8. Some of the pears in a pack may not be ripe, and the supermarket manager claims that the number of unripe pears in a pack can be modelled by the distribution B(8, 0.15).

A random sample of 150 packs was selected and the number of unripe pears in each pack was recorded. The following table shows the observed frequencies together with some of the expected frequencies using the manager's binomial distribution.

Number of unripe pears per pack	0	1	2	3	4	5	$\geqslant 6$
Observed frequency	35	48	43	15	6	3	0
Expected frequency	40.874	p	35.641	12.579	2.775	0.392	q

(a) Find the values of p and q. [2]

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(b) Carry out a goodness of fit test, at the 5% significance level, to test whether the manager's claim is justified. [6]

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4 Manet has developed a new training course to help athletes improve their time taken to run 800 m. Manet claims that his course will decrease an athlete's time by more than 2 s on average. For a random sample of 10 athletes the times taken, in seconds, before and after the course are given in the following table.

Athlete	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
Before	150	146	131	135	126	142	130	129	137	134
After	145	138	129	135	122	135	132	128	127	137

Use a t-test, at the 5% significance level, to test whether Manet's claim is justified, stating any assumption that you make. [8]

5 Nine balls labelled 1, 2, 3, 4, 5, 6, 7, 8, 9 are placed in a bag. Kai selects three balls at random from the bag, without replacement. The random variable X is the number of balls selected by Kai that are labelled with a multiple of 3.

(a) Find the probability generating function $G_X(t)$ of X. [3]

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The balls are replaced in the bag.

Jacob now selects two balls at random from the bag, without replacement. The random variable Y is the number of balls selected by Jacob that are labelled with an even number.

(b) Find the probability generating function $G_Y(t)$ of Y. [2]

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The random variable Z is the sum of the number of balls that are labelled with a multiple of 3 selected by Kai and the number of balls that are labelled with an even number selected by Jacob.

(c) Find the probability generating function of Z, expressing your answer as a polynomial. [3]

(d) Use the probability generating function of Z to find $E(Z)$. [2]

6 The blood cholesterol levels, measured in suitable units, of a random sample of 11 women and a random sample of 12 men are shown below.

Women	51	55	242	167	152	256	75	137	98	238	235	
Men	311	262	170	302	175	320	220	260	72	351	86	333

Carry out a Wilcoxon rank-sum test, at the 5% significance level, to test whether, on average, there is a difference in cholesterol levels between women and men. [9]