1 The heights of the members of a large sports club are normally distributed. A random sample of 11 members of the club is chosen and their heights, x cm, are measured. The results are summarised as follows, where $\bar{x}$ denotes the sample mean of x.

$$\bar{x} = 176.2 \qquad \Sigma(x-\bar{x})^2 = 313.1$$

Test, at the 5% significance level, the null hypothesis that the population mean height for members of this club is equal to 172.5 cm against the alternative hypothesis that the mean differs from 172.5 cm. [5]

2 A large school is holding an essay competition and each student has submitted an essay. To ensure fairness, each essay is given a mark out of 100 by two different judges. The marks awarded to the essays submitted by a random sample of 12 students are shown in the following table.

Student	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*
Judge 1	62	74	52	48	68	55	56	64	37	70	81	59
Judge 2	65	70	47	49	76	74	67	54	50	77	72	75

(a) One of the students claims that Judge 2 is awarding higher marks than Judge 1.

Carry out a Wilcoxon matched-pairs signed-rank test at the 5% significance level to test whether the data supports the student's claim. [7]

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It is discovered later that the marks awarded to student *A* have been entered incorrectly. In fact, Judge 1 awarded 65 marks and Judge 2 awarded 62 marks.

(b) By considering how this change affects the test statistic, explain why the conclusion of the test carried out in part **(a)** remains the same. [2]

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3 A random sample of 200 observations of the continuous random variable X was taken and the values are summarised in the following table.

Interval	$0 \leqslant x < 0.5$	$0.5 \leqslant x < 1$	$1 \leqslant x < 1.5$	$1.5 \leqslant x < 2$	$2 \leqslant x < 2.5$	$2.5 \leqslant x < 3$
Observed frequency	5	23	40	41	46	45

It is required to test the goodness of fit of the distribution with probability density function f given by

$$f(x) = \begin{cases} \frac{1}{9}x(4-x) & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

Most of the relevant expected frequencies, correct to 2 decimal places, are given in the following table.

Interval	$0 \leqslant x < 0.5$	$0.5 \leqslant x < 1$	$1 \leqslant x < 1.5$	$1.5 \leqslant x < 2$	$2 \leqslant x < 2.5$	$2.5 \leqslant x < 3$
Expected frequency	p	q	37.96	43.52	43.52	37.96

(a) Show that $p = 10.19$ and find the value of q. [3]

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(b) Carry out a goodness of fit test, at the 5% significance level, to test whether f is a satisfactory model for the data. [4]

4 The continuous random variable X has cumulative distribution function F given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 2, \\ \frac{1}{60}x^2 - \frac{1}{15} & 2 \leqslant x \leqslant 8, \\ 1 & x > 8. \end{cases}$$

(a) Find $\mathrm{P}(3 \leqslant X \leqslant 6)$. [1]

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(b) Find $\mathrm{E}\left(\sqrt{X}\right)$. [3]

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(c) Find $\text{Var}(\sqrt{X})$. [2]

(d) The random variable Y is defined by $Y = X^3$. Find the probability density function of Y. [3]

5 The random variable X has the binomial distribution $\mathrm{B}(n, p)$.

(a) Write down an expression for $\mathrm{P}(X = r)$ and hence show that the probability generating function of X is $(q + pt)^n$, where $q = 1 - p$. [3]

(b) Use the probability generating function of X to prove that $\mathrm{E}(X) = np$ and $\mathrm{Var}(X) = np(1-p)$. [5]

6 Nassa is researching the lengths of a particular type of snake in two countries, A and B.

(a) He takes a random sample of 10 snakes of this type from country A and measures the length, x m, of each snake. He then calculates a 90% confidence interval for the population mean length, μ m, for snakes of this type, assuming that snake lengths have a normal distribution. This confidence interval is $3.36 \leqslant \mu \leqslant 4.22$.

Find the sample mean and an unbiased estimate for the population variance. [4]

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(b) Nassa also measures the lengths, y m, of a random sample of 8 snakes of this type taken from country B. His results are summarised as follows.

$$\Sigma y = 27.86 \qquad \Sigma y^2 = 98.02$$

Nassa claims that the mean length of snakes of this type in country B is less than the mean length of snakes of this type in country A. Nassa assumes that his sample from country B also comes from a normal distribution, with the same variance as the distribution from country A.

Test at the 10% significance level whether there is evidence to support Nassa's claim. [8]

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