1 Find the Maclaurin's series for $e^x \tan x$ from first principles up to and including the term in x^2. [5]

2 The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} -1 & 2 & 12 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

Use the characteristic equation of **A** to show that

$$\mathbf{A}^4 = p\mathbf{A}^2 + q\mathbf{I},$$

where p and q are integers to be determined. [6]

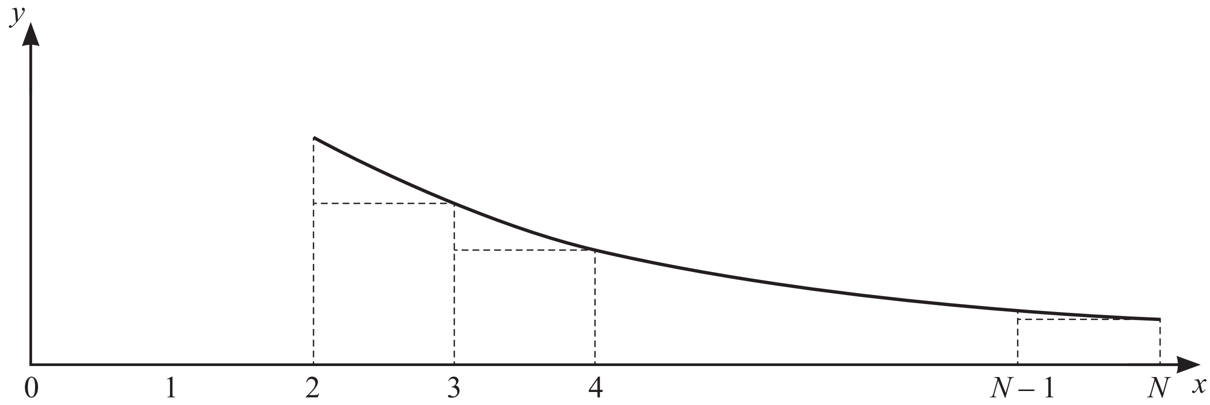
3 The curve C has equation

$$xy^3 - 4x^3y = 3.$$

(a) Show that, at the point $(-1, 1)$ on C, $\frac{dy}{dx} = 11$. [3]

(b) Find the value of $\frac{d^2y}{dx^2}$ at the point $(-1, 1)$. [5]

4



The diagram shows the curve with equation $y = \dfrac{\ln x}{x^2}$ for $x \geqslant 2$, together with a set of $(N-2)$ rectangles of unit width.

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^{N} \frac{\ln r}{r^2} < \frac{2 + 3\ln 2}{4} - \frac{1 + \ln N}{N}.$$ [7]

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(b) Use a similar method to find, in terms of N, a lower bound for $\sum_{r=1}^{N} \frac{\ln r}{r^2}$. [3]

5 Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\cos x,$$

given that, when $x = 0$, $y = -4$ and $\frac{dy}{dx} = 3$. [11]

6 **(a)** Use de Moivre's theorem to show that

$$\operatorname{cosec} 5\theta = \frac{\operatorname{cosec}^5\theta}{5\operatorname{cosec}^4\theta - 20\operatorname{cosec}^2\theta + 16}.$$ [6]

(b) Hence obtain the roots of the equation

$$x^5 - 10x^4 + 40x^2 - 32 = 0$$

in the form $\operatorname{cosec}(q\pi)$, where q is rational. [4]

7 **(a)** Show that an appropriate integrating factor for

$$\sqrt{x^2-1}\frac{dy}{dx}+y=x^2-x\sqrt{x^2-1}$$

is $x+\sqrt{x^2-1}$. [4]

(b) Hence find the solution of the differential equation

$$\sqrt{x^2-1}\,\frac{dy}{dx}+y=x^2-x\sqrt{x^2-1}$$

for which $y = 1$ when $x = \frac{5}{4}$. Give your answer in the form $y = f(x)$. [7]

8 **(a)** Starting from the definition of cosh in terms of exponentials, prove that

$$2\cosh^2 A = \cosh 2A + 1.$$ [3]

The curve C has parametric equations

$$x = 2\cosh 2t + 3t, \quad y = \tfrac{3}{2}\cosh 2t - 4t, \quad \text{for } -\tfrac{1}{2} \leqslant t \leqslant \tfrac{1}{2}.$$

The area of the surface generated when C is rotated through 2π radians about the y-axis is denoted by A.

(b) **(i)** Show that $A = 10\pi \int_{-\frac{1}{2}}^{\frac{1}{2}} (2\cosh 2t + 3t)\cosh 2t \, dt$. [4]

(ii) Hence find A in terms of π and e. [7]