

1 It is given that

$$\alpha + \beta + \gamma = 3, \quad \alpha^2 + \beta^2 + \gamma^2 = 5, \quad \alpha^3 + \beta^3 + \gamma^3 = 6.$$

The cubic equation $x^3 + bx^2 + cx + d = 0$ has roots α, β, γ .

Find the values of b, c and d .

[6]

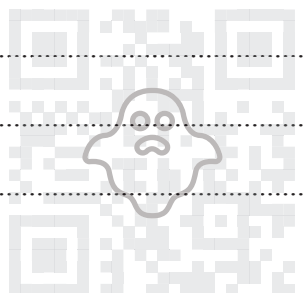
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2 (a) Use standard results from the list of formulae (MF19) to find $\sum_{r=1}^n r(r+1)(r+2)$ in terms of n , fully factorising your answer. [3]

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(b) Express $\frac{1}{r(r+1)(r+2)}$ in partial fractions and hence use the method of differences to find

$$\sum_{r=1}^n \frac{1}{r(r+1)(r+2)}$$
 [5]

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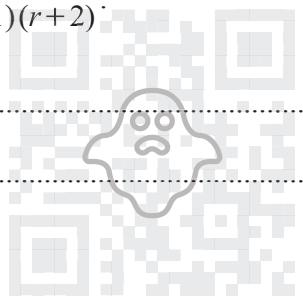
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(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{r(r+1)(r+2)}$. [1]

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3 The sequence of real numbers a_1, a_2, a_3, \dots is such that $a_1 = 1$ and

$$a_{n+1} = \left(a_n + \frac{1}{a_n}\right)^3.$$

(a) Prove by mathematical induction that $\ln a_n \geq 3^{n-1} \ln 2$ for all integers $n \geq 2$. [6]

[You may use the fact that $\ln\left(x + \frac{1}{x}\right) > \ln x$ for $x > 0$.]

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(b) Show that $\ln a_{n+1} - \ln a_n > 3^{n-1} \ln 4$ for $n \geq 2$. [2]

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4 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$.

(a) The matrix **M** represents a sequence of two geometrical transformations.

State the type of each transformation, and make clear the order in which they are applied. [2]

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(b) Find the values of θ , for $0 \leq \theta \leq \pi$, for which the transformation represented by **M** has exactly one invariant line through the origin, giving your answers in terms of π . [9]

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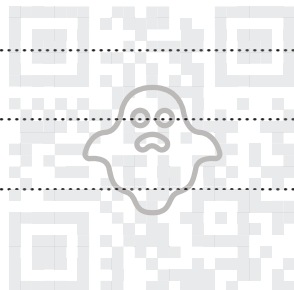
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Handwriting practice area consisting of 25 horizontal dotted lines.



5 The plane Π has equation $\mathbf{r} = -2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} + \mathbf{k}) + \mu(2\mathbf{i} + 3\mathbf{j})$.

(a) Find a Cartesian equation of Π , giving your answer in the form $ax + by + cz = d$. [4]

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The line l passes through the point P with position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ and is parallel to the vector \mathbf{k} .

(b) Find the position vector of the point where l meets Π . [3]

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- (c) Find the acute angle between l and Π . [3]

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- (d) Find the perpendicular distance from P to Π . [3]

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(b) Sketch C .

[2]

(c) Find the area of the region bounded by C and the initial line, giving your answer in exact form. [6]

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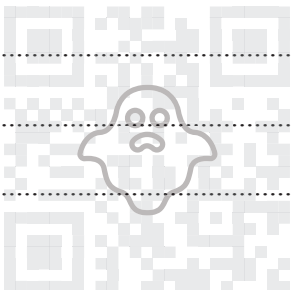
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7 The curve C has equation $y = \frac{4x+5}{4-4x^2}$.

(a) Find the equations of the asymptotes of C .

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(b) Find the coordinates of any stationary points on C .

[4]

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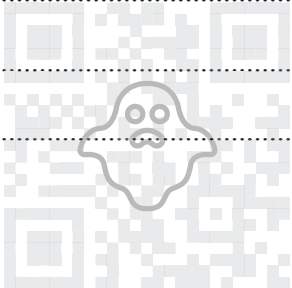
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(c) Sketch C , stating the coordinates of the intersections with the axes.

[3]

(d) Sketch the curve with equation $y = \left| \frac{4x+5}{4-4x^2} \right|$ and find in exact form the set of values of x for which $4|4x+5| > 5|4-4x^2|$.

[6]

