		•••••
		•••••
about the <i>x</i> -axis.	he area of the surface generated when the curve is rotated	

(00)			
	 •	• • • • • • • • • • • • • • • • • • • •	• • • • • • • • • • • • • • • • • • • •
\sim			

.....

4 Find the solution of the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = \mathrm{e}^x$$

for which $y = 3$ when $x = 1$. Give your answer in the form $y = f(x)$.	[8]

5 The curve C has equation

$$y^2 + (xy+1)^2 = 5.$$

(a) Show that, at the point (1,1) on C, $\frac{dy}{dx} = -\frac{2}{3}$. [3]

 •	 	
 •	 	

(b) Find the value of $\frac{d^2y}{dx^2}$ at the point (1,1). [5]

6 Find	d the particular	solution	of the diff	erential	equation
--------	------------------	----------	-------------	----------	----------

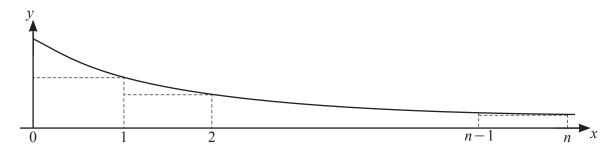
$$\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 15x = 102\cos 3t,$$

	$\mathbf{G}t$	
given that, when $t = 0$, $x = 1$ and $\frac{dx}{dt}$	$\frac{1}{2}=0$.	[11]
	502	

Show that $\sum_{r=1}^{\infty} z^{2r} = \frac{z^{2n+1} - z}{z - z^{-1}}$,	
	 •••••
	 •••••
	 •••••
	 •••••

$1 + 2\sum_{r=1}^{n} \cos(2r\theta) =$	$\frac{\sin(2n+1)\theta}{\sin\theta}$.	[5]

8



The diagram shows the curve $y = \frac{1}{\sqrt{x^2 + x + 1}}$ for $x \ge 0$, together with a set of *n* rectangles of unit width. By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^{n} \frac{1}{\sqrt{r^2 + r + 1}} < \ln\left(\frac{1}{3} + \frac{2}{3}n + \frac{2}{3}\sqrt{n^2 + n + 1}\right).$$
 [10]

- 9 It is given that a is a positive constant.
 - (a) Show that the system of equations

$$ax + (2a+5)y + (a+1)z = 1,$$

 $-4y = 2,$
 $3y-z = 3,$

has a unique solution and interpret this situation geometrically. [3]]
	•

The matrix \mathbf{A} is given by

$$\mathbf{A} = \begin{pmatrix} a & 2a+5 & a+1 \\ 0 & -4 & 0 \\ 0 & 3 & -1 \end{pmatrix}.$$

•••••			
•••••			
211			
ind	a matrix P such that	(
		$\mathbf{A} = \mathbf{P} \begin{pmatrix} a & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -4 \end{pmatrix} \mathbf{P}^{-1}.$	
		(0 0 -4)	
	•••••		
